

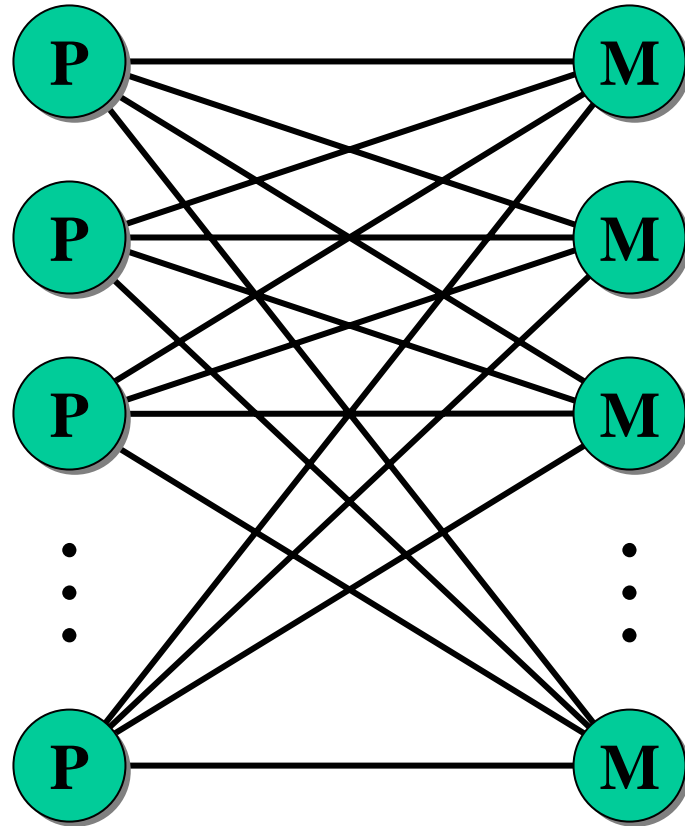
# Hypercubic Networks

Charles E. Leiserson

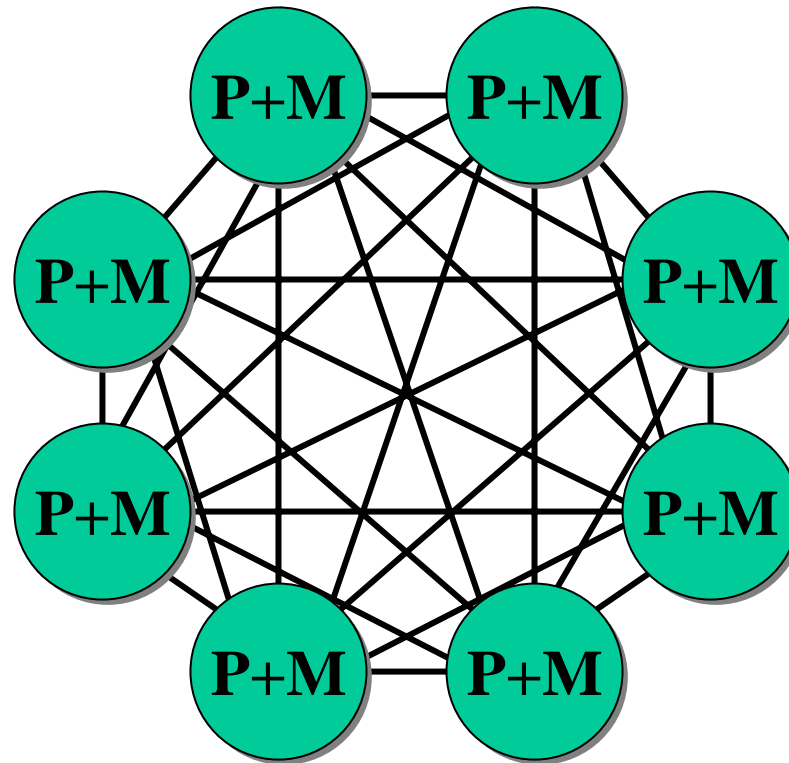
*6.895 Theory of Parallel Systems*

November 12, 2003

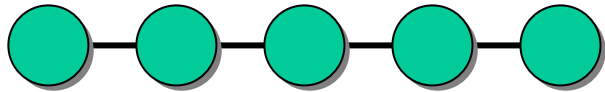
# “Ideal” Parallel Computer



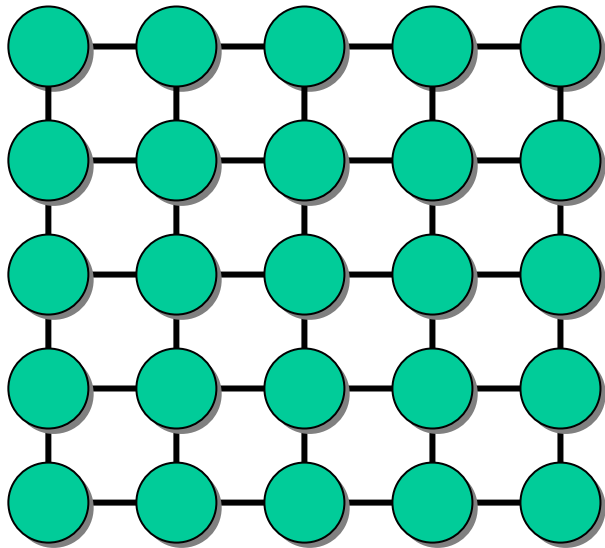
# “Ideal” Distributed-Memory Parallel Computer



