Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.685 Electric Machines

Problem Set 3 Solutions September 26, 2013

Problem 1: This proof is a good reason to own **Electric Power Principles**. Figure 9.5, on Page 151, reproduced here as Figure 1 shows graphically that:

$$L_d I \sin \delta_i = \Lambda_t \sin \delta$$

then, noting that

$$\begin{aligned} \omega \Lambda_t &= V \\ E_{af} &= \omega M I_f \\ X_d &= \omega L_d \end{aligned}$$

Direct substitution gives:

$$\begin{aligned} \frac{3}{2}\frac{p}{\omega}\frac{V E_{af}}{X_d}\sin\delta &= \frac{3}{2}\frac{p}{\omega}\frac{\omega\Lambda_t \sin\delta \omega M I_f}{\omega L_d} \\ &= \frac{3}{2}\frac{p}{\omega}\frac{\omega L_d I \sin\delta_i \omega M I_f}{\omega L_d} \\ &= \frac{3}{2} p M I I_f \sin\delta_i \end{aligned}$$

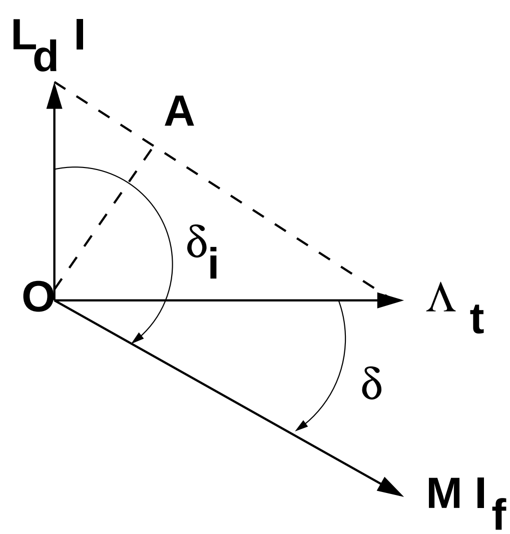

Figure 1: Vector Diagram for Torque Theorem Proof

Which was to be proven.

Problem 2: Note there was a bug in the problem statement, so I suggested assuming that $2g =$ 1mm. Then, if $\mu_0 = 1/800,000$,

$$L = \frac{\mu_0 N^a A}{2g} = \frac{1}{800,000} \times \frac{100^2 \times 8 \times 10^{-4}}{.001} = \frac{8}{800} = 10\text{mH}$$

If the core has permeability of $100\mu_0$ (this is actually quite a small permeability), the inductance could be calculated by:

$$L = \frac{N^2 A}{\frac{2g}{\mu_0} + \frac{2\ell_c}{\mu}} = \frac{\mu_0 N^2 A}{2g + \frac{\mu_0}{\mu} 2\ell_c}$$

Here, I am using ℓ_c to be the average length of one core, which is just about 0.14 m. For $\mu_r = \frac{\mu}{\mu_0} = 100$, the inductance evaluates to:

$$L = \frac{10^4 \times 8 \times 10^{-4}}{800 + .28 \times 8000} = \frac{8}{800 + 2240} = 2.6mH$$

Magnetic flux density is

$$B = \frac{NI}{\frac{2g}{\mu_0} + \frac{2\ell_c}{\mu}}$$

which leads to:

$$NI = \frac{B}{\mu_0}\left(2g + \frac{2\ell_c}{\mu_r}\right) = 2 \times 800,000 \times \left(.001 + \frac{.28}{100}\right) = 6080$$

Then current is $I = 6080/100 = 60.8$A.

Note if we let μ_r go to a very large number,

$$NI = 2 \times 800,000 \times .001 = 1600$$

and $I = 16$A.

The heating limit is fairly simple:

$$NI = JA_c = 3 \times 10^6 \times 2 \times .02 \times .04 = 4,800\text{A T}$$

Or $I = 48$A.

