MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

Receivers, Antennas, and Signals – 6.661

Issued: 4/08/03
Problem Set No. 9 Due: 4/17/03

Problem 9.1

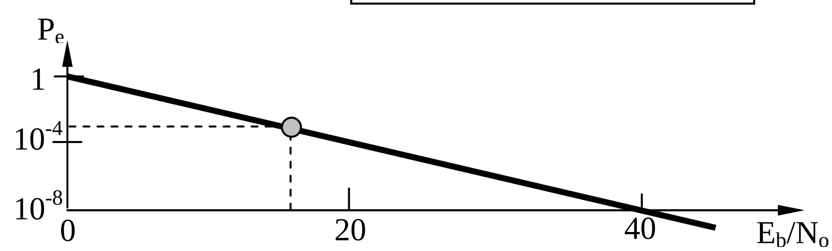
a) *Let* $E_b = S/C = S/[B\ log_2(1 + S/N_oB)] = E_bR/[(R/Q)\ log_2(1 + RE_bQ/kTR)]$*, so* $Q = log_2(1 + E_bQ/kT)$ *and* $2^Q - 1 = E_bQ/kT$*. Therefore*
$E_b = kT(2^Q - 1)/Q = kT$ *if* $Q = 1$*. For small Q,* E_b *diminishes only slightly*

b) *Numerically,* $Q = 2 \Rightarrow E_b = 1.5kT$*;* $Q = 0.3 \Rightarrow 0.77kT$*,* $Q = 0.1 \Rightarrow 0.72kT$*.* $Q = 0.001 \Rightarrow 0.69kT$*. Lower values for* E_b *are generally unattainable in practice, so* $E_b > \sim 0.7kT$ *is a fairly hard limit, where T is the system noise temperature characterizing the channel, not the signal. Thus the desired range is:* $0 < Q < 2$*.*

Problem 9.2

We can use up to 12 parity check bits. Using (4.5.6) in the text, where K+R = 24, we find:
$$12 \geq log_2[1 + \binom{24}{1} + \binom{24}{2} + \binom{24}{3} + ?] \text{ where } \binom{24}{n} \equiv 24!/n!(24-n)!;$$
Evaluating $\Rightarrow$
$12 \geq log_2[1 + 24 + 276 + 2024] = log_2 2325$*, so we can correct up to 3 errors.*

Problem 9.3

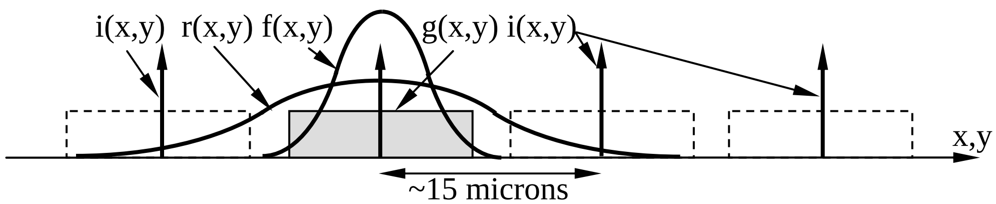


We may refer to (4.5.15) and surrounding text for help here. We lose 2.4dB in E_b/N_o *because we have to speed up to make room for the parity bits, but we reduce the probability of block error from* 10^{-3} *to* $Z = (1-10^{-3})^5 P_e^2 \times 7 \times 6/2!$ *Since a block error would result on average in half the message bits being in error, the probability of message bit error would be (2/7)Z, or* $\sim 6 \times 10^{-6}$*. Thus* P_e *would be reduced by a factor of 168, corresponding to a change in* E_b/N_o *of 22.2/2 = 11.1 dB (Note: log 168 = 22.2 dB, and 40 dB change in* E_b/N_o *corresponds to 80 dB in* P_e*). The coding gain is the difference between the penalty of 2.4 dB and the reward of 11.1 dB, or* $G_c \cong 11.1 - 2.4 = 8.7dB$

Problem 9.4

b) The noise $n_1(t)$ can be considered to be the environmental noise, while $n_2(t)$ is post-smoothing and can include the sensor noise, photon shot noise, read-out noise, and quantization noise. Lumping the sensor noise with $n_1(t)$ is awkward because $n_1(t)$ precedes convolution.

a) Let x,y be coordinates referenced to the focal plane. We can consider f(x,y) to correspond to the lens blurring function (arbitrarily chosen to be narrow) and g(x,y) to be the focal plane response function of each CCD pixel; i(x,y) represents the spacing of the pixels and r(x,y) corresponds to the viewer's visual response function (assumed blurry). Thus we might have:



Problem 9.5

The received power must support the desired output SNR, which is 20 + 40 dB ($\Rightarrow 10^6$). But for SSBSC, which performs the same as DSBSC, the necessary S/N_{out} is $<s^2(t)>(P_c/2N_oW)$ where $N_o = kT_s/2$, $T_s = 4000K$, $k = 1.38\times10^{-23}$, and $W = 10^4$ Hz. $<s^2(t)> = 0.5$ for pure sine waves at maximum amplitude, and 1 for square waves. The maximum average received power is then $P_c<s^2(t)> = 2N_oW\times10^6$ $= 2\times1.38\times10^{-23}\times4000\times10^4\times10^6 = 1.1\times10^{-9}$ [W]. If we allow for a 70-dB path loss, then the average transmitter power is $\sim1.1\times10^{-2}$W, or ~10 milliwatts, which is reasonable.

Problem 9.6

a) *Referring to Figure 4.7-10 and associated text, the FM threshold S/N for $\beta^* = 10$ is approximately 18 dB, so $P_c/BkT = 10^{1.8}$ and $P_r = P_c > 63\times2W(1+\beta^*)kT$ $= 63\times2\times10^4(1+10)1.38\times10^{-23}\times4000 =$ 7.7×10^{-13} W received and 7.7×10^{-6} W transmitted*

b) *The output SNR requirement is 20+40 = 60 dB, where $S_{out}/N_{out} = P_c<s^2>3\beta^{*2}/2N_oW$ $= 10^6$, so $P_c \cong 10^6\times2\times1.38\times10^{-23}\times4000\times10^4/(0.5\times3\times100) = 7.36\times10^{-12}$W; $P_t \cong 7.4\times10^{-5}$W.*

c) *The requirements for P_t are a factor of ~9.6 greater than the FM threshold, so we could reduce β^* slightly to x, where $10^2/x^2 = 9.6$, so the new x = β^* could be 3.23, but then the only margin left would be due to the fact that the FM threshold drops slightly with β^*.*