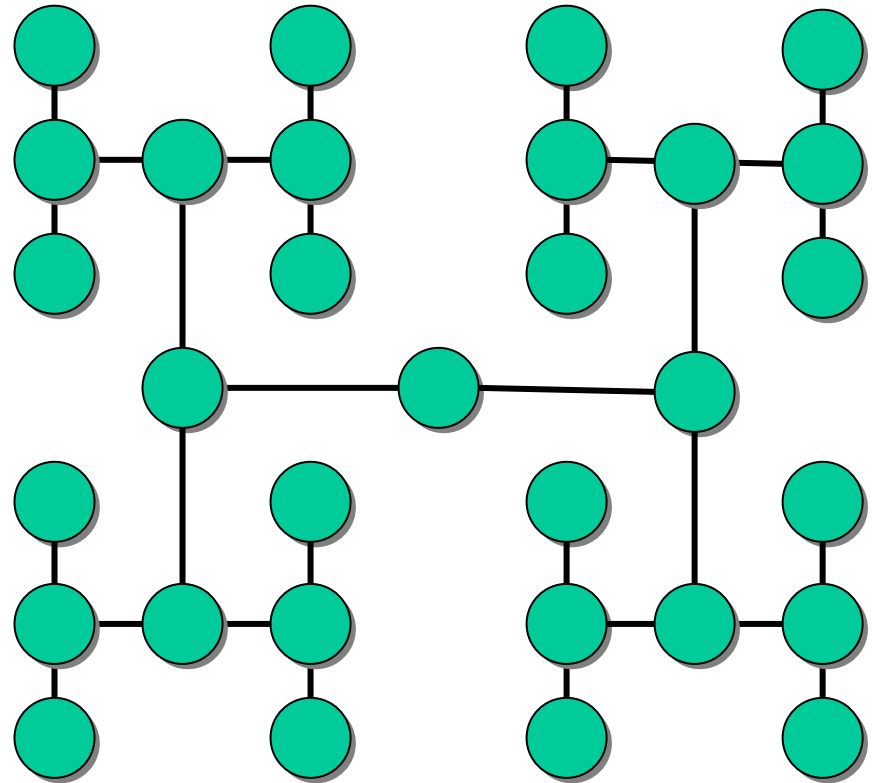
# Interconnection Networks



linear array



2D mesh



binary tree

