Generalized Josephson Junctions

Outline

1. Junctions with Resistive Channel
2. RCSJ Model
3. DC Current Drive
    - Overdamped and Underdamped Junctions
    - Return Current
    - Dynamical Analysis
4. Pendulum Model

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Junctions with Resistive Channel

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Please see: Figure 9.1, page 450, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

Tunneling between two superconductors

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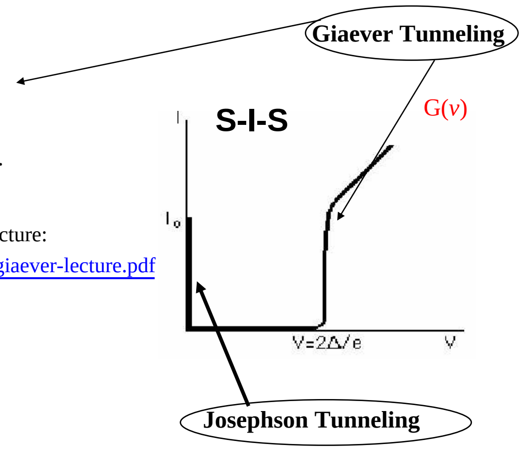
Please see: p. 148 of Giaever's 1973 Nobel lecture:
http://nobelprize.org/physics/laureates/1973/giaever-lecture.pdf



Normal and Superconducting Analogy

Superconductor

$$\frac{\partial \mathbf{J}}{\partial t} = \frac{n^\star (q^\star)^2}{m^\star}\mathbf{E}$$

$$\sigma_s(\omega) = \frac{n^\star (q^\star)^2}{m^\star}\left(\frac{1}{j\omega}\right) = \frac{1}{j\omega \ell_s}$$

Normal metal

$$\frac{\partial \mathbf{J}}{\partial t} + \frac{\mathbf{J}}{\tau_{tr}} = \frac{n^\star (q^\star)^2}{m^\star}\mathbf{E}$$

for dc drive

$$\sigma_n(\omega = 0) = \frac{ne^2 \tau_{tr}}{m}$$

and

$$\ell_s/\rho_n = 1/\tau_{\text{tr}}$$

Superconducting Josephson Junction

$$\frac{di}{dt} = I_c \frac{d\varphi}{dt}\cos\varphi = \Big(\underbrace{\frac{2\pi I_c}{\Phi_o}}_{L_J^{-1}} \cos\varphi(t)\Big)\, v(t)$$

For a normal junction, the phase is constantly being driven back to zero so linearize near zero and add a damping time

$$\frac{di}{dt} + \frac{i}{\tau_i} = \frac{2\pi I_c}{\Phi_o} v(t)$$

for dc drive

$$G_s(\omega = 0) = \frac{2\pi I_c \tau_i}{\Phi_o}$$

and

$$L_J/R_n = 1/\tau_i$$

$I_C R_n$ Product

The condition $L_J / R_n = 1/\tau_i$ is equivalent to $I_c R_n = \frac{\Phi_o}{2\pi\tau_i} = \text{constant}$

Experimentally, $I_c R_n = \frac{\pi \Delta_o}{2e}$

For Nb at 2K, $I_c R_n = 1.9\,\text{mV}$

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Please see: Figure 9.2, page 454, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity.* Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

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Please see: Figure 9.3, page 456, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity.* Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

$$G(v) \approx \begin{cases} 0 & \text{if } |v| < 2\Delta_o/e \\ \frac{1}{R_n} & \text{otherwise} \end{cases}$$

Capacitance of a Josephson Junction

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Please see: Figure 8.4, page 399, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity.* Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

$$C = \frac{\epsilon A}{2a}$$

Generalized Josephson Junction

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Please see: Figure 9.4, page 457, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

$$i = I_c \sin\varphi + vG(v) + C\frac{dv}{dt} \qquad \text{and} \qquad v = \frac{\Phi_o}{2\pi}\frac{d\varphi}{dt}$$

Therefore,

$$i = I_c \sin\varphi + G(v)\frac{\Phi_o}{2\pi}\frac{d\varphi}{dt} + C\frac{\Phi_o}{2\pi}\frac{d^2\varphi}{dt^2}$$

RCSJ Model

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Please see: Figure 9.6, page 459, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

$$i = I_c \sin\varphi + \frac{v}{R} + C\frac{dv}{dt} \qquad \text{and} \qquad v = \frac{\Phi_o}{2\pi}\frac{d\varphi}{dt}$$

