Linear Programming

Lots of parameters, "few" constraints
does not apply to our problem (2,3 parameters, 250,000 constraints)

Perceptron "Learning" Algorithm

$\rightarrow \underline{t}^{(n)} \cdot \bar{\underline{s}}_n \geq 0 ? \longrightarrow \boxed{\text{YES}}$ $n{+}{+}$

$\downarrow$

$\boxed{\text{NO}}$

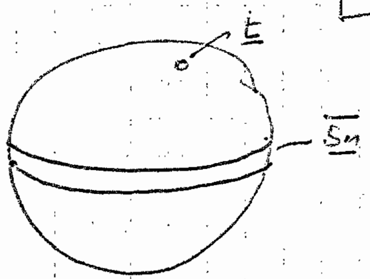
$k . \bar{\underline{s}}_n$ is a good choice

(i) $(\underline{t}^{(n)} + \delta t^{(n)}) \cdot \bar{\underline{s}}_n = 0$

(ii) $\min \| \delta t^{(n)} \|$

(iii) $t^{(n+1)} = t^{(n)} - \dfrac{\underline{t}^{(n)} \cdot \bar{\underline{s}}_n}{\bar{\underline{s}}_n \cdot \bar{\underline{s}}_n} \bar{s}_n$

If there is a solution, it converges.



Ambiguity in the general case

$\underline{t}, \underline{\omega}, Z(x,y)$
$\underline{t}_2, \underline{\omega}_2, Z_2(x,y)$
same motion field ?

$$\dot{\underset{\sim}{r}} = \underbrace{[\underline{r}\ \underline{\omega}\ \hat{\underline{z}}]\,\underline{r} - \underline{r} \times \underline{\omega}}_{\text{rotational}} + \underbrace{\frac{1}{Z}\left((\underline{t} \cdot \hat{\underline{z}})\underline{r} - \underline{t}\right)}_{\text{translational}}$$

$\underline{\delta\omega} = \underline{\omega}_2 - \underline{\omega}_1$:

$$\frac{1}{z_1}\left((\underline{t}_1 \cdot \hat{\underline{z}})\underline{r} - \underline{t}_1\right) - \frac{1}{z_2}\left((\underline{t}_2 \cdot \hat{\underline{z}})\underline{r} - \underline{t}_2\right) = [\underline{r}\ \underline{\delta\omega}\ \underline{z}]\,\underline{r} - \underline{r} \times \underline{\delta\omega}$$

multiply by : $\underline{t}_2 \times \underline{r}$: $\frac{1}{z_1}[\underline{t}_2\ \underline{t}_1\ \underline{r}] + (\underline{r} \times \underline{\delta\omega}) \cdot (\underline{t}_2 \times \underline{r}) = 0$

idem for $\underline{t}_1 \times \underline{r}$

↳ z_1 as a function of $\underline{r}$ $\quad \underline{r} = (x, y, f)^T$ $\quad$ we want $Z(\underline{R})$

$\underline{R} = \frac{z_1}{f}\underline{r}$ $\quad$ thus : $(\underline{R} \times \underline{\delta\omega}) \cdot (\underline{t}_2 \times \underline{R}) + [\underline{t}_2\ \underline{t}_1\ \underline{R}] = 0$

$$(R \cdot t_2)(\delta\omega \cdot R) - (t_2 \cdot \delta\omega)(R \cdot R) + (t_2 \times t_1) \cdot R = 0 \qquad (*)$$

$t_2 \to k t_2 \quad \Rightarrow$ no change on z_1

$t_1 \to k t_1 \quad \Rightarrow z_1 \to k z_1$

$\underline{R} = 0$ is a solution (surface through COP) $\quad$ but there must be some others.

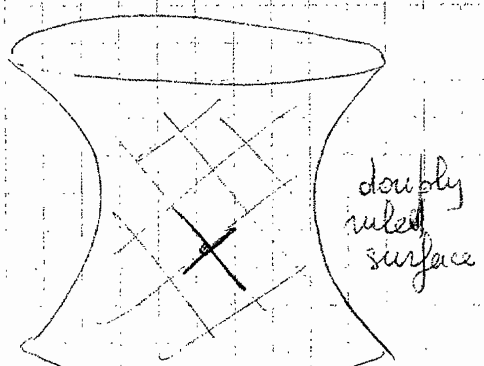
Quadric Surfaces

$$\pm\left(\frac{x'}{A}\right)^2 \pm \left(\frac{y'}{B}\right)^2 \pm \left(\frac{z'}{C}\right)^2 = 1 \qquad \text{(that's } (*) \text{ after trans \& rot)}$$

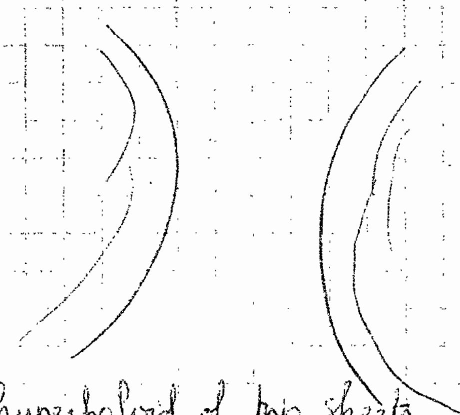
(removing first order & const term)

+	+	+	ellipsoid
+	+	−	hyperboloid of 1 sheet
+	−	−	hyperboloid of 2 sheets

— — — imaginary ellipsoid



hyperboloid of one sheet



hyperboloid of two sheets

Degenerate cases : elliptical cone

hyperbolic paraboloid

two intersecting planes!

"Critical surfaces"

Hyperboloid of one sheet