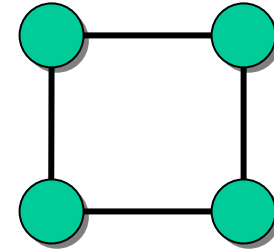
# Hypercube



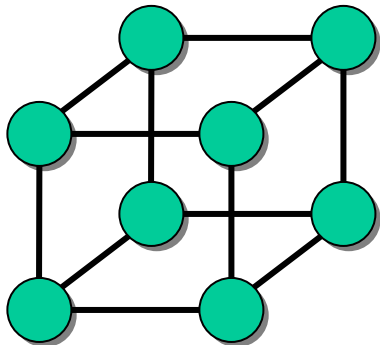
$d = 0$   
 $N = 1$



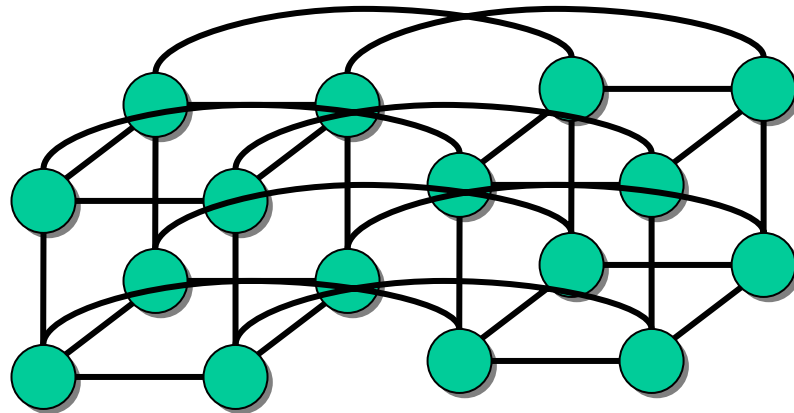
$d = 1$   
 $N = 2$



$d = 2$   
 $N = 4$

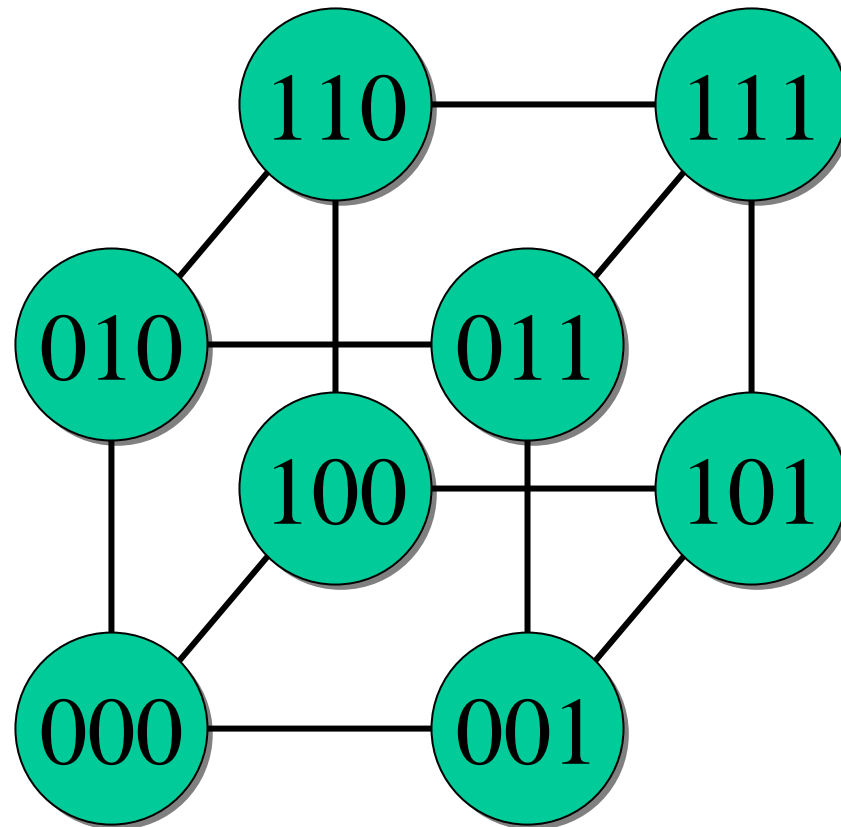


