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Massachusetts Institute of Technology
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6.685 Electric Machines

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Problem Set 6   Issued October 7, 2013
                                                                Due October 16, 2013

Problem 1: Induction Motor

This concerns a real, 10 horsepower induction motor. The rotor diameter is 5.7" and the active length is 6". The air-gap dimension (rotor to stator spacing) is 0.0185". The stator has 48 slots. Here are some details on the winding:

This is a four pole motor but the winding is called 'consequent pole'. This means that each phase winding is wound around only two poles. Thus each phase winding has two groups, and in this case each group has six coils. The coils are 'concentric', so that if you consider the six groups, they all link flux in the same axis.

The coils have, respectively, 6, 12, 18, 18, 12 and 6 turns each, and span 17, 15, 13, 11, 9 and 7 slots.

You will note that the winding scheme has some overlap from phase to phase: the coils with 8 and 9 turns each overlap with the adjacent phase with 9 and 8 turns, so the total number of wires in each slot is a uniform 17.

1. Create a winding plan that shows how many turns are in each slot, phase half by phase half. Convince yourself that there are indeed 18 conductors in each slot.

2. Compute the winding factor for the space fundamental, for the two 'belt' harmonics (order 5 and 7) and for the 'zigzag' harmonics (orders 23 and 25).

3. Now create a winding plan for a 'regular' (two layer) winding with lapped end turns. Compute the 'belt' harmonic winding factors (pitch X breadth) for full-pitch, 5/6 pitch and 2/3 pitch versions of that winding.

4. Back to the concentric winding: Compute the magnetizing reactance for this machine, assuming it is to be operated at 60 Hz.

5. If all coils of a phase are connected in series and if the three-phase winding is connected in 'star' (same as 'wye'), and if the machine is operated with line-line voltage of 480 V, RMS, what is the *peak* value of flux density in the air-gap when the machine is running at no load? (Assume air-gap voltage is equal to terminal voltage).

Problem 2: Retarder

Figure 1 shows a retarder used in amusement park rides (roller coasters) and some material handling equipment. Two rows of permanent magnets are mounted on 'back iron', which you can consider to be perfectly permeable, and separated by a gap. A conducting fin, usually made of copper or aluminum is mounted on the vehicle to be braked and therefore moves between the permanent magnets. Eddy currents flow in that fin, dissipating power and therefore slowing down the vehicle.
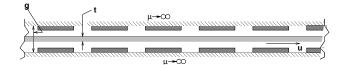
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Figure 1: Cartoon of a Permanent Magnet Retarder

For the purpose of this exercise, assume the gap to be 'small', meaning that $kg << 1$. The conductive fin is moving to the right with velocity in the x direction u. The permanent magnets produce a flux density at the location of the fin of: $B_y = B_0 \cos kx$. The eddy currents in the fin will produce some reaction magnetic field. Find the retarding force density per unit area as a function of fin speed u.

Problem 3: Fields in Free Space

Now we are going to solve a magnetic field problem without the assumption of a narrow air-gap. This one is static. The situation is shown in Figure 2. A row of permanent magnets, alternately polarized, is located on top of a perfectly permeable surface. It faces, across some air-gap, another perfectly permeable surface.
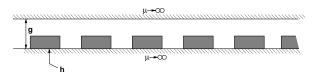

Figure 2: Magnetic Problem

We will assume that the problem is linear so that we can treat magnetization of the magnets as a fourier series, and then we will solve only for the space fundamental. The magnets will have magnetization and incremental permeability, so that the y-directed flux density is:

$$B_y = B_r \cos kx + \mu_m H_y$$

Magnetization in the magnets is purely in the y- direction, so that x- directed field is:

$$B_x = \mu_m H_x$$

Magnetic field in the air gap is, as one would expect:

$$\vec{B} = \mu_0 \vec{H}$$

This simplified situation is shown in Figure 3.

This problem could be solved in a variety of ways: you could simply write down the form of the magnetic field in the air-gap and in the magnet, realizing that the curl of $\vec{H}$ and the divergence of B are both zero (it would not take long to discover the proper form of these fields). Alternatively, you could realize that $\vec{H}$ can be found to be the gradient of a scalar potential ($\vec{H} = -\nabla \psi$), and that this scalar potential satisfies Laplace's equation:
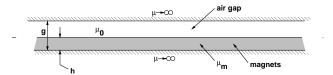
$$\nabla^2 \psi = 0$$

Figure 3: Magnetic Problem Cut Down To Size

In either case, you can use the boundary conditions that, at the upper and lower boundaries (the iron boundaries), $H_x = 0$ (why?). And at the boundary between the magnets and the air-gap, H_x must be continuous and B_y must be continuous.

In working this problem, you may find it convenient to set the value of y to be zero at the boundary between the magnets and the air-gap.

1. First, assume that $kg \ll 1$ and find the magnetic flux density in the vertical (y) direction. What is H_y in the magnet? Is this what you expect?

2. Now relax that assumption and find magnetic field H_y at the upper ferromagnetic surface.

3. Finally, let the gap dimension go to infinity (that is, assume that there is no upper boundary). Find expressions for magnetic field in the region above the magnets.

Problem 4: Non-Magnetic LIM

A prototypical linear induction motor is shown in Figure 4. The 'stator' of this motor is the lower element, and it carries a surface current of:

$$K_z = \mathrm{Re}\left\{ K_0 e^{j(\omega t - kx)} \right\}$$

The 'shuttle', which is the equivalent of the rotor of an ordinary induction motor, consists of a layer of conductive material. You should consider the layer of material to be thin, and the region 'behind' (above) that layer has permeability μ_0, zero conductivity, and is deep enough that there is no magnetic interaction with anything above the layer The 'shuttle' is moving to the right (the positive x direction) with velocity u.
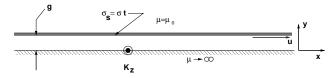

Figure 4: Cartoon of a notional linear induction motor

1. Derive an expression for the y- directed magnetic field in the air-gap as a function of speed.

2. Use that expression to find force in the x- direction as a function of speed. Be sure to consider speeds that are both smaller than and greater than the phase velocity $u_{ph} = \frac{\omega}{k}$.

3. Find the vertical ('lift' or the opposite of 'lift'), also as a function of speed.

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