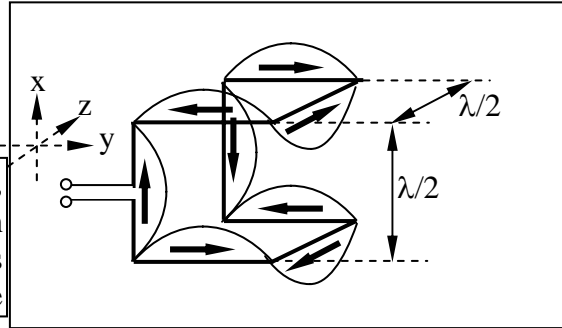Solutions -- Problem Set No. 6   April 1, 2003
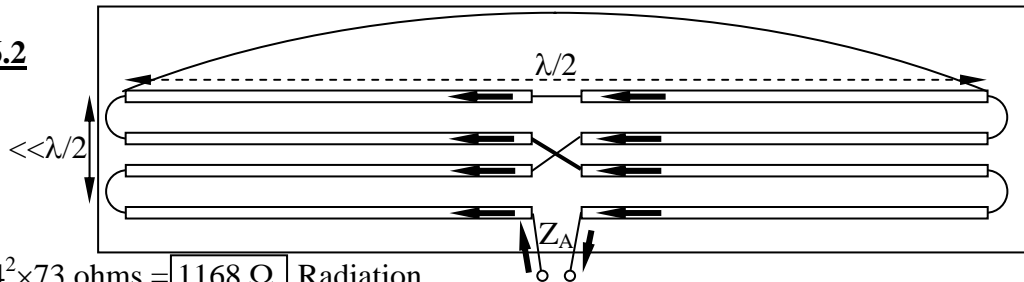
Problem 6.1

a)   Instantaneous currents could be:

b)   Maximum gain along ±x and ±z axes, nulls along ±y axis. The x-axis radiation is z-polarized, and the z-axis radiation is x-polarized. Off-axis radiation is more easily determined using a computer.

c)   There are nulls along the y axis.

Problem 6.2

a)

b) $Z_A \cong 4^2 \times 73$ ohms = 1168 Ω. Radiation resistance is proportional to radiated power (see (3.2.26) in text), and we have quadrupled the current normally flowing in a half-wave dipole, thus increasing the radiated power and radiation resistance by a factor of 16. The R_r for a half-wave dipole is ~73 ohms (see p 3-17 of text).
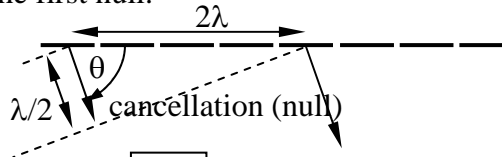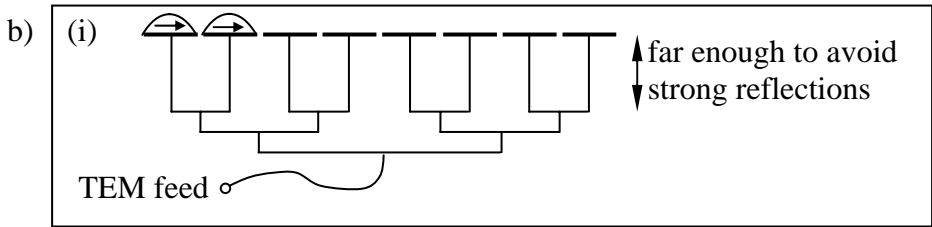
Problem 6.3