$d = 3$   
 $N = 8$

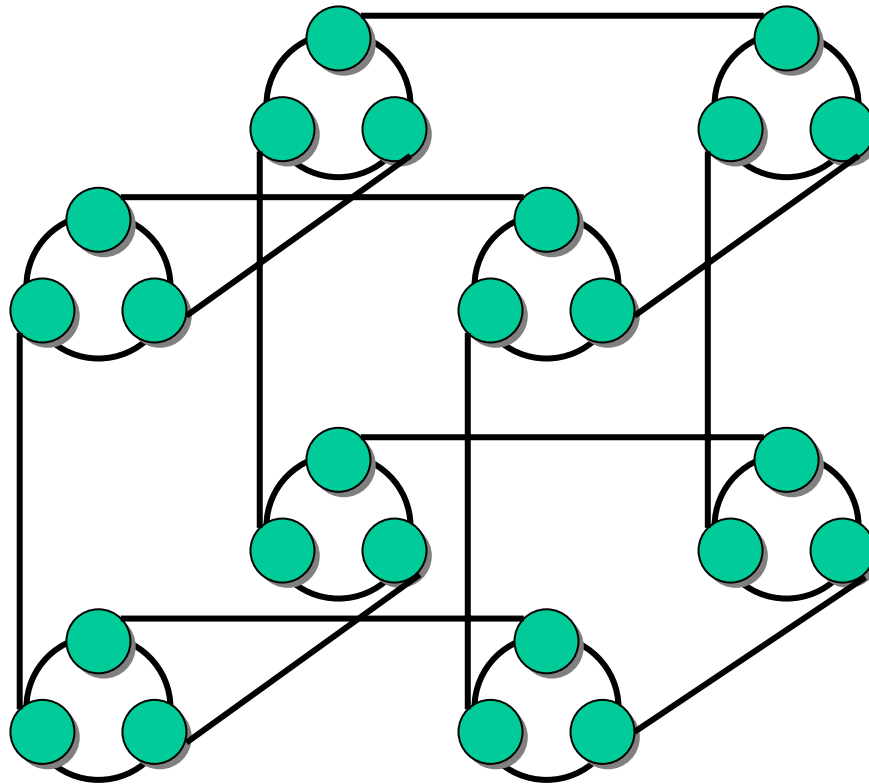


$d = 4$   
 $N = 16$

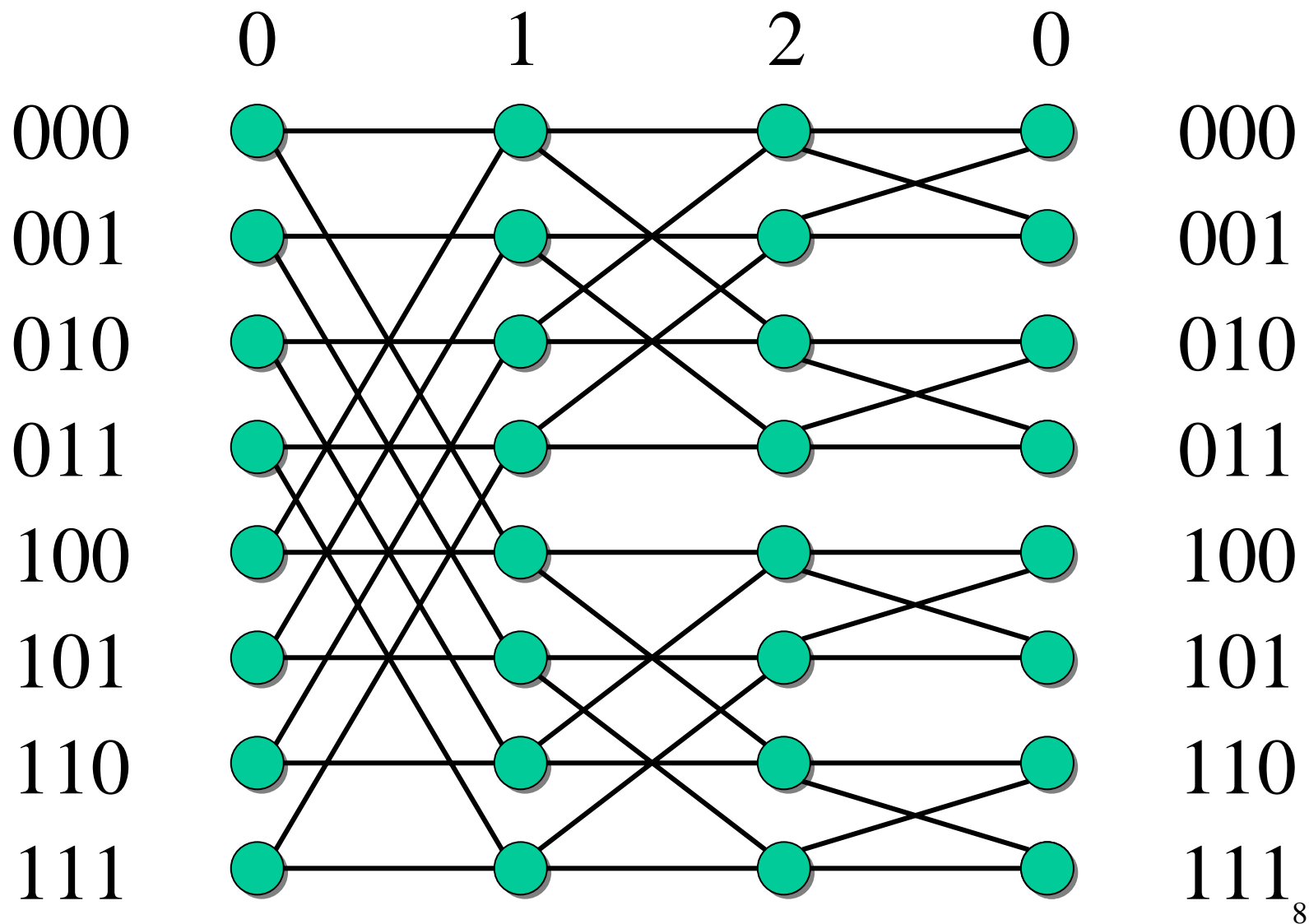
