Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.685 Electric Machinery

Quiz 2 **Two Crib Sheets Allowed** **December 4, 2013**
(Your crib sheet from Quiz 1 and another one)

There is space for you to write your answers on this quiz.

Problem 1: Permanent Magnet Machines (15 points)

This concerns a surface mounted permanent magnet machine, for which the direct- and quadrature axis inductances are equal:

$$L_d = L_q = 5\text{mH}$$

For the purpose of this problem, assume that resistance of the stator is negligible.

This is a three-phase, four-pole motor. The following test is run on the machine: It is set up in a fixture that allows the shaft to be turned slowly and the torque produced by the motor measured. When one of the phases is driven by a current of 10 A (DC), the torque produced is sinusoidal and has a peak value of ± 10N-m

Now, if the machine is operated with a balanced set of voltages:

$$\begin{aligned} v_a &= 100\cos(\omega t) \\ v_b &= 100\cos(\omega t - \frac{2\pi}{3}) \\ v_c &= 100\cos(\omega t + \frac{2\pi}{3}) \end{aligned}$$

where $\omega = 200$Radians/Second

1. What is the maximum value for torque that this machine can make with that excitation?

   $T =$ N-m

2. What is the maximum mechanical power produced by the motor?

   $P =$ W

Problem 2 Synchronous Reluctance Machines (25 points)

A synchronous reuluctance machine is like a buried magnet permanent magnet machine, with a high degree of saliency but without the magnets. This particular machine has four poles and three phases. The direct and quadrature inductances are:

$$\begin{aligned} L_d &= 3\text{mH} \\ L_q &= 12\text{mH} \end{aligned}$$

Assume that this machine is operated at an electrical frequency of $\omega_e = 1000$ radians/second and a balanced three-phase current with a *peak* amplitude of 10 A/phase.

1. Assuming phase currents are driven with phase angles to make the maximum torque, how much torque is it making?

   $T =$ N-m

2. How much *real* power is it drawing from the supply?

   $P =$ W

3. How much *reactive* power is it drawing from the supply?

   $Q =$ VAR

Problem 3 Doubly Fed Generator (25 points)

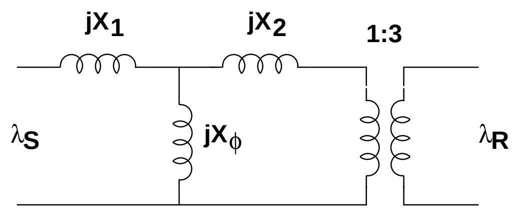

Figure 1: Doubly Fed Machine Magnetic Model

This problem is about doubly fed machines such as might be used as generators in wind turbines. The magnetic model of the machine is shown, for reference, in Figure 1. For the purposes of this problem, you should assume that the magnetizing branch of this magnetic circuit has a very large value ($X_\phi \to \infty$). The sum of the two 'leakage' elements $X_1 + X_2 = 1\Omega$. The rotor is wound with more turns than the stator so the ideal transformer has a turns ratio of three:

$$\frac{N_A k_A}{N_a k_a} = 3$$

This is a three-phase, wye connected six pole machine, and it is operating with a terminal phase voltage of 1,000 volts, *RMS* into a 60 Hz power system.

The machine is operating as a generator, with a *stator* output power of 1 MW at a power factor of unity.

1. What are the real and reactive power *into* the rotor terminals if the rotor speed is:
   N=900 RPM

   $P_r =$ W $Q_r =$ VAR

   N=1500 RPM

   $P_r =$ W $Q_r =$ VAR

2. What are the rotor voltage and current for the same two speeds? You should be able to check your answers to the first part of the problem. Put your answers in the form $V = V_r + jV_i$.
   N=900 RPM

   $V_r =$ V $I_r =$ A

   N=1500 RPM

   $V_r =$ V $I_r =$ A

Problem 4 Surface Mount Machine (35 points)

Figure 2 shows the cross-section of a surface mount Permanent Magnet Motor. The relative rotation gap is $g = 0.5$mm. The magnet radial dimension is $h_m = 4.0$mm. The stator inner radius is $R = 10$cm and the active length is $L = 20$cm. The magnet *electrical* angular width is $\theta_{me} = \frac{2\pi}{3}$ (120°). Remanent flux density of the magnet is $B_{\text{rem}} = 1.0$T. There is a single winding in the stator. Assume that the rotor is shown in the figure in the position $\phi = 0$.

