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High Speed Communication Circuits and Systems
Lecture 12
Low Noise Amplifiers

Massachusetts Institute of Technology

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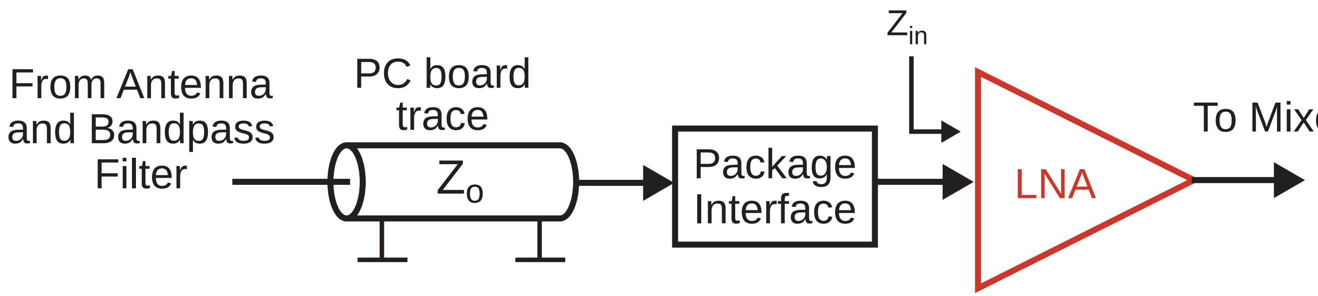
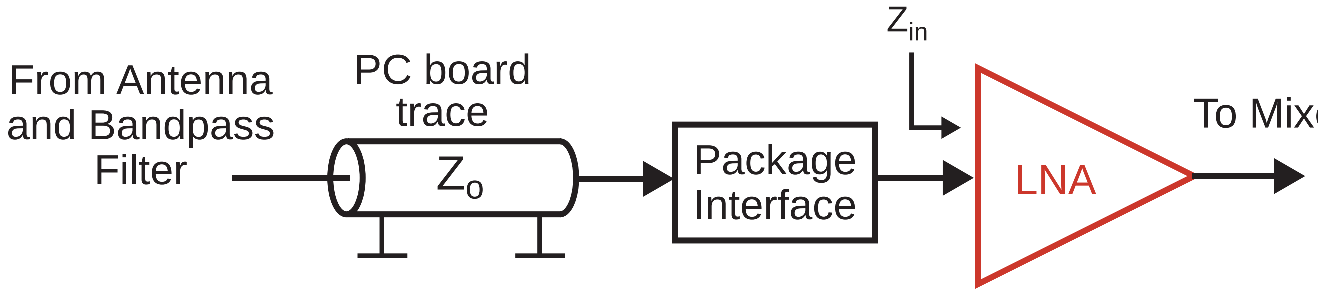
Narrowband LNA Design for Wireless Systems



- **Design Issues**
  - **Noise Figure – impacts receiver sensitivity**
  - **Linearity (IIP3) – impacts receiver blocking performance**
  - **Gain – high gain reduces impact of noise from components that follow the LNA (such as the mixer)**
  - **Power match – want $Z_{in} = Z_o$ (usually = 50 Ohms)**
  - **Power – want low power dissipation**
  - **Bandwidth – need to pass the entire RF band for the intended radio application (i.e., all of the relevant channels)**
  - **Sensitivity to process/temp variations – need to make it manufacturable in high volume**

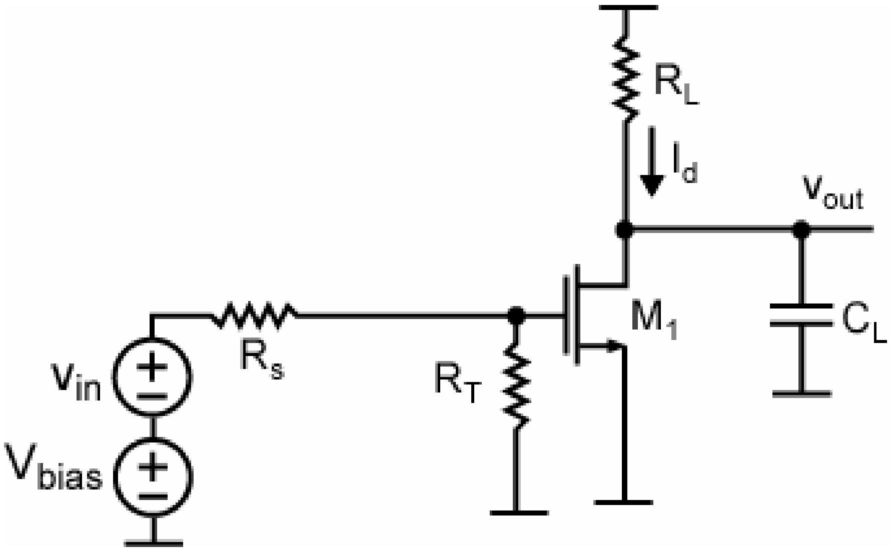
Our Focus in This Lecture



- Designing for low Noise Figure
- Achieving a good power match
- Hints at getting good IIP3
- Impact of power dissipation on design
- Tradeoff in gain versus bandwidth

Direct Input Termination of CS Amplifier

- For power match, terminate the input of CS amplifier



$$R_T = R_s = R$$

- Voltage gain is halved
- Termination resistor adds thermal noise
- More appropriate for broadband than narrowband (maybe)

Noise Factor (Low Frequency)

- Noise factor at low frequencies (thus ignoring gate current noise) looking at short-circuit output current

$$\overline{i_{n1}^2} = \frac{1}{4} g_m^2 4kTR\Delta f = kTg_m(g_m R)\Delta f$$

$$\overline{i_{n2}^2} = 2kTg_m(g_m R)\Delta f + 4kT\gamma g_{do}\Delta f$$

$$F = \frac{2kTg_m(g_m R)\Delta f + 4kT\gamma g_{do}\Delta f}{kTg_m(g_m R)\Delta f}$$

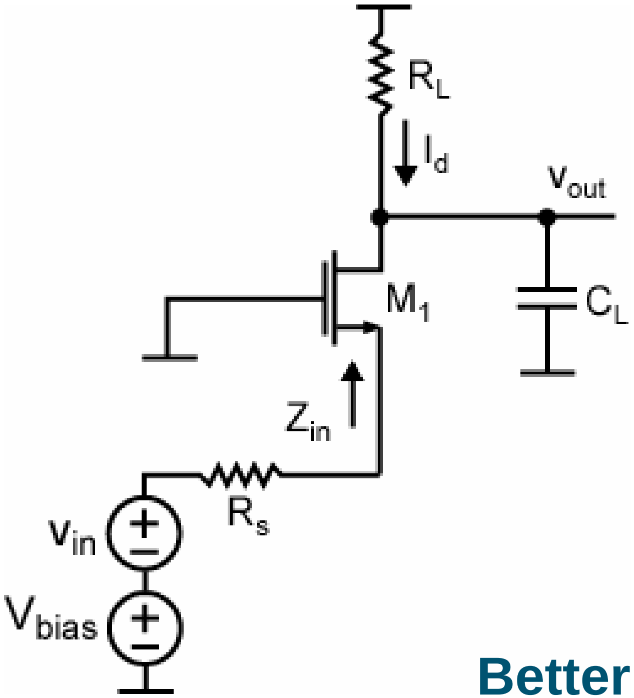
$$= 2 + 4\frac{\gamma g_{do}}{g_m} \cdot \frac{1}{g_m R}$$

$$= 2 + 4\frac{\gamma}{\alpha} \cdot \frac{1}{g_m R}$$

- Noise factor too high even at low frequencies.
- Generally not an acceptable circuit for LNA