To make these equal,we need to have

$$g_{\text{eff}} = 2g + \frac{2\ell_c}{\mu_r}$$

to satisfy:

$$NI = \frac{B}{\mu_0} g_{\text{eff}}$$

For this problem, then:

$$g_{\text{eff}} = NI \times \frac{B}{\mu_0} = \frac{4,800}{1,600,000} = 3 \times 10^{-3}$$

For $\mu_r = 100$, this means the physical gap must be pretty small:

$$2g = g_{\text{eff}} - \frac{2\ell_c}{mu_r} = .003 - .0028 = .0002\text{m}$$

To make a 10 mH inductor, use

$$L = \frac{\mu_0 N^2 A}{g_{\text{eff}}}$$

which means we must make

$$N = \sqrt{\frac{g_{\text{eff}} L}{\mu_0 A}} = \sqrt{\frac{3 \times 10^{-3} \times .01 \times 800,000}{8 \times 10^{-4}}} = 173\text{turns}$$

and it is good to check this by substituting it back into the original expression to make sure it does, indeed evaluate to 10 mH.

Problem 3: This problem, too, had a bug in the description. It was intended to use flux density of 1.25 T, *peak*. We have no good data for RMS flux density as high as 1.25 T, RMS.

The core area is $A_c = .006m^2$, so flux per turn is $\Phi = .0075\text{Wb}$. So volts/turn is

$$\frac{V}{N} = 377 \times .0075 = 2.8275\text{V, peak} \approx 2.00\text{V, RMS}$$

then we require 4,000 turns in the high voltage coil and 120 turns in the low voltage coil.

Currents in the two coils will then be:

$$\begin{aligned} I_H &= \frac{50,000}{8,000} = 6.25\text{A, RMS} \\ I_L &= \frac{50,000}{240} = 208 \quad 1/3\text{A, RMS} \end{aligned}$$

Since window area is $A_w = .01m^2$, current density in the window will be:

$$J_w = \frac{4000 \times 6.25 = 120 \times 208 \quad 1/3}{.01} = 2.5 \times 10^6 A/m^2$$

to find core reactance, we will actually try two different calculations. The first is to estimate the permeability of the core at the intended operating point. My reading of the data sheet is that at 60 Hz, magnetic field is, at $B = 1.2T$, $H = 137\text{A/M}$ and at $B = 1.3A/m$ it is $H = 177\text{A/m}$. A linear interpolation to $B = 1.25T$ gives $H = 157\text{A/m}$. That means that core permeability is:

$$\mu_c = \frac{1.25}{157} \approx .008\text{H/m}$$

Note that this is just about $6,400\mu_0$.

My estimate of effective core length is illustrated in Figure 2, and amounts to:

$$\ell_c = 2h + 2W + wW_c = 400 + 200 + 200 = 800\text{mm} = .8\text{m}$$

then core inductance is

$$\begin{aligned} L = \frac{\mu A N^2}{\ell_c} = \frac{.008 \times .006}{.8} \times 4000^2 7 \quad &= 6 \times 10^{-5} \times 4000^2 \quad = 960\text{H, High Side} \\ &= 6 \times 10^{-5} \times 120^2 \quad = .864\text{H, Low Side} \end{aligned}$$

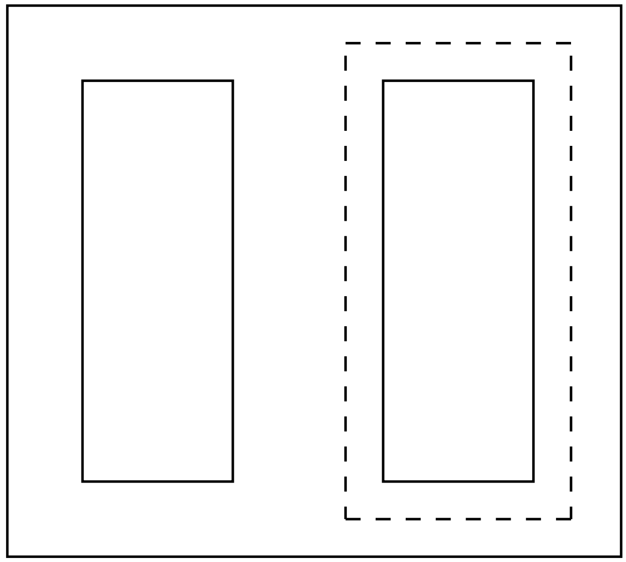

Figure 2: Core Length Estimate

To get core loss, we look at the material data sheet and do the same sort of interpolation as we did for finding magnetic field H. Here we find that, at 60 Hz, for 1.2 T the loss density is 1.89 W/kg and for 1.3 T it is 2.73 W/kg. Averaging these we surmise that for 1.25 T, the loss density is 2.31 W/kg. Density of the core material is 7650 kg/m^3. Core volume is:

