Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.685 Electric Machines

Problem Set 11 — Issued November 17, 2013

Due November 27, 2013

Quiz 2 : Remember Quiz 2 will be on December 4, 2013 at normal class time. Closed Book; 2 Crib sheets allowed. Calculators OK but should be un-necessary.

Problem 1: Variable Reluctance Motor

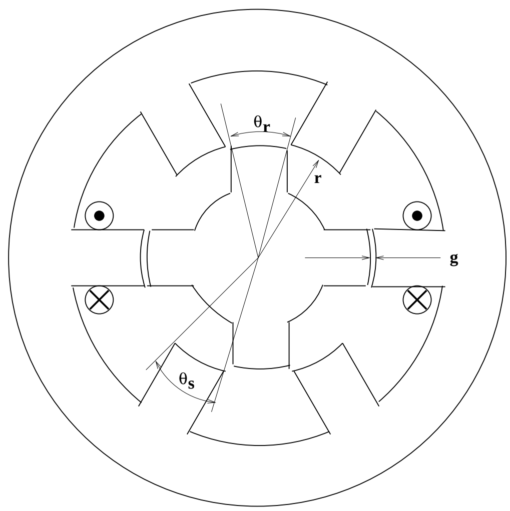

Figure 1: Crude Cartoon of a Variable Reluctance Machine

Figure 1 is a very crude picture of a "6-4" variable reluctance motor. The objective of this problem is to take a first-order look at operation of this machine. To start, assume that the flux vs. current pattern of the machine is as shown in Figure 2. For currents less than some "saturation" level, inductance of one winding of the motor is as shown in Figure 3. For currents greater than the "saturation" level, the incremental inductance is some lower level *Lsat* which is not a function of rotor position, so that, noting current is positive, flux is:

$$\begin{aligned} \text{if} I < I_s \quad \lambda(\theta, I) &= L(\theta) I \\ \text{else} \quad \lambda(\theta, I) &= L(\theta) I_s + L_{\text{min}} (I - I_s) \end{aligned}$$

Assume the following set of parameters:

Maximum Inductance	L_{max}	0.3 Hy
Minimum Inductance	L_{min}	0.075 Hy
Saturation Current	I_s	5 A
Overlap Angle	θ_{overlap}	$\frac{9\pi}{20} = 81^\circ$

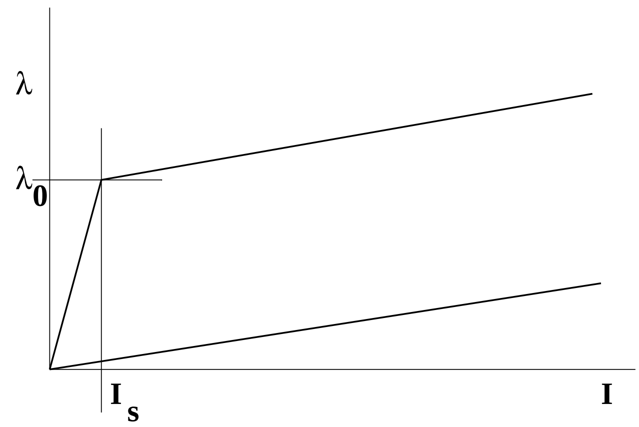

Figure 2: VRM Saturation Characteristic

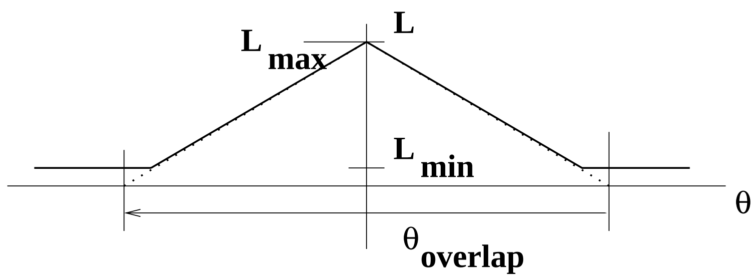

Figure 3: Inductance vs. Alignment Angle

The machine is to be operated as shown in Figure 4. For each phase, voltage is applied, driving flux up to the maximum at which point the switches are turned off. Current continues to flow through the diodes of the drive circuit (not shown here), putting negative voltage across the phase until current (and flux) goes to zero. Note that the angle at which the voltage pulse starts, θ_0, could vary over a wide range and is a control parameter. For the purposes of this problem assume that the width of the triangular pulse of flux is the same as the overlap angle $(9\pi/20)$.