Common-Gate Amplifier

Offers the possibility of power match without an explicit termination resistor



For power match, make

$$Z_{in} = \frac{1}{g_m} = R_s$$

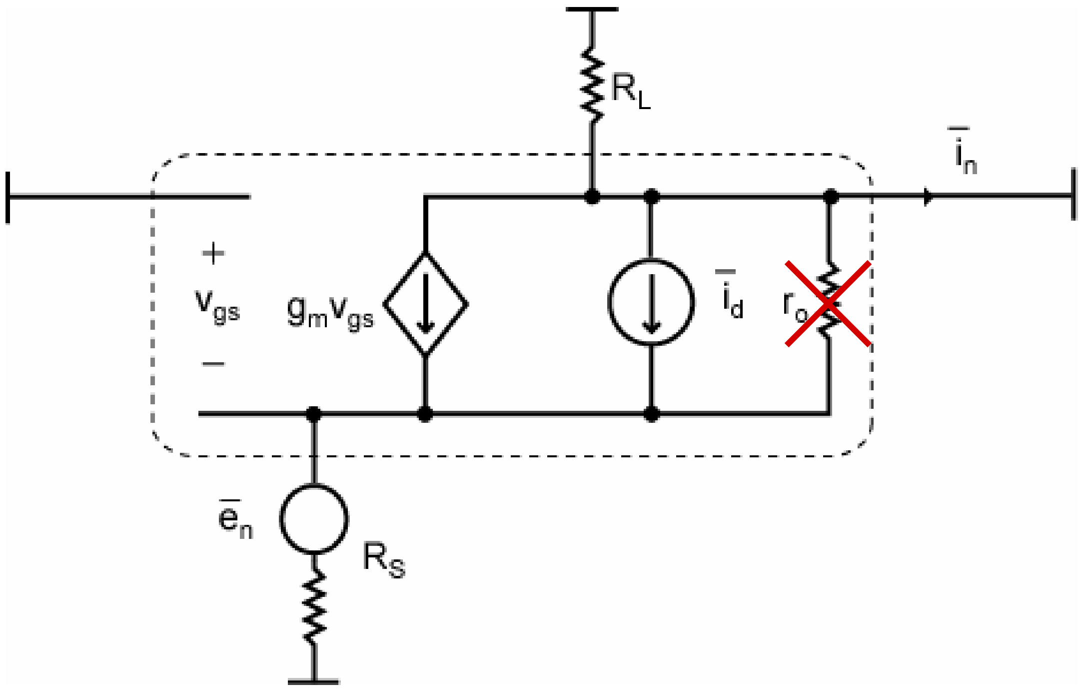
At low frequencies, it can be shown

$$F = 1 + \frac{\gamma}{\alpha} \cdot \frac{1}{g_m R_s} = 1 + \frac{\gamma}{\alpha}$$

Better than the CS, but noise factor degrades significantly at high frequencies

Noise Factor Analysis of CG Amp (Low Freq.)

- **Equivalent Circuit**



Noise Factor Calculation

- Noise factor at low frequencies (thus ignoring gate current noise) looking at short-circuit output current

  First calculate the output noise due to the source resistance only (e_n) using voltage divider:

$$\overline{i_{n1}^2} = \frac{(\frac{1}{g_m})^2}{(R_s + \frac{1}{g_m})^2} g_m^2 4kTR_s\Delta f$$

$$= \frac{g_m^2}{(1 + g_m R_s)^2} 4kTR_s\Delta f$$

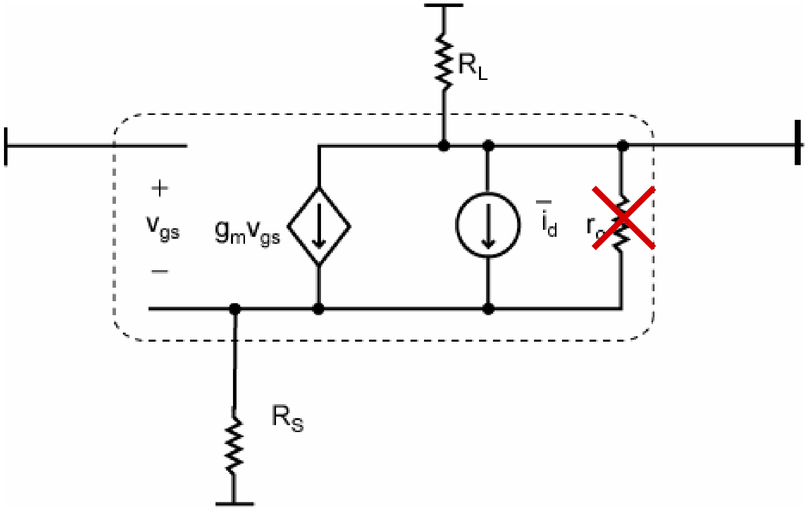
The next step is finding the total output noise due to e_n and i_d. We can use superposition as before.

Noise Factor Calculation, Continued

First, let's compute output noise due to i_d only. We'll need to use the instantaneous value of i_d for now:

$$v_{gs} = -(i_d + g_m v_{gs}) R_s$$

$$g_m v_{gs} = -\frac{g_m R_s}{1 + g_m R_s} i_d$$



The instantaneous value of the output noise due to i_d

$$i_n(i_d) = i_d + g_m v_{gs} = \frac{1}{1 + g_m R_s} i_d$$

The mean-square value of the output noise due to i_d

$$\overline{i_n(\overline{i_d})^2} = \frac{1}{(1 + g_m R_s)^2} 4kT\gamma g_{do} \Delta f$$

Noise Factor of CG Amplifier

$$\overline{i_{n2}^2} = \overline{i_{n1}^2} + \overline{i_n(i_d)^2}$$

$$F = \frac{\overline{i_{n2}^2}}{\overline{i_{n1}^2}} = \frac{\overline{i_{n1}^2} + \overline{i_n(i_d)^2}}{\overline{i_{n1}^2}}$$

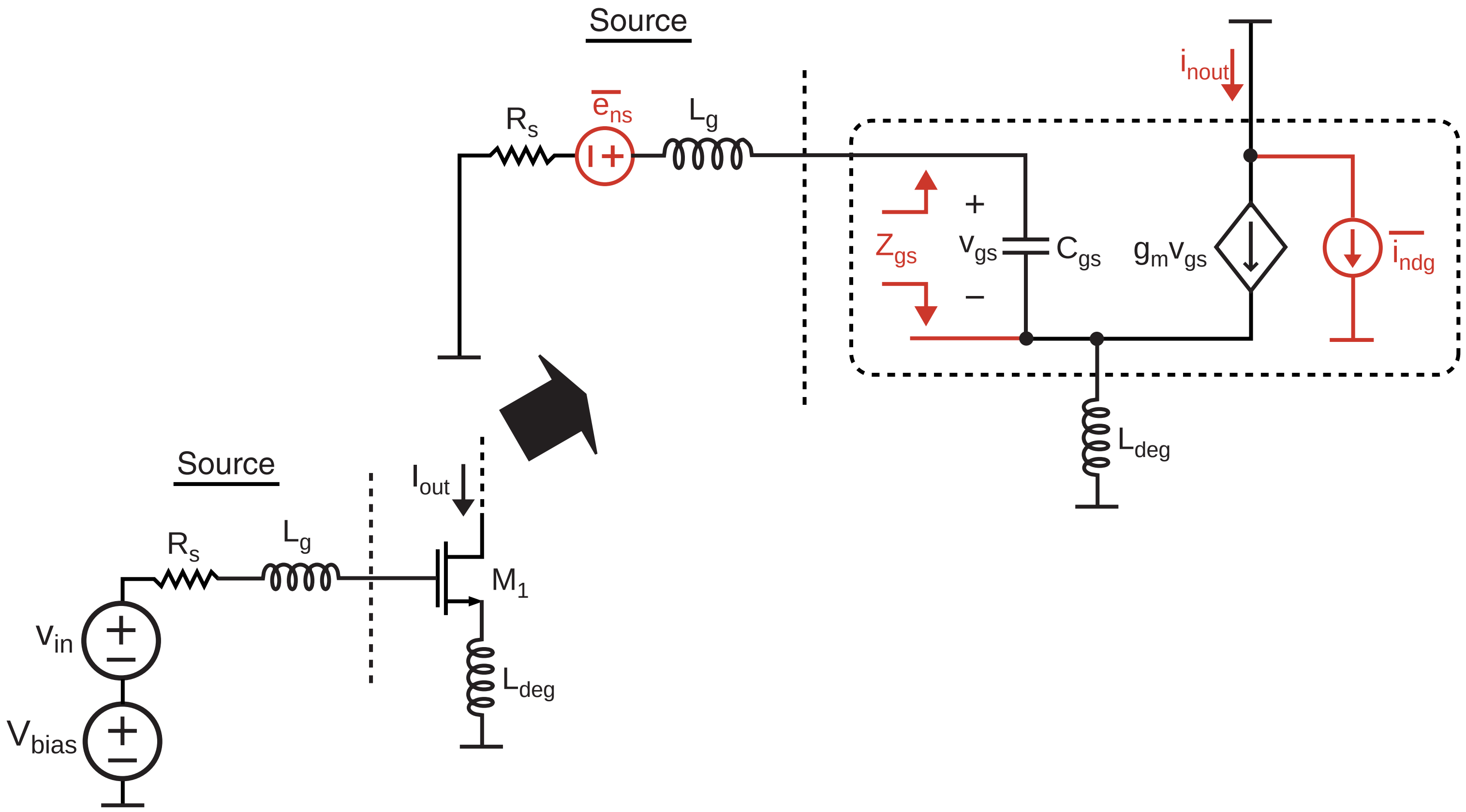
$$F = 1 + \frac{\gamma}{\alpha} \cdot \frac{1}{g_m R_s}$$