$$\begin{aligned}
V_c &= \left((2W + 2W_c)\left(h + W_c\right) - 2hW\right) D \\
&= (.4 \times .3 - 2 \times .2 \times .1) \times .06 \\
&= (.12 - .04) \times .06 \\
&= .048m^3
\end{aligned}$$

Multiplied by the density, we find core mass is 36.72 kilograms. Then core power loss is

$$P_c = 36.73 \times 2.31 = 85\text{watts}$$

To check on our rough inductance calculation, we look at 'apparent power' density for the core, which, at 60 Hz is 4.04 VA/kg and 5.66 VA/kg at 1.2 and 1.3 T, respectively, or 4.85 VA/kg at 1.25 T. The reactive part of this is:

$$\sqrt{4.85^2 - 2.31^2} = 4.21\text{VAR/kg}$$

This gives us a reactive power drawn by the core of

$$Q_c = 36.73 \times 4.26 = 156\text{VARs}$$

We can estimate the VARs that would be drawn by our initially estimated core inductance of 960 H, to find

$$Q_c = \frac{8000^2}{277 \times 960} = 177\text{VARs}$$

This gives us some idea of the accuracy we might expect from this sort of estimate.

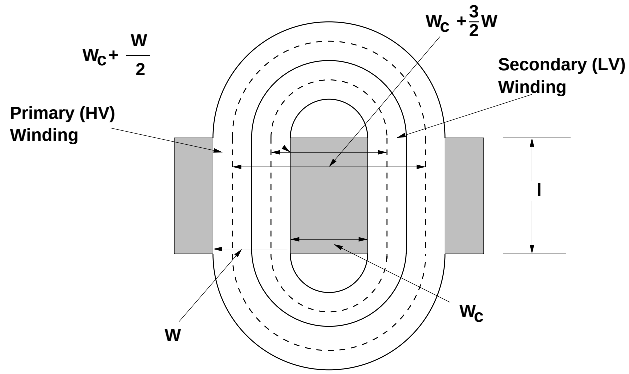

Figure 3: Winding Length Estimate

Leakage Calculation: Start from the formula we derived in class:

$$L_\ell = \mu_0 \frac{D}{h}\left(\frac{w_1 + w_2}{3} + w_s\right) N^2$$

where w_1 and $w_2$are the 'width's of the windings, D is core length, h is window height and N is number of turns. Since we must account for two windows, it is convenient to just multiply D by two. Here, I will be lazy and ignore the insulation and space between windings. The permeance is:

$$\mathcal{P} = \mu_0 \frac{2D}{h}\frac{w_1 + w_2}{3} = \mu_0 \frac{.12}{.2}\frac{.1}{3} \approx 2.5132 \times 10^{-9}$$

Then, leakage inductance seen from the high side is $L_\ell = \mathcal{P} \times 4000^2 \approx 40.2$mH and from the low side it is $L_\ell = \mathcal{P} \times 120^2 \approx 36\mu$H. As a check, these amount to 15.15Ω from the high side and 13.6$m\Omega$ from the low side. That is about 1.18% using the rating of the transformer (8 kV/240V and 50 kVA).