Therefore,

$$i = I_c \sin\varphi + \frac{\Phi_o}{2\pi R}\frac{d\varphi}{dt} + C\frac{\Phi_o}{2\pi}\frac{d^2\varphi}{dt^2}$$

DC Current drive in the RSCJ Model

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Please see: Figure 9.6, page 459, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

$$i = I_c \sin\varphi + \frac{v}{R} + C\frac{dv}{dt}$$

and $$v = \frac{\Phi_o}{2\pi}\frac{d\varphi}{dt}$$

Therefore, $$i = I_c \sin\varphi + \frac{\Phi_o}{2\pi R}\frac{d\varphi}{dt} + C\frac{\Phi_o}{2\pi}\frac{d^2\varphi}{dt^2}$$

The equation of motion can be rewritten as

$$\frac{i}{I_c} = \sin\varphi + \frac{d\varphi}{d\tilde{\tau}} + \beta_c\frac{d^2\varphi}{d\tilde{\tau}^2} \quad \text{where} \quad \tilde{\tau} = \frac{t}{\tau_J}$$

$$\tau_J = \frac{L_J}{R} = \frac{\Phi_o}{2\pi}\frac{1}{I_c R} \qquad \beta_c = \frac{\tau_{RC}}{\tau_J} = \frac{R^2 C}{L_J} = \frac{2\pi R^2 C}{\Phi_o}$$

Josephson Time Constant *Stewart-McCumber Parameter* $\beta_c = \mathbf{Q}^2$

Overdamped Junction $\beta_c << 1$

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Please see: Figure 9.6, page 459, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

$$\frac{i}{I_c} = \sin\varphi + \frac{d\varphi}{d\tilde{\tau}}$$

A. Static Solution: $\varphi = \sin^{-1}\frac{i}{I_c}$ for $i \le I_c$

B. Dynamical Solution for $i > I_c$

$$d\tilde{\tau} = \frac{d\varphi}{\frac{i}{I_c} - \sin\varphi} \Rightarrow \varphi(t) = 2\tan^{-1}\left[\sqrt{1-\left(\frac{I_c}{i}\right)^2}\tan\left(\frac{t\sqrt{(i/I_c)^2-1}}{2\tau_J}\right) - \frac{I_c}{i}\right]$$

This is periodic with period $$\Theta = \frac{2\pi\tau_J}{\sqrt{(i/I_c)^2 - 1}}$$

Overdamped Junction $\beta_c << 1$

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Please see: Figure 9.7, page 462, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

The time averaged voltage is

$$\langle v(t) \rangle = \frac{1}{\Theta} \int_0^{\Theta} v(t)\, dt$$

Use the voltage-phase relation,

$$\langle v(t) \rangle = \frac{\Phi_o}{\Theta}$$

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Please see: Figure 9.8, page 463, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

Therefore,

$$\langle v(t) \rangle = iR\sqrt{1 - \left(\frac{I_c}{i}\right)^2} \qquad \text{for } i > I_c$$

Non-hysteretic

Underdamped Junction $\beta_c >> 1$

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Please see: Figure 9.6, page 459, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

A. Static Solution: $\varphi = \sin^{-1} \frac{i}{I_c} \qquad \text{for} \quad i \leq I_c$

B. Dynamical Solution

The phase changes quickly compared to RC, so the voltage is just from R and C. Therefore,

$\langle v(t) \rangle = i R$

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Please see: Figure 9.9, page 464, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

Hysteretic

Junction with arbitrary β_c

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A. Static Solution: $\varphi = \sin^{-1}\frac{i}{I_c}$ for $i \leq I_c$

B. Dynamical Solution

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Please see: Figure 9.11, page 465, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

Return Current

Energy Loss per cycle = Energy supplied by source

$$\frac{1}{2}CV^2\frac{1}{Q} = I^2R\tau$$

where V= IR and $\tau = \Phi_0 / (2\pi I R)$, therefore

$$\frac{1}{2}C(IR)^2\frac{1}{Q} = I^2R\frac{\Phi_o}{2\pi IR}$$

So that

$$\frac{I}{I_c} = 2Q\frac{\Phi_0}{2\pi I_c R^2 C} = \frac{2}{\sqrt{\beta_c}}$$

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Please see: Figure 9.11, page 465, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