- Using Figures 2 and 3, calculate current in the one phase winding as a function of rotor angle and with flux as a parameter. Use values of flux of 0.2, 0.4, ... 2.0 Wb, and a range of angles from $-\pi/2$ to $\pi/2$.
- Using the principal of virtual work (Coenergy), calculate torque produced by that one phase as a function of flux and angle. Plot the results for the same range of angles and flux values as you used for the first part.
- Now the machine is to be operated at a steady speed of 1000 RPM. Find and calculate the time average torque as a function of the starting angle θ_0. Assume the machine is operated as shown in Figure 4 with flux having a triangular pulse form and consequently voltage having positive and negative square pulses. Remember your voltage pulse may overlap more than one variation in inductance.

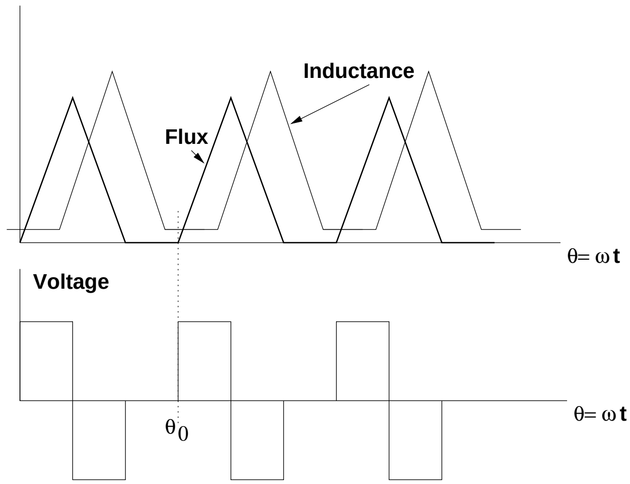

Figure 4: Operating Strategy

- Pick two starting angles, one near peak motoring torque and one near peak generating torque. Plot, vs. angle (corresponding to time), inductance, flux, voltage, current and instantaneous torque for each of these two cases.

Problem 2: Damping

In this problem, assume a synchronous machine with only two rotor windings: a field winding on the d- axis and a damper winding on the quadrature axis. Further, assume that the field winding has a time constant that is long enough that field resistance can be neglected in computing transient torque. Our objective is to find a damping coefficient so that we could write a mechanical differential equation in the form:

$$\frac{d^2\delta}{dt^2} = \frac{2H}{\omega_0}\left(T_e(\delta) - B\frac{d\delta}{dt}\right)$$

The damping coefficient B represents dissipation from only the q-axis damper. You should be able to find this coefficient by:

1. Assume the machine is driven into having a torque angle:

$$\delta = \delta_0 + \delta_1 \sin \Omega t$$

2. Using the electrical equations for the machine, calculate dissipation in the q- axis damper,
3. Do the same for the simplified mechanical equation given in this problem statement,
4. And then using the comparison to deduce the damping coefficient B in terms of the electrical parameters (reactances and time constants) of the machine. Do you think this is a valid procedure?

Problem 3: Single Phase Motor

Assume, somewhat unrealistically, that a single phase motor has identical running and starting windings, oriented perpendicular to each other. This is a two pole motor to be operated with a terminal voltage of 120 V, RMS, 60 Hz.

Magnetizing Reactance	x_m	98	Ω
Stator Leakage	X_1	1.25	Ω
Rotor Leakage	X_2	0.75	Ω
Stator Resistance	R_1	2.0	Ω
Rotor Resistance	R_2	1.20	Ω

Calculate and plot (on the same figure), the torque vs. speed curve for this motor over the speed range of zero to 3,600 RPM, for the following conditions:

1. Running: only the main winding is connected to the power source.
2. Capacitive Start: Assume the starting winding is in series with a 50 microfarad capacitor. the same speed range.
3. Resistive split-phase starting: Assume the starting winding is in series with a 10 ohm resistance.

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