To get winding loss, we must first made an estimate of the length of the wires in the windings. Figure 3 shows how we might make an estimate of this length. We assume semicircular end turns. While it makes no substantive difference, we assume the low voltage winding is the 'inner' winding and the high voltage winding is the 'outer' winding. The two winding average lengths are:

$$\begin{aligned}
\ell_s &= 2D + \pi\left(W_c + \frac{W}{2}\right) = .59\text{m} \\
\ell_p &= 2D + \pi\left(W_c + \frac{3W}{2}\right) = .90\text{m}
\end{aligned}$$

Yet another bug in the problem statement was the omission of a 'space factor' for the winding itself. We will her assume a space factor of $\lambda_a = 0.5$. If the windings occupy half of the window with a .5 mm insulation space around, the area occupied by winding is

$$A_w = (50-1)\left((200-1\right) = .49 * .199 = .09751 m^2$$

Winding resistance will be $R = \frac{\ell N^2}{A_w \lambda_a \sigma}$, so that:

$$\begin{aligned}
R_p &= \frac{.9 \times 4000^2}{.09751 \times .5 \times 5.81 \times 10^7} = 5.08\Omega \\
R_s &= \frac{.59 \times 120^2}{.09751 \times .5 \times 5.81 \times 10^7} = 3m\Omega
\end{aligned}$$

Referred to the primary side, the secondary resistance is:

$$R_{sp} = .003 \times (\frac{4000}{120})^2 = 3.33\Omega$$

Then winding dissipation at rated operation is

$$P_d = 6.25^2 \times (5.08 + 3.33) = 329\text{watts}$$

Nominal efficiency at rated conditions would then be:

$$\eta = \frac{50,000}{50,000 + 329 + 85} \approx .992$$

Problem 4: The situation is shown in Figure 4. Magnetic flux density is in the $\hat{z}$ direction and currents are confined to the $\hat{x}$ direction. The sheet has thickness h.

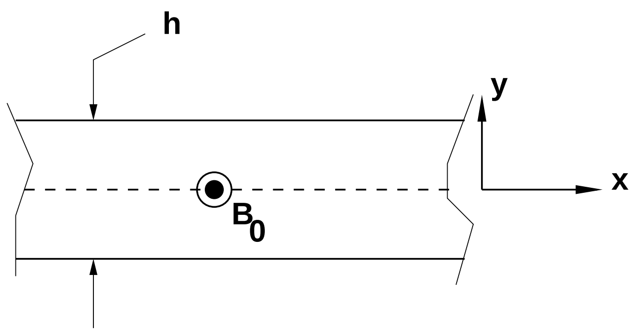

Figure 4: Loss Model and Coordinate System

Magnetic flux density is

$$B_z = \sqrt{2} B_0 \cos \omega t = \Re \sqrt{2} B_0 e^{j\omega t}$$

Electric field is found using Faraday's Law:

$$\frac{\partial E_x}{\partial y} = -j\omega B_z$$

and if that is uniform:

$$E_x == j\omega B_z y$$

Loss density is

$$P_d = \frac{1}{2}\sigma |E_x|^2 = \omega^2 B_0^2 y^2 \sigma$$

Average loss density is:

$$< P_d >= \frac{2}{h}\int_0^{\frac{h}{2}} \omega^2 B_0^2 \sigma y^2 dy = \frac{\omega^2 B_0^2 h^2 \sigma}{12}$$

This is, for half millimeter this material:

$$\frac{377^2 \times 1^2 \times .0005^2 \times 3 \times 10^6}{12} \approx 8883 W/m^3$$

and if material density is 7,200 kg/m^3,

$$< P_d >= 1.2337 W/kg$$

Now, if the 1/2 mm steel is stacked with 1/20 mm separators, the stacking density is

$$\lambda_s = \frac{.5}{.55} = \frac{1}{1.1}$$

Flux density in the material is 1.1 T, and power density is:

$$< P_d >= 1.1^2 \times 1.2337 \approx 1.4928 W/kg$$

To investigate loss vs. sheet thickness, note that flux density is:

$$B = \frac{h + h_f}{h} B_0$$

And our simple model is, if hysteresis loss is also proportional to flux density squared:

$$P_d = \left(P_{e0} \left(\frac{h}{h_B} \right)^2 + P_{h0} \right) \left(\frac{h + h_f}{h} \right)^2$$

To find the minimum of this, we need to set the partial derivative of it with respect to h to zero:

$$0 = \frac{\partial P_d}{\partial h} = 2 P_{e0} \frac{h + h_f}{h_B^2} + 2 P_{h0} \frac{h + h_f}{h^2} - 2 P_{h0} \frac{(h + f_f)^2}{h^3}$$