Dynamical Analysis

$$\dot{\varphi} = v$$

$$\dot{v} = i - \sin\varphi - \alpha v$$

where $\alpha = \frac{1}{\sqrt{\beta_C}}$ and $V = I_c\sqrt{L_J/C}\,v$

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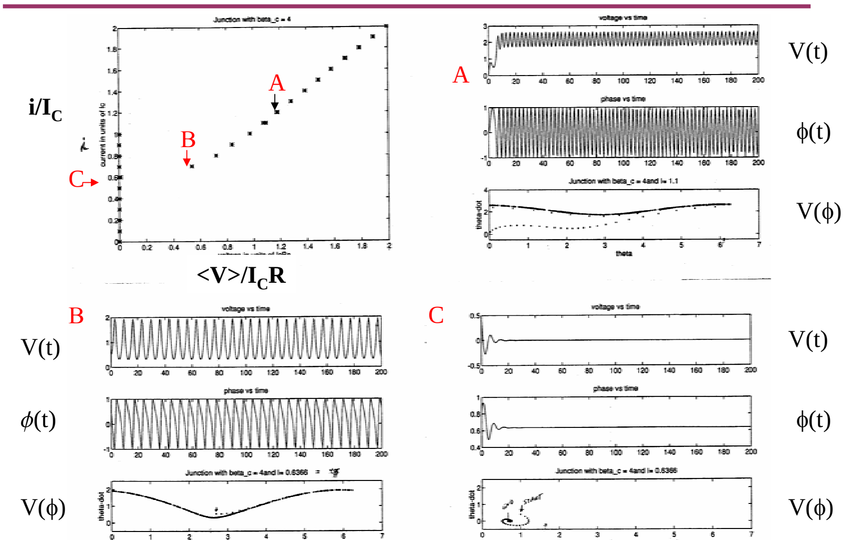
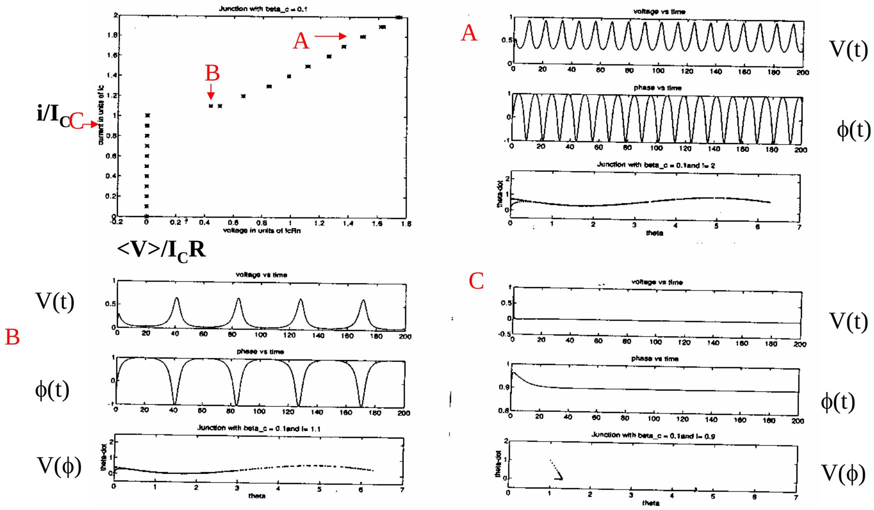
Please see: Figure 8.5.10, page 272, from Strogatz, S. *Nonlinear Dynamics and Chaos.* 1st ed. Cambridge, MA: Perseus Books Group, January 15, 2001. ISBN: 0738204536.

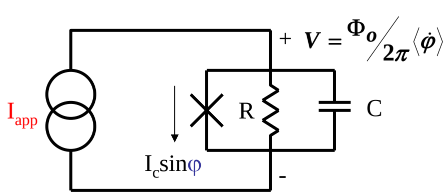
$\beta_C = 4$

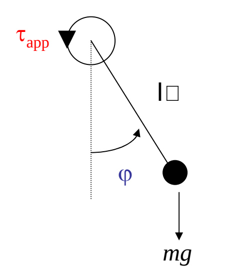


$\beta_c = 0.5$



Pendulum Model for a Josephson Junction





$$I_{app} = C\dot{V} + \frac{1}{R}V + I_c \sin\varphi$$

$$\tau_{app} = ml^2\ddot{\varphi} + D\dot{\varphi} + mgl\sin\varphi$$

$$\boxed{F = \ddot{\varphi} + \Gamma\dot{\varphi} + \sin\varphi}$$

- Single junction (RCSJ model) ⟺ pendulum (damped)
- Coupled junctions – can support non-linear excitations (breathers and moving vortices)

Pendulum Model for a vortex

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Please see: Figure 5.42, page 237, from Kardin, A. *Introduction to Supercomputing Circuits.* 1st ed. New York, NY: Wiley-Interscience, March 11, 1999. ISBN: 0471314323.