MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

Receivers, Antennas, and Signals – 6.661

Issued: 02/20/03
Problem Set No. 3 Due: 02/27/03

Problem 3.1

Consider the correlation radiometer of Figure 2.2-11 in the course notes. By taking the following steps, derive the expression $\Delta T_{rms} = T_{eff}/(B\tau)^{0.5}$ where $T^2_{eff} = T^2_A + 2T_AT_R + 2T^2_R$.

a) Find $\phi_m(\tau)$ in terms of $\phi_s(0)$, $\phi_s(\tau)$, and $\phi_n(\tau)$, assuming $E[n_an_b] = 0$ and s and n are gaussian with zero mean.

b) Find $\Phi_m(f)$ in terms of $\phi_s(0)$, $\Phi_s(f)$, and $\Phi_n(f)$.

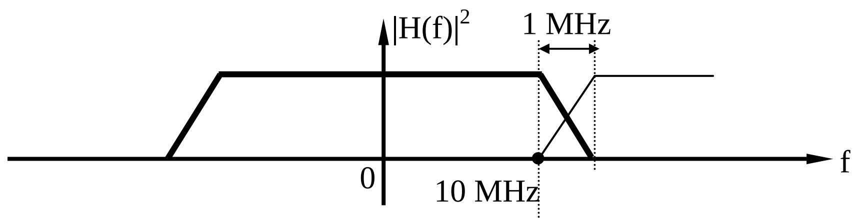
c) The expression produced in (b) has three parts, i.e. a "signal x signal" term, a "signal x noise" term and a "noise x noise" term. Graph and fully dimension the power density spectra corresponding to these three parts (using double-sided spectra in terms of T_A, T_R, etc.).

d) Find the DC power and the AC power that emerges in the output $v_o(t)$.

e) Find ΔT_{rms}.

Problem 3.2

A digital 1-bit autocorrelation spectrum analyzer is to be designed with a frequency resolution of 10 kHz. The autocorrelation function is to be apodized with a weighting function that reduces spectral resolution by a factor of 1.4 relative to uniform weighting, while significantly reducing spectral sidelobes. Approximately 1000 independent spectral intervals are desired at the output, distributed over a 10-MHz band. The effects of aliasing can be made acceptably low for the given input r.f. filter if the 1-MHz spectral region just above the desired 1-MHz band is not aliased into that desired band. Note that both apodization and aliasing increase the number of taps required.



a) What is the slowest sampling rate consistent with the foregoing requirements?

b) How many stages are required for the shift register?

Problem 3.3

a) Prove that a conjugate match $\underline{Z}_L = R - jX$ maximizes the power transferred from a Thevenin source characterized by the impedance $R + jX$. This maximum is the "available power" if $R > 0$. Assume the source impedance is fixed, and that of the load is varied to yield the desired maximum.

b) Recalling that the wave reflection coefficient from a load at the end of a TEM line with characteristic impedance Z_o is $\underline{\Gamma} = \underline{V}_-/\underline{V}_+ = (\underline{Z}_{Ln} - 1)/(\underline{Z}_{Ln} + 1)$, what is the exchangeable gain G_e of a negative-resistance load $-R_L$? We define $\underline{Z}_{Ln} = \underline{Z}_L/Z_o$. This amplifier works by reflecting a signal in a TEM line from a negative resistance load which is preceded by a three-port circulator (discussed later in the course; see Eqn. 2.3.37 in the text) that sends the reflected signal out a different TEM port than the port through which the input signal entered the amplifier. The negative-resistance load is often a tunnel diode biased to its negative resistance point.