a)   8 half-wave dipoles end-to-end are collectively 4 wavelengths long. The figure below suggests the location of the first null.
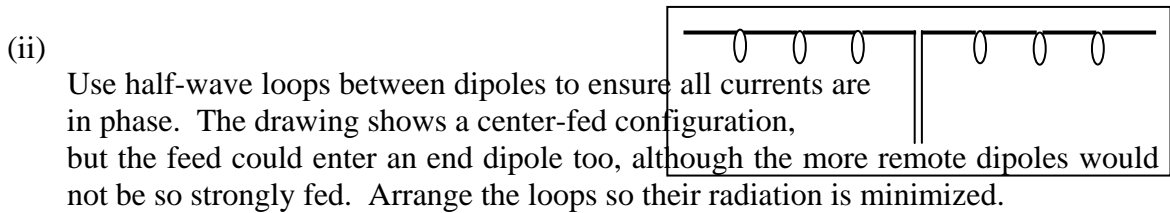
The first null occurs at
$\theta = \cos^{-1}([\lambda/2]/[2\lambda])$
$= \cos^{-1} 0.25 = 75.5°$
so the 3-dB beamwidth is ~2(90 – 77.5) = 14.4°, broadside.

b)   (i)

This feed system balances impedances and ensures in-phase excitation of all dipoles.

(ii)
Use half-wave loops between dipoles to ensure all currents are
in phase. The drawing shows a center-fed configuration,
but the feed could enter an end dipole too, although the more remote dipoles would
not be so strongly fed. Arrange the loops so their radiation is minimized.

Problem 6.4

a)   We simply want the power received P_{rec} to exceed the receiver noise kTB plus the given margins. The user antenna gain should be ~1 for general use.

$P_{rec} = (G_t P_t/4\pi r^2)(G_r\lambda^2/4\pi)/1000$ (dividing by 1000 provides the 30-dB path margin)
$\cong 2.5\times10^{-15}$ watts if $G_r = 3$
$100kTB = 100\times1.38\times10^{-23}\times600\times3000 = \boxed{2.48\times10^{-15} \text{ watts per channel}}$

Thus the uplink (link to the base station) has adequate margins if the base station $\boxed{\text{antenna has a gain of 3 or more.}}$ The gain of the antenna of Problem 6.3 is approximately 8, so a vertical stack of 4 such dipoles end-to-end should provide 360-degree coverage out to ~20 km. Although lower gains would be directed toward users closer to the base station antenna, less gain should then suffice.   The calculation for the down-link from the base station is identical, except that up to (1.5 MHz)/(3 kHz) = 500 users could be active at once, requiring up to $\boxed{500 \text{ watts}}$ transmitter power at one watt per user. This is a modest requirement.

b)   Frequency reuse requires independent beams.   Orthogonal polarizations don't separate well in complex multipath environments, so multiple beams are better.   If digital modulation is used, $>~12$-dB isolation between beams generally suffices. If 4× reuse is desired, then at least 8 beams should be used to provide both adequate separation and coverage. A possible 12-beam configuration is pictured here, where each element radiates only over 180° of azimuth due to reflecting surfaces behind. The elements would be arranged in three rows of four each, where each 4-element antenna array would produce 4 beams, as shown. Frequencies could be reused in non-adjacent beams over ×6 reuse, in principal. Because users are often not symmetrically distributed, ×6 offers a little margin if the real goal is ×4.
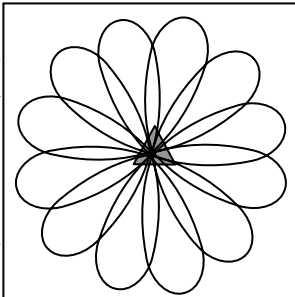
Problem 6.5

The solution to this problem failed to consider that the ground-plane produced image of the entire helical antenna corresponds to a wave moving in a direction opposite to the original, although it is rotating in the same direction; image currents are equal and opposite to the originals. The image current could radiate 'backward' and thus contribute to the forward wave, but that would imply the real helix also radiates in the backward direction. This backward radiation can be reduced by ensuring that the radiated waves from adjacent turns of the helix cancel in the backward direction while they add in the forward direction. This can be achieved by placing successive turns of the helix $\lambda/4$ apart, which implies that the length of wire per turn is 1.25λ. In this way, two consecutive and adjacent radiated contributions can add forward and cancel backward. Where the helix passes through the ground plane is irrelevant, because all positions are equivalent. If adjacent turns are $\lambda/4$ apart, then that first turn of the wire which is furthest from the entry point must be $\lambda/8$ from the ground plane (half of the $\lambda/4$ needed per turn).