For power match, $1/g_m = R_s$

$$F = 1 + \frac{\gamma}{\alpha}$$

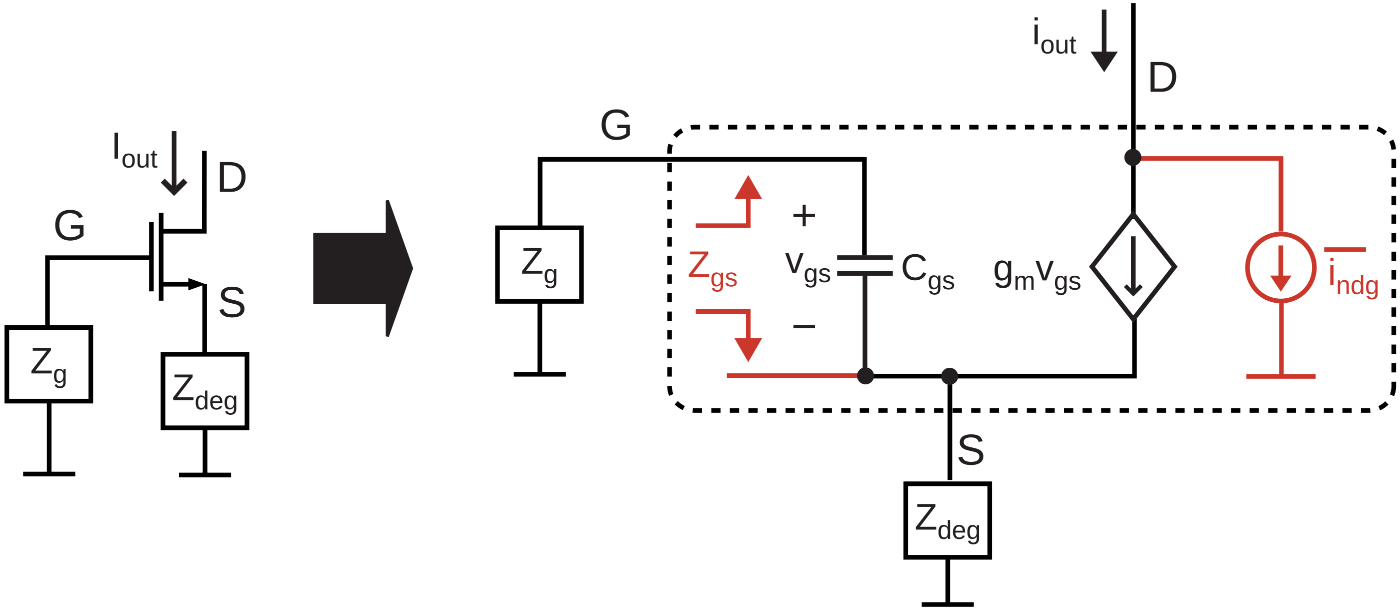
This may be o.k. for broadband amplifiers. For narrowband amplifiers, we can improve the noise factor considerably while maintaining power match: inductors!

Inductor Degenerated CS Amp



- **Same as amp in Lecture 10 except for inductor degeneration**
  - **Note that noise analysis in Tom Lee's book does not include inductor degeneration (i.e., Table 12.1 (11.1))**

Recall Small Signal Model for Noise Calculations

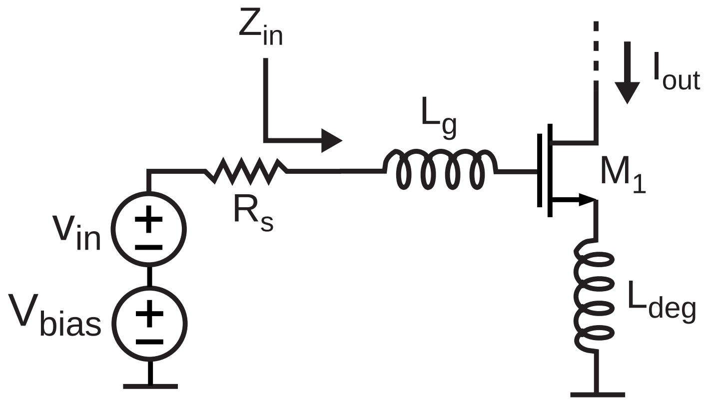


$$\frac{\overline{i_{ndg}^2}}{\Delta f} = \frac{\overline{i_{nd}^2}}{\Delta f}\left(|\eta|^2 + 2Re\left\{c\chi_d\eta^* Z_{gsw}\right\} + \chi_d^2|Z_{gsw}|^2\right)$$

where: $\frac{\overline{i_{nd}^2}}{\Delta f} = 4kT\gamma g_{do},\ \chi_d = \frac{g_m}{g_{do}}\sqrt{\frac{\delta}{5\gamma}},\quad Z_{gsw} = wC_{gs}Z_{gs}$

$$Z_{gs} = \frac{1}{jwC_{gs}} \,\Big|\Big|\, \frac{Z_{deg}+Z_g}{1+g_m Z_{deg}} \qquad \eta = 1 - \left(\frac{g_m Z_{deg}}{Z_{deg}+Z_g}\right) Z_{gs}$$

Key Assumption: Design for Power Match



Real $=R_T$

- **Input impedance (from Lec 9)**

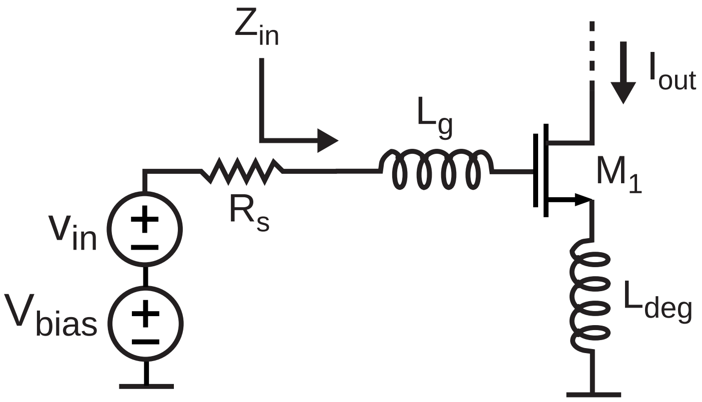
$$Z_{in}(s) = \frac{1}{sC_{gs}} + s(L_{deg} + L_g) + \frac{g_m}{C_{gs}} L_{deg}$$