Clearing the common factor $2(h + h_f)$,

$$0 = P_{e0} \frac{1}{h_B^2} + P_{h0} \frac{1}{h^2} - P_{h0} \frac{h + h_f}{h^3}$$

Then, removing a common term, we are left with:

$$P_{e0} \frac{1}{h_B^2} = P_{h0} \frac{h_f}{h^3}$$

which leaves us with:

$$h^3 = \frac{P_{h0}}{P_{e0}} h_f h_B^2$$

This evaluates to $h \approx 3.03 \times 10^{-4} m = .303 mm$

The actual losses have been evaluated for this situation and the results are shown in Figure 5

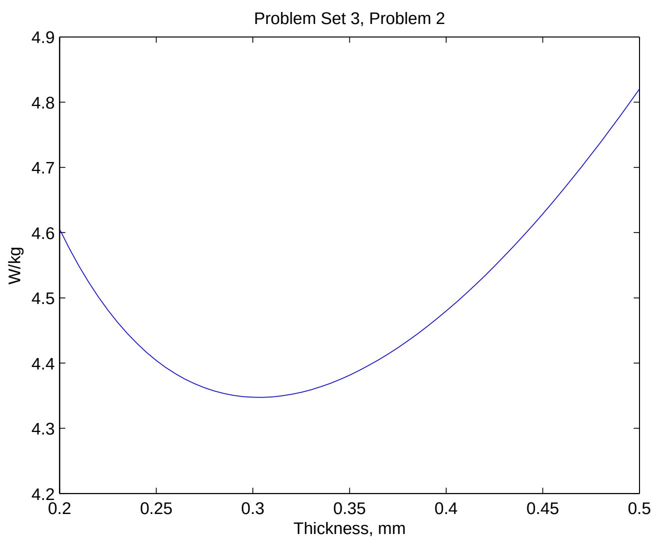

Figure 5: Loss as a function of sheet thickness

Problem 5 Armature current rating is:

$$I_a = \frac{VA}{3V_{ph}} = \frac{10^9}{3 \times 15,011} \approx 22,205\text{A, RMS}$$

Since, in RMS,

$$E_{af} = \frac{\omega M I_{fnl}}{\sqrt{2}}$$

Mutual inductance is:

$$M = \frac{\sqrt{2}E_{af}}{\omega I_{fnl}} = \frac{\sqrt{2} \times 15011}{377 \times 2501} \approx 22.5\text{mH}$$

To find synchronous inductance, note that:

$$\omega L_d I_a = \frac{\omega M I_{fsi}}{\sqrt{2}}$$

So that:

$$L_d = \frac{M}{\sqrt{2}} \frac{I_{fsi}}{I_a} = \frac{22.5}{\sqrt{2}} \frac{5003}{22,205} \approx 2.585\text{mH}$$

Idiot Check: Since 'base impedance is:

$$Z_B = \frac{15,011}{22,205} = .676\Omega$$

Per-Unit impedance should be:

$$x_d = \frac{X_d}{Z_B} = \frac{377 \times .003585}{.676} = 2.0 = \frac{AFSI}{AFNL}$$

as expected.

```
% 6.685 2011 Problem Set 3, Problem 2

% base case
om = 2*pi*60;
sig = 3e6;
rho = 7200;
h_b = .0005;
h_f = .00005;

P_eb = om^2*h_b^2*sig/(12*rho);
P_hb = 2.75;

h_opt = ((P_hb/P_eb)*h_f*h_b^2)^(1/3);

fprintf('Base eddy current dissipation = %g W/kg\n', P_eb)
fprintf('Base hysteresis dissipation =   %g W/kg\n', P_hb)
fprintf('Optimal Thickness           =   %g mm\n', 1000*h_opt)

% now look at dissipation vs. thickness

h = 4*h_f:.1*h_f:h_b;

B = (h+h_f) ./ h;

P = (P_eb .* (h ./ h_b) .^2  + P_hb) .* B .^2;

figure(1)
plot(1000.*h, P)
title('Problem Set 3, Problem 4')
ylabel('W/kg')
xlabel('Thickness, mm')
```

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