# Hypercube



# Cube-Connected Cycles

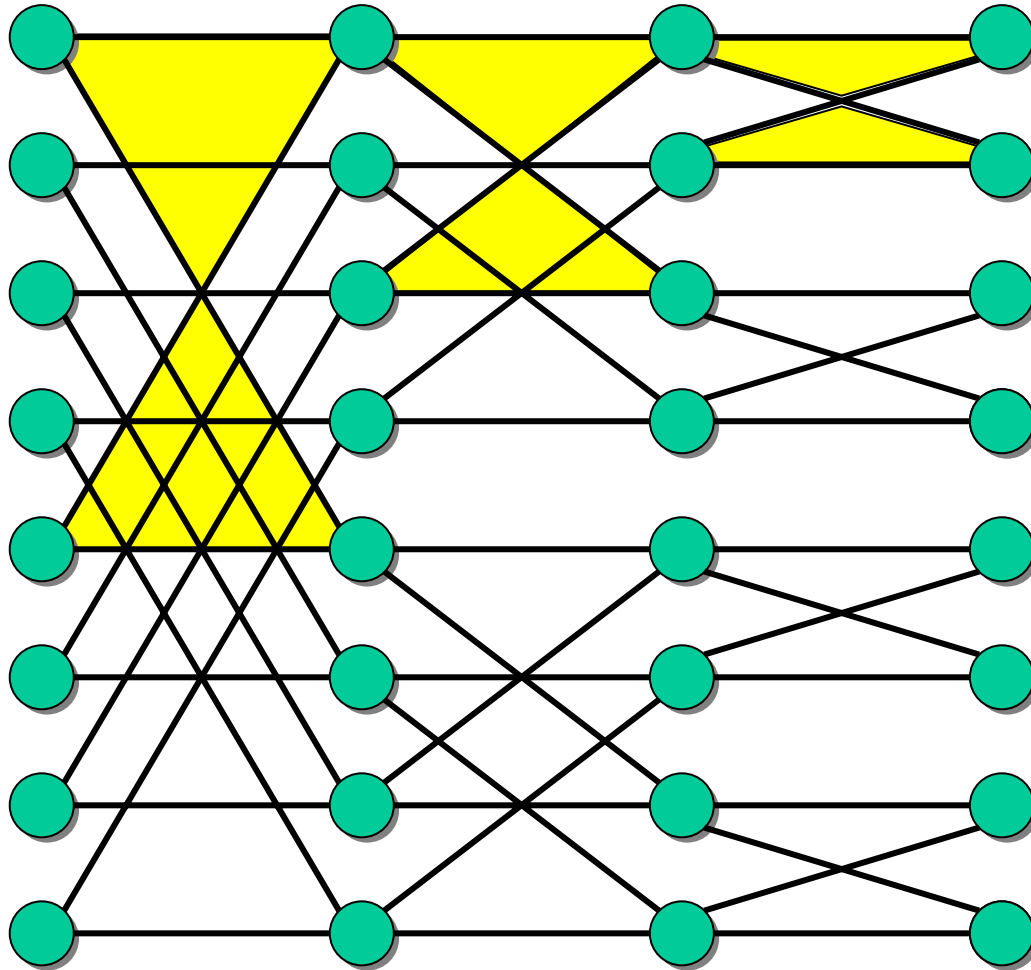


# Butterfly (FFT) Network