In this problem you are going to make two sketches of voltage on templates provided. Please either do it in pencil or think carefully about what you will draw before putting pen to paper. Use the simplest possible model of magnetic flux density in the gap, which is considered to be valid for $g + h_m << $ pole pitch.

1. If the stator winding has a total of 20 turns and is concentrated (in a single slot per pole) and the rotor is turning at a speed of $\Omega = 500$Radians/second, find and sketch the voltage induced in the winding. Use Figure 3 and plot voltage against rotor position. Note: this is *physical* rotor angle.
2. Now assume that the stator winding still has 20 turns, but is short pitched so make it a 5/6 pitch winding. Find and sketch the voltage on Figure 4

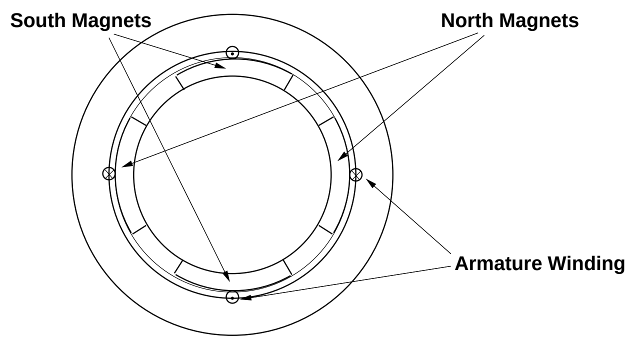

Figure 2: Cartoon of a Permanent Magnet Motor

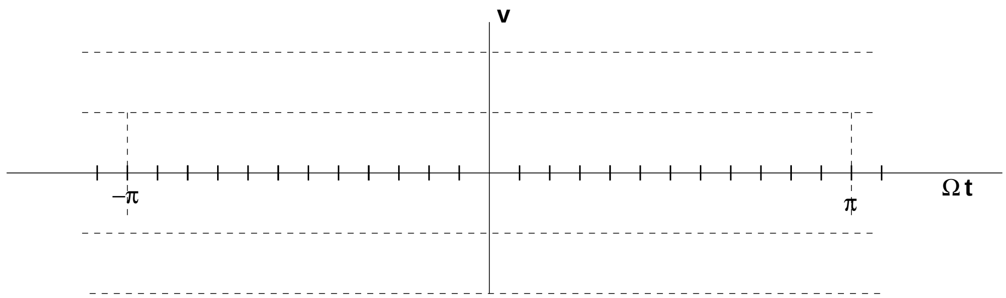

Figure 3: Your answer for voltage vs. rotor position for a concentrated winding

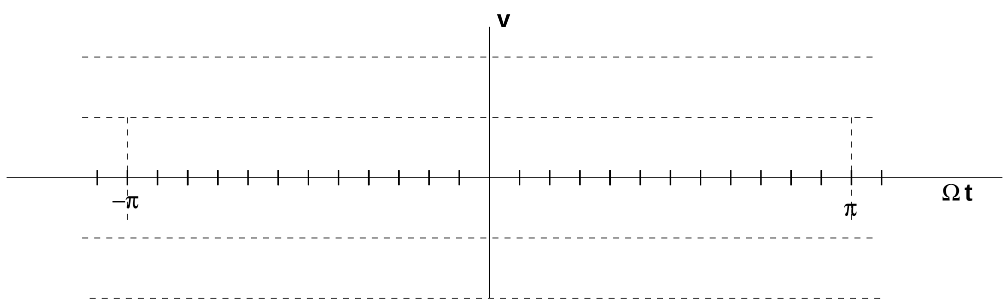

Figure 4: Your answer for voltage vs. rotor position for a 5/6 pitch winding

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Fall 2013