- **Set to achieve pure resistance = R_s at frequency ω_o**

$$\Rightarrow \quad \frac{1}{\sqrt{(L_g + L_{deg})C_{gs}}} = w_o, \qquad R_T = \frac{g_m}{C_{gs}} L_{deg} = R_s$$

Adding L_g gives additional degree of freedom to set ω_o and R_T independently

Key Assumption: Design for Power Match



- The total impedance of the RLC circuit (including the source resistance)

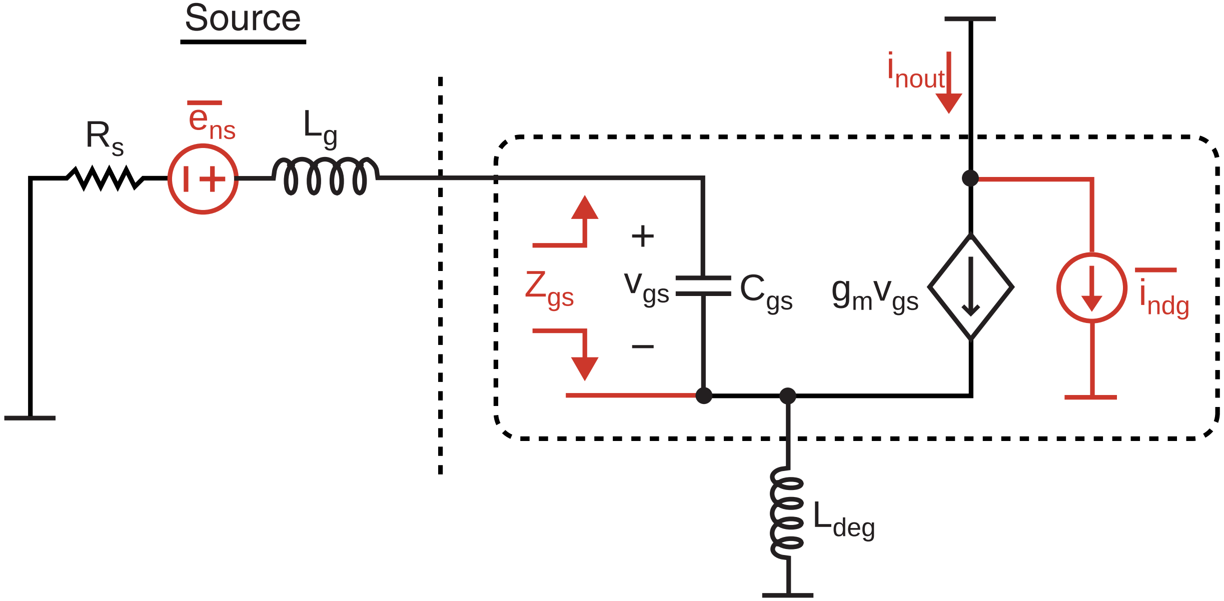
$$Z(s) = \frac{1}{sC_{gs}} + s(L_{deg} + L_g) + \frac{g_m}{C_{gs}} L_{deg} + R_s$$

Thus Q is then

R_s

$$Q = \frac{1}{w_o C_{gs} 2R_s} = \frac{w_o(L_g + L_{deg})}{2R_s}$$

Process and Topology Parameters for Noise Calculation



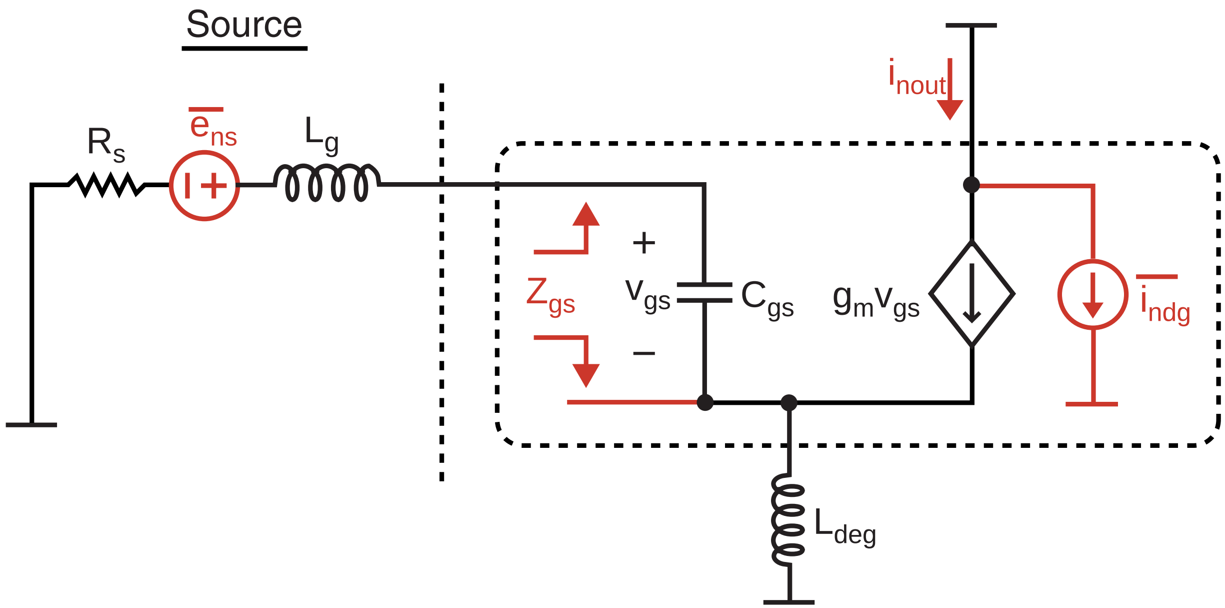
- **Process parameters**
  - **For 0.18μ CMOS, we will assume the following**

$$c = -j0.55, \quad \gamma = 3, \quad \delta = 2\gamma = 6, \quad \frac{g_m}{g_{do}} = \frac{1}{2} \Rightarrow \chi_d = 0.32$$