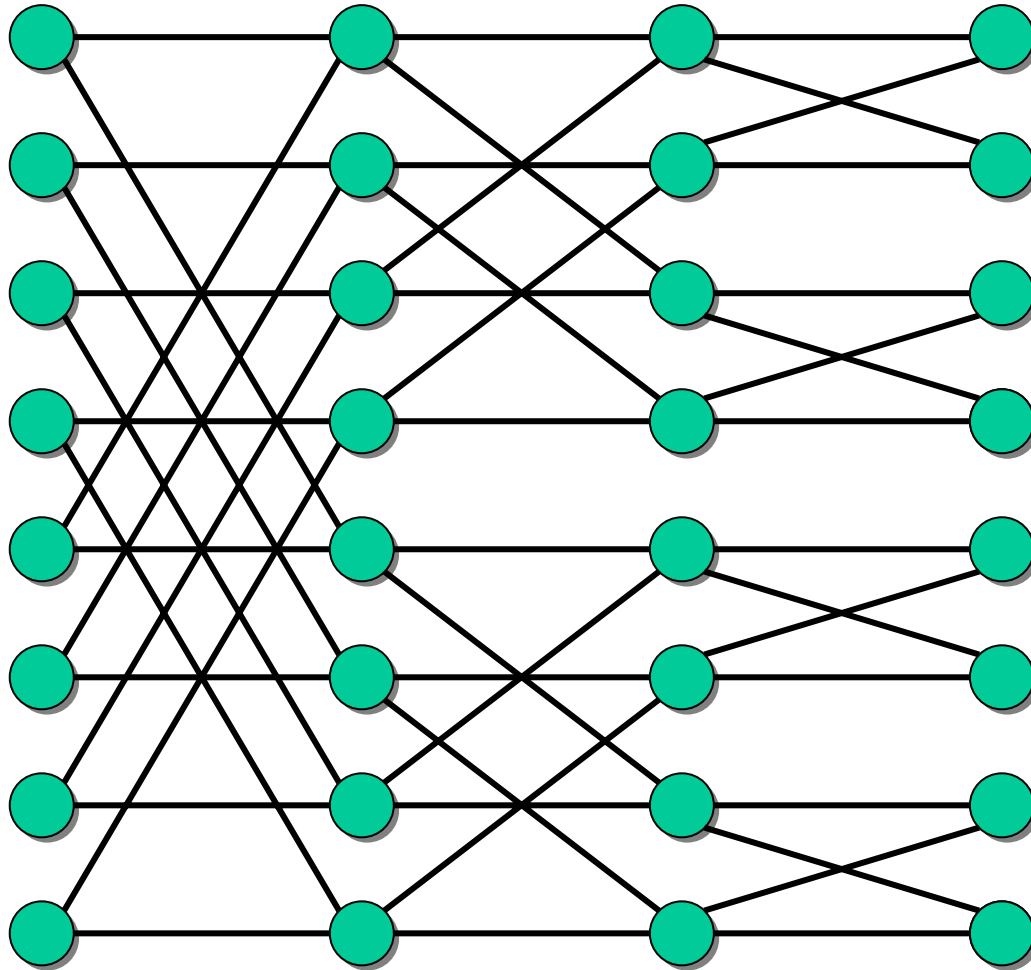




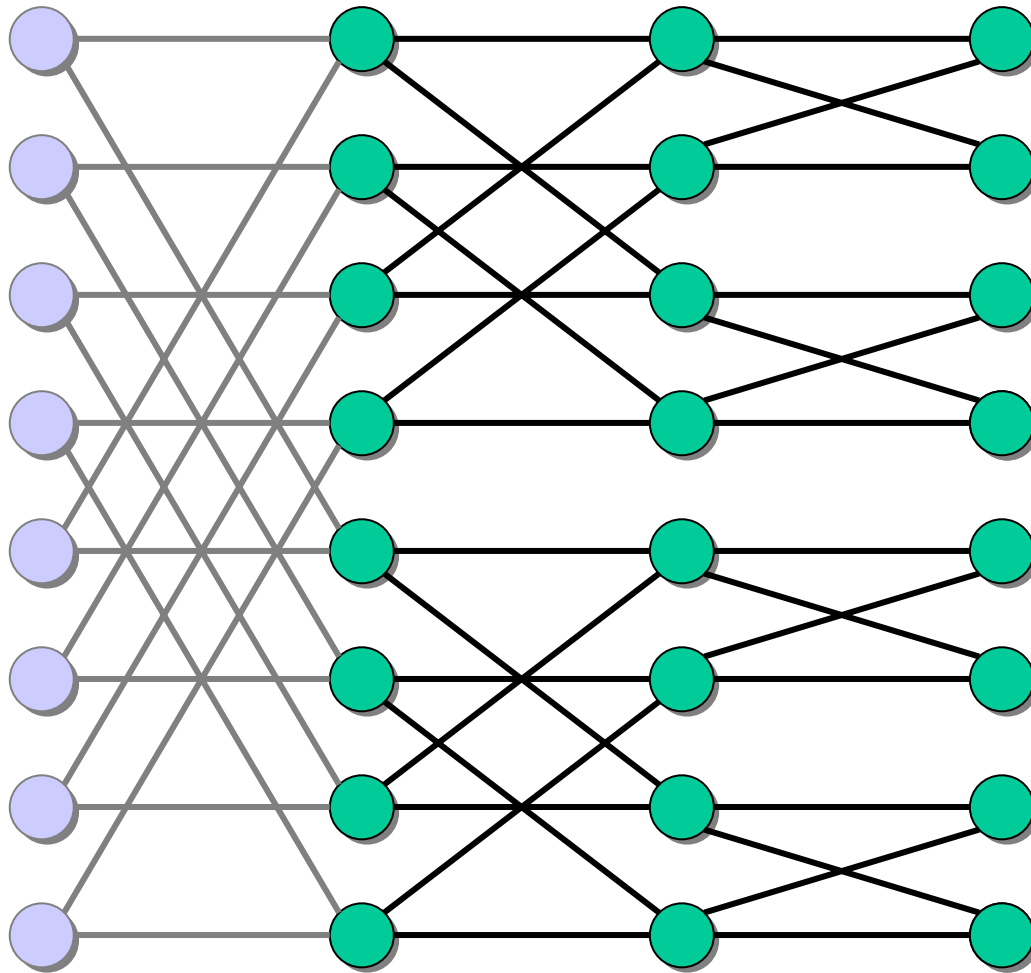
# Butterflies