- **Circuit topology parameters Z_g and Z_{deg}**

$$Z_g = R_s + jwL_g, \quad Z_{deg} = jwL_{deg}$$

Calculation of Z_{gs}

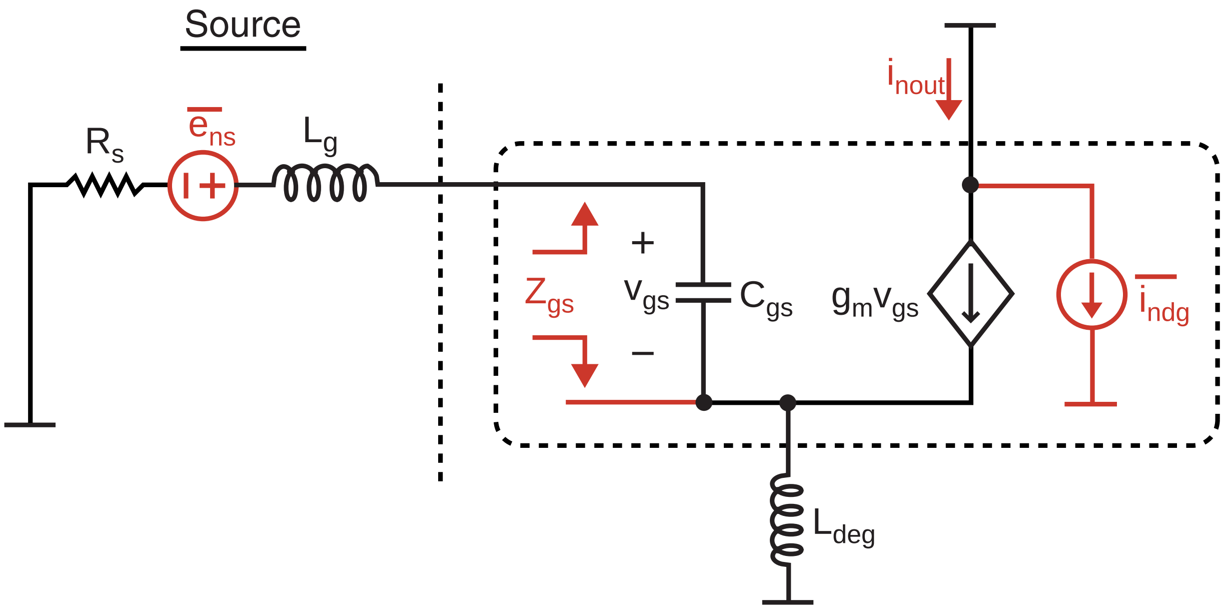


$$Z_{gs} = \frac{1}{jwC_{gs}} \bigg|\bigg| \frac{Z_{deg} + Z_g}{1 + g_m Z_{deg}} = \frac{1}{jw_oC_{gs}} \bigg|\bigg| \frac{jw_o(L_{deg} + L_g) + R_s}{1 + g_m jw_o L_{deg}}$$

$$= \frac{jw_o(L_{deg} + L_g) + R_s}{1 - w_o^2 C_{gs}(L_{deg} + L_s) + jw_o(g_m L_{deg} + R_s C_{gs})}$$

$$= \frac{jw_o(L_{deg} + L_g) + R_s}{jw_o(g_m L_{deg} + R_s C_{gs})}$$

Calculation of η



$$\eta = 1 - \left(\frac{g_m Z_{deg}}{Z_{deg} + Z_g}\right) Z_{gs} = 1 - \frac{g_m j w_o L_{deg}}{j w_o (L_{deg} + L_g) + R_s} Z_{gs}$$

$$= 1 - \frac{g_m j w_o L_{deg}}{j w_o (g_m L_{deg} + R_s C_{gs})} = 1 - \frac{(g_m/C_{gs}) L_{deg}}{\underbrace{(g_m/C_{gs}) L_{deg}}_{= R_s} + R_s}$$

$$= 1 - \frac{R_s}{R_s + R_s} = \boxed{\frac{1}{2}}$$

Calculation of Z_{gsw}

- **By definition**

$$Z_{gsw} = w_o C_{gs} Z_{gs} \quad \left(Q = \frac{1}{w_o C_{gs} 2R_s} = \frac{w_o (L_g + L_{deg})}{2R_s} \right)$$

- **Calculation**

$$Z_{gsw} = w_o C_{gs} \frac{j w_o (L_{deg} + L_g) + R_s}{j w_o (g_m L_{deg} + R_s C_{gs})}$$

$$= \frac{j w_o^2 C_{gs} (L_{deg} + L_g) + w_o C_{gs} R_s}{j w_o (g_m L_{deg} + R_s C_{gs})}$$

$$= \frac{j1 + 1/(2Q)}{j w_o (g_m L_{deg} + R_s C_{gs})}$$

$$= \frac{j1 + 1/(2Q)}{j w_o C_{gs} ((g_m / C_{gs}) L_{deg} + R_s)}$$

$$= \frac{j1 + 1/(2Q)}{j w_o C_{gs} (R_s + R_s)} = \frac{j1 + 1/(2Q)}{j1/Q} = \boxed{\frac{1}{2}(2Q - j)}$$

Calculation of Output Current Noise

- **Step 3: Plug in values to noise expression for i_{ndg}**

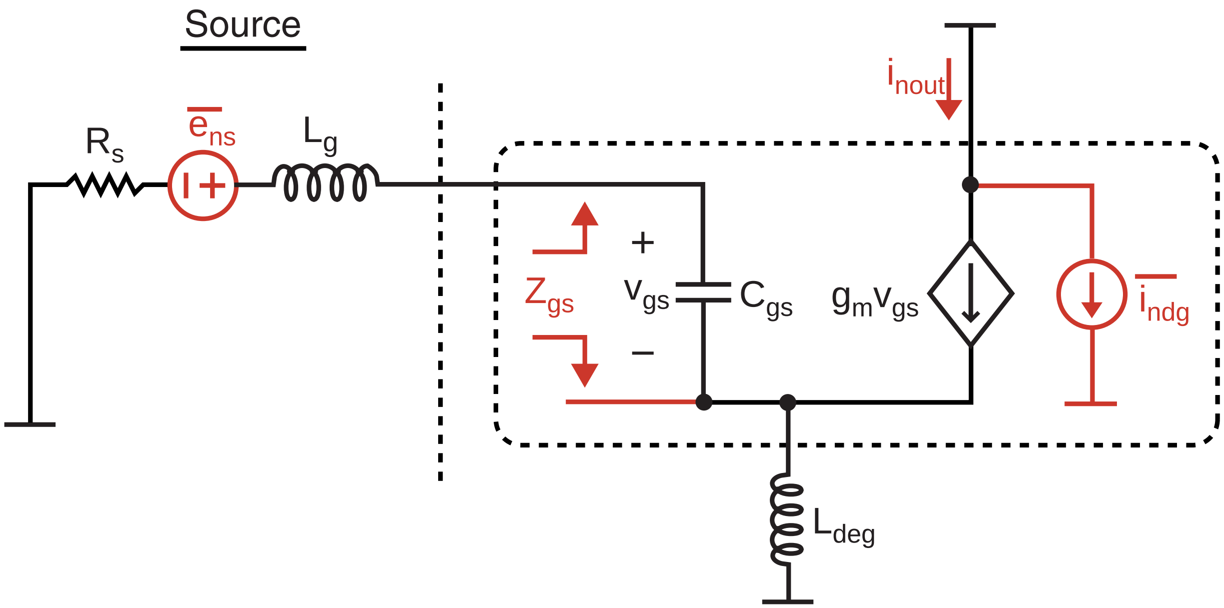
$$\frac{\overline{i_{ndg}^2}}{\Delta f} = \frac{\overline{i_{nd}^2}}{\Delta f}\left(|\eta|^2 + 2Re\{-j|c|\chi_d \eta^* Z_{gsw}\} + \chi_d^2 |Z_{gsw}|^2\right)$$

$$\text{where } \eta = \frac{1}{2}, \qquad Z_{gsw} = \frac{1}{2}(2Q - j)$$

$$\Rightarrow \frac{\overline{i_{ndg}^2}}{\Delta f} = \frac{\overline{i_{nd}^2}}{\Delta f}\left(\frac{1}{4} + 2Re\left\{-j|c|\chi_d \frac{1}{4}(2Q - j)\right\} + \chi_d^2 \frac{1}{4}|2Q - j|^2\right)$$

$$= \frac{\overline{i_{nd}^2}}{\Delta f}\frac{1}{4}\left(1 - 2|c|\chi_d + \chi_d^2(4Q^2 + 1)\right)$$

Compare Noise With and Without Inductor Degeneration



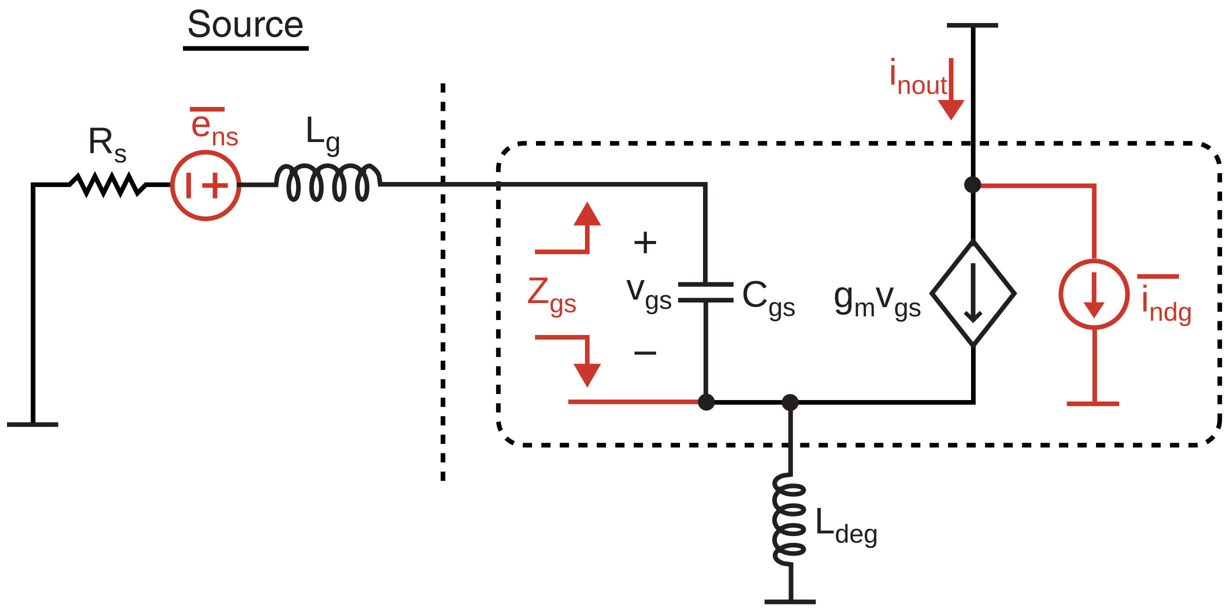
- **From Lecture 10, we derived for $L_{deg} = 0$, $\omega_o^2 = 1/(L_g C_{gs})$**

$$\frac{\overline{i_{ndg}^2}}{\Delta f} = \frac{\overline{i_{nd}^2}}{\Delta f}\left(1 - 2|c|\chi_d + \chi_d^2(Q^2 + 1)\right)$$

- **We now have for $(g_m/C_{gs})L_{deg} = R_s$, $\omega_o^2 = 1/((L_g + L_{deg})C_{gs})$**

$$\frac{\overline{i_{ndg}^2}}{\Delta f} = \frac{\overline{i_{nd}^2}}{\Delta f}\frac{1}{4}\left(1 - 2|c|\chi_d + \chi_d^2(4Q^2 + 1)\right)$$

Derive Noise Factor for Inductor Degenerated Amp

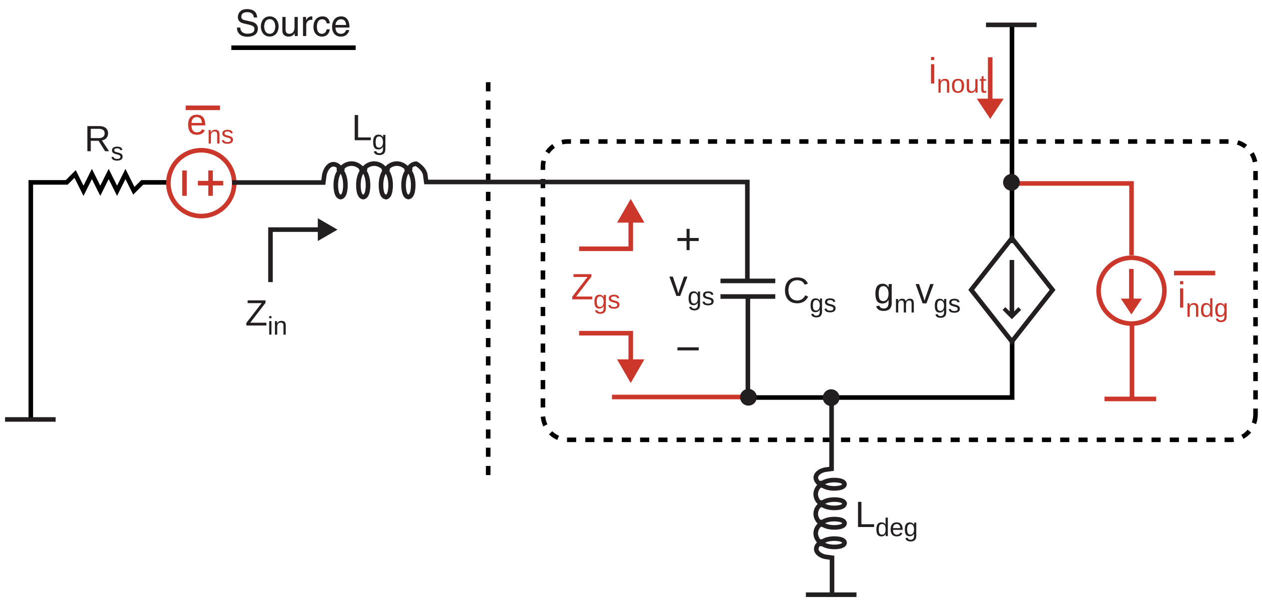


- Recall the alternate expression for Noise Factor derived in Lecture 11

$$F = \frac{\text{total output noise power}}{\text{output noise due to input source}} = \frac{\overline{i^2_{nout(tot)}}}{\overline{i^2_{nout(in)}}}$$

- We now know the output noise due to the transistor noise
  - We need to determine the output noise due to the source resistance

Output Noise Due to Source Resistance



$$Z_{in} = \frac{1}{jw_o C_{gs}} + jw_o(L_{deg} + L_g) + \frac{g_m}{C_{gs}} L_{deg} = R_s$$

$$\Rightarrow \quad v_{gs} = \frac{\overline{e_{ns}}}{R_s + Z_{in}}\left(\frac{1}{jw_o C_{gs}}\right) = \frac{\overline{e_{ns}}}{2R_s}\left(\frac{1}{jw_o C_{gs}}\right) = \left(\frac{Q}{j}\right)\overline{e_{ns}}$$

($Z_{in} \rightarrow R_s$)

$$\Rightarrow \quad i_{nout} = g_m \left(\frac{Q}{j}\right)\overline{e_{ns}}$$

$$\Rightarrow \quad \overline{i^2_{nout}} = (g_m Q)^2 \overline{e^2_{ns}}$$

Noise Factor for Inductor Degenerated Amplifier

$$\text{Noise Factor} = \frac{(g_m Q)^2 \overline{e_{ns}^2} + \overline{i_{ndg}^2}/\Delta f}{(g_m Q)^2 \overline{e_{ns}^2}} = 1 + \frac{\overline{i_{ndg}^2}/\Delta f}{(g_m Q)^2 \overline{e_{ns}^2}}$$

$$= 1 + \frac{4kT\gamma g_{do}(1/4)(1 - 2|c|\chi_d + \chi_d^2(4Q^2+1))}{(g_m Q)^2 4kTR_s}$$

$$= 1 + \left(\frac{1}{g_m Q R_s}\right)\gamma\left(\frac{g_{do}}{g_m}\right)\frac{1}{4Q}\left(1 - 2|c|\chi_d + (4Q^2+1)\chi_d^2\right)$$