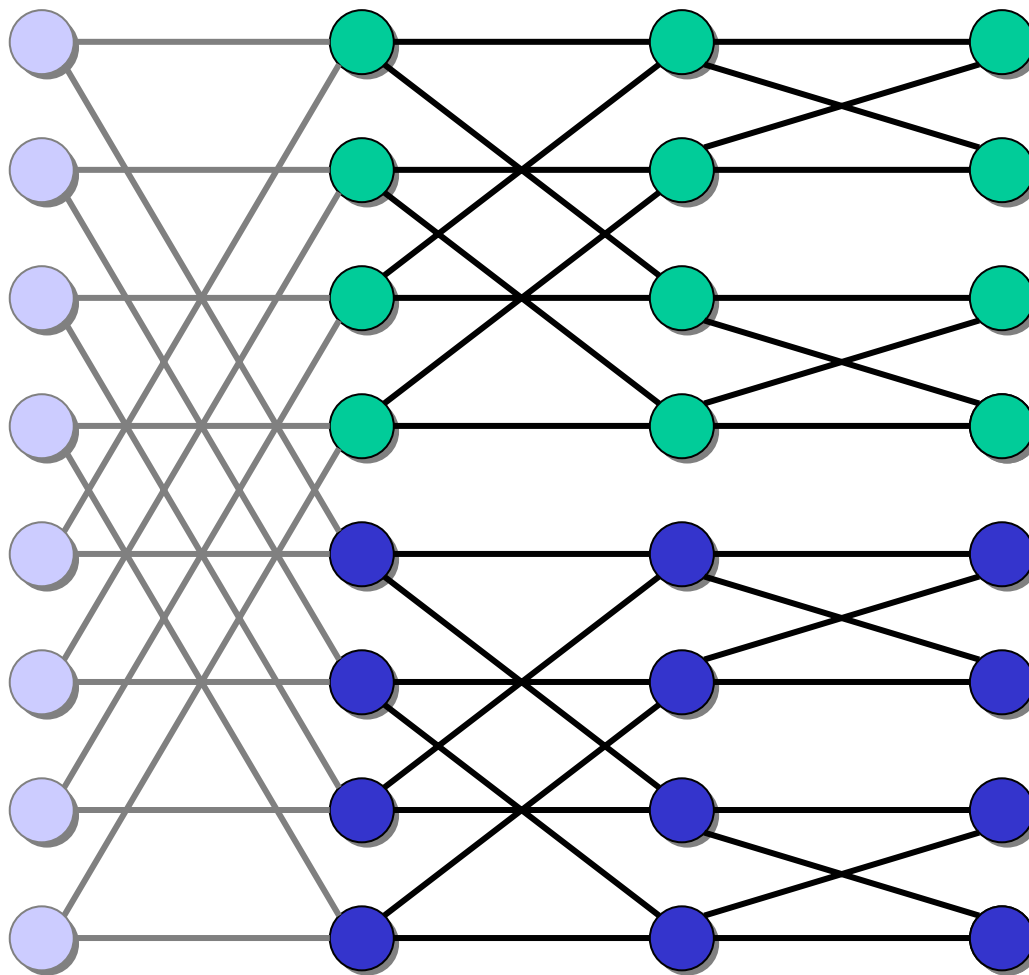
# Decomposing a Butterfly



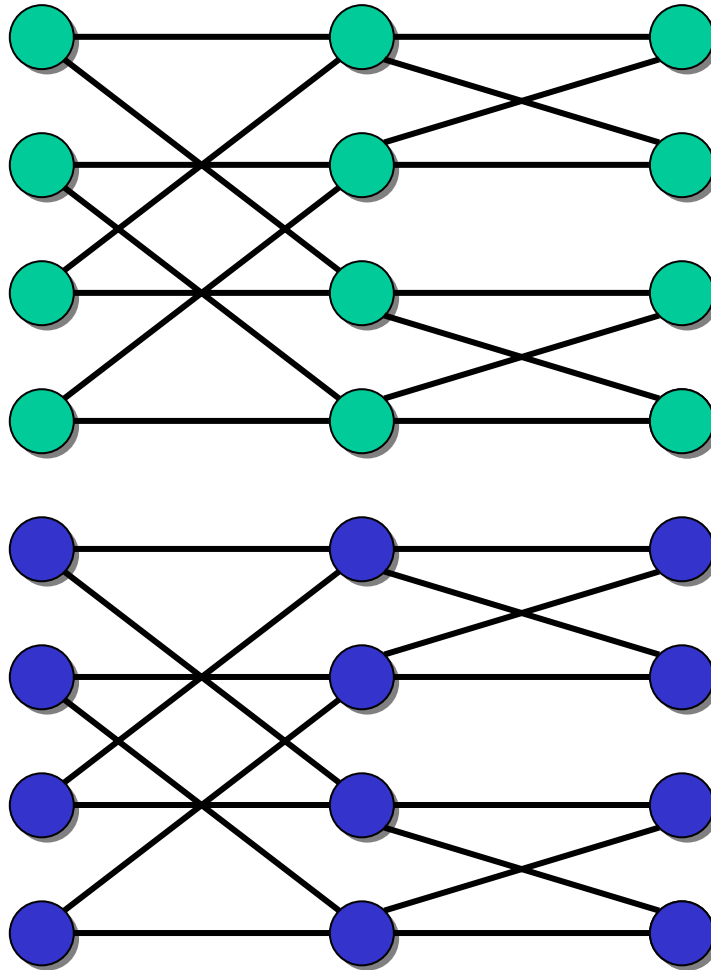
# Decomposing a Butterfly