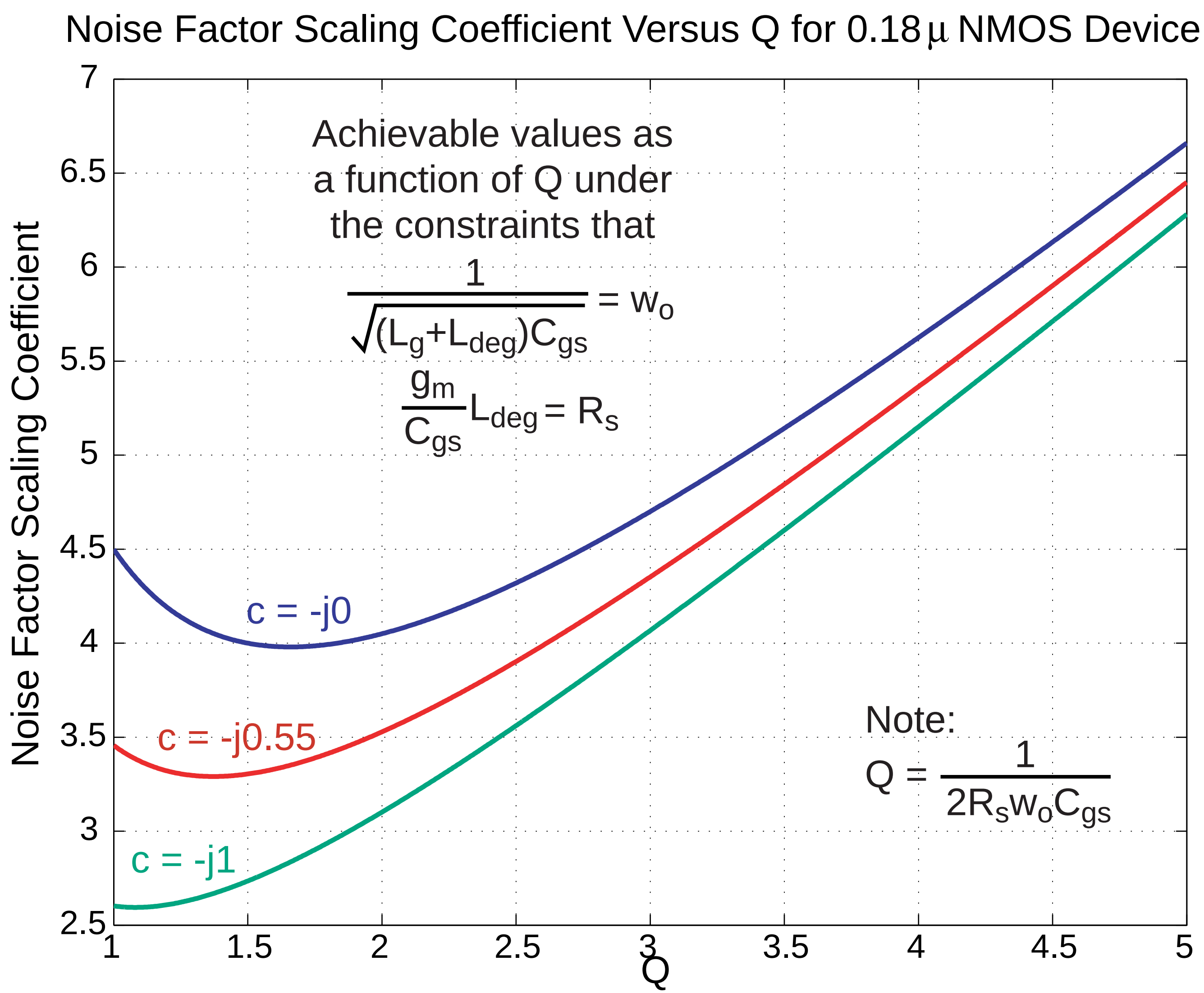
$1/w_t$

$$= 1 + \left(\frac{2w_o R_s C_{gs}}{g_m R_s}\right)\gamma\left(\frac{g_{do}}{g_m}\right)\frac{1}{4Q}\left(1 - 2|c|\chi_d + (4Q^2+1)\chi_d^2\right)$$

$$= 1 + \left(\frac{w_o}{w_t}\right)\gamma\left(\frac{g_{do}}{g_m}\right)\frac{1}{2Q}\left(1 - 2|c|\chi_d + (4Q^2+1)\chi_d^2\right)$$

Noise Factor scaling coefficient

Noise Factor Scaling Coefficient Versus Q



Noise Factor Comparison

- **Inductor Degenerated CS**

$$\text{Noise Factor} = 1 + \left(\frac{w_o}{w_t}\right)\gamma\left(\frac{g_{do}}{g_m}\right)\frac{1}{2Q}\left(1 - 2|c|\chi_d + (4Q^2+1)\chi_d^2\right)$$

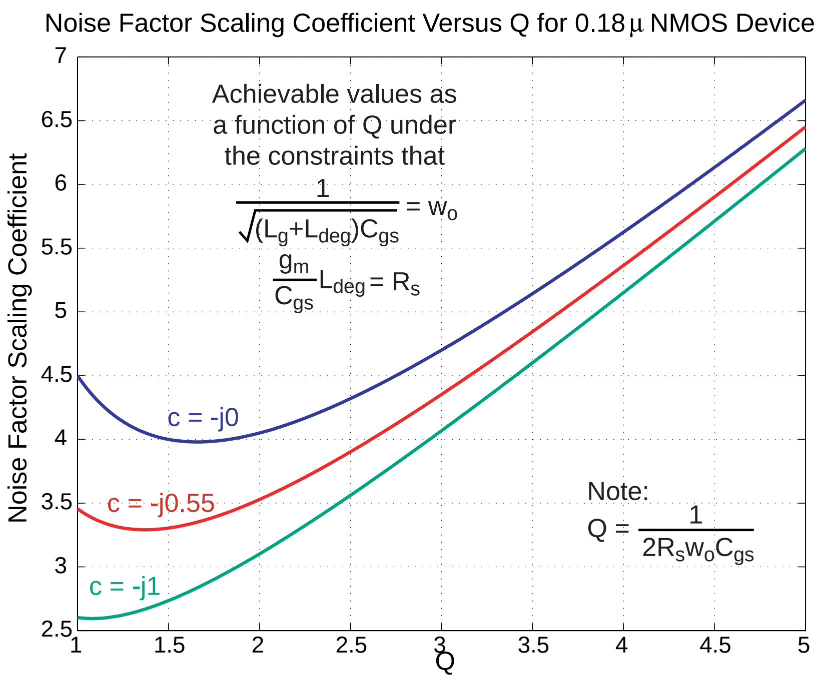
Noise Factor scaling coefficient

- **Non degenerated (From Lecture 10)**

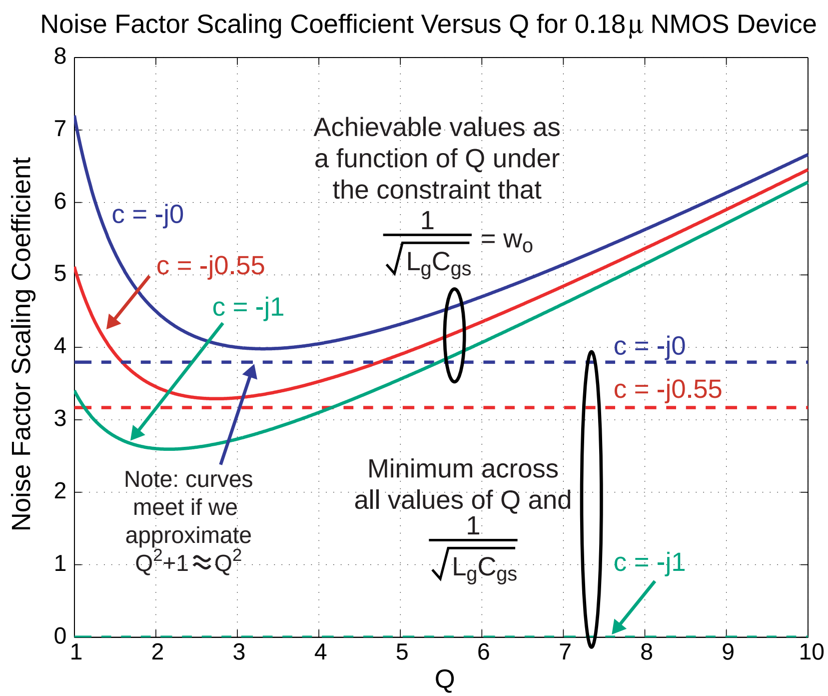
$$\text{Noise Factor} = 1 + \left(\frac{w_o}{w_t}\right)\gamma\left(\frac{g_{do}}{g_m}\right)\frac{1}{Q}\left(1 - 2|c|\chi_d + (Q^2+1)\chi_d^2\right)$$

Noise Factor scaling coefficient

Noise Factor Scaling Coefficient Comparison



With degeneration



Without degeneration

Achievable Noise Figure in 0.18μ with Power Match

- **Suppose we desire to build a narrowband LNA with center frequency of 1.8 GHz in 0.18µ CMOS (c=-j0.55)**
  - **From Hspice – at V_{gs} = 1 V with NMOS (W=1.8µ, L=0.18µ)**
    - measured g_m =871 µS, C_{gs} = 2.9 fF

$$\Rightarrow \quad w_t \approx \frac{g_m}{C_{gs}} = \frac{871 \times 10^{-6}}{2.9 \times 10^{-15}} = 2\pi(47.8GHz)$$

$$\Rightarrow \quad \frac{w_o}{w_t} = \frac{2\pi 1.8e9}{2\pi 47.8e9} \approx \frac{1}{26.6}$$

  - **Looking at previous curve, with Q ≈ 1.4 we achieve a Noise Factor scaling coefficient ≈ 3.25**

$$\Rightarrow \quad \text{Noise Factor} \approx 1 + \frac{1}{26.6} 3.25 \approx 1.12$$

$$\Rightarrow \quad \text{Noise Figure} = 10\log(1.13) \approx 0.49\ dB$$

Component Values for Minimum NF with Power Match

- Assume R_s = 50 Ohms, Q = 1.4, f_o = 1.8 GHz, f_t = 47.8 GHz
  - C_{gs} calculated as

$$Q = \frac{1}{2R_s w_o C_{gs}}$$

$$\Rightarrow C_{gs} = \frac{1}{2R_s w_o Q} = \frac{1}{2(50)2\pi 1.8e9(1.4)} = \boxed{631fF}$$

  - L_{deg} calculated as

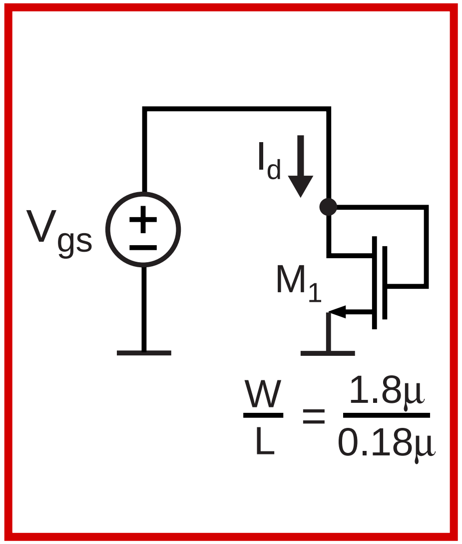
$$\frac{g_m}{C_{gs}} L_{deg} = R_s \quad \Rightarrow \quad L_{deg} = \frac{R_s}{w_t} = \frac{50}{2\pi 47.8e9} = \boxed{0.17nH}$$

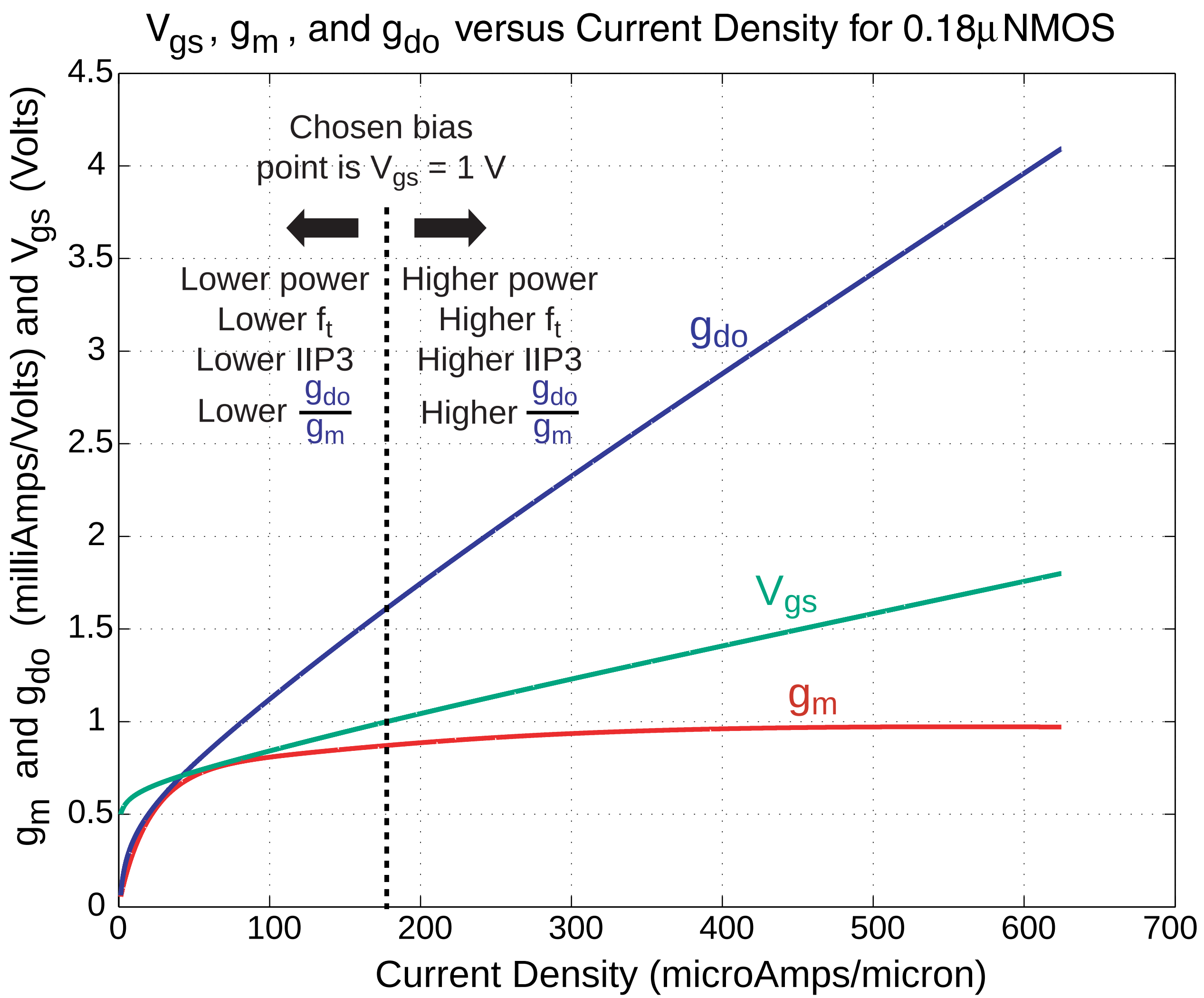
  - L_g calculated as

$$\frac{1}{\sqrt{(L_g + L_{deg})C_{gs}}} = w_o \Rightarrow L_g = \frac{1}{w_o^2 C_{gs}} - L_{deg}$$

$$\Rightarrow L_g = \frac{1}{(2\pi 1.8e9)^2 631e\text{-}15} - 0.17e\text{-}9 = \boxed{12.2nH}$$

Have We Chosen a Good Bias Point? (V_{gs} = 1V)





- **Note: IIP3 is also a function of Q**

Calculation of Bias Current for Example Design

- **Calculate current density from previous plot**

$$V_{gs} = 1V \;\Rightarrow\; I_{dens} \approx 175\mu A/\mu m$$

- **Calculate W from HSpice simulation (assume L=0.18 μm)**

$$C_{gs} = 2.9fF \text{ for } W = 1.8\mu m \;\Rightarrow\; W = \frac{631fF}{2.9fF}1.8\mu m \approx 392\mu m$$

  - **Could also compute this based on C_{ox} value**

- **Calculate bias current**

$$I_{bias} = I_{den}W = (175\mu A/\mu m)(392\mu m) \approx \boxed{69mA}$$

  - **Problem: this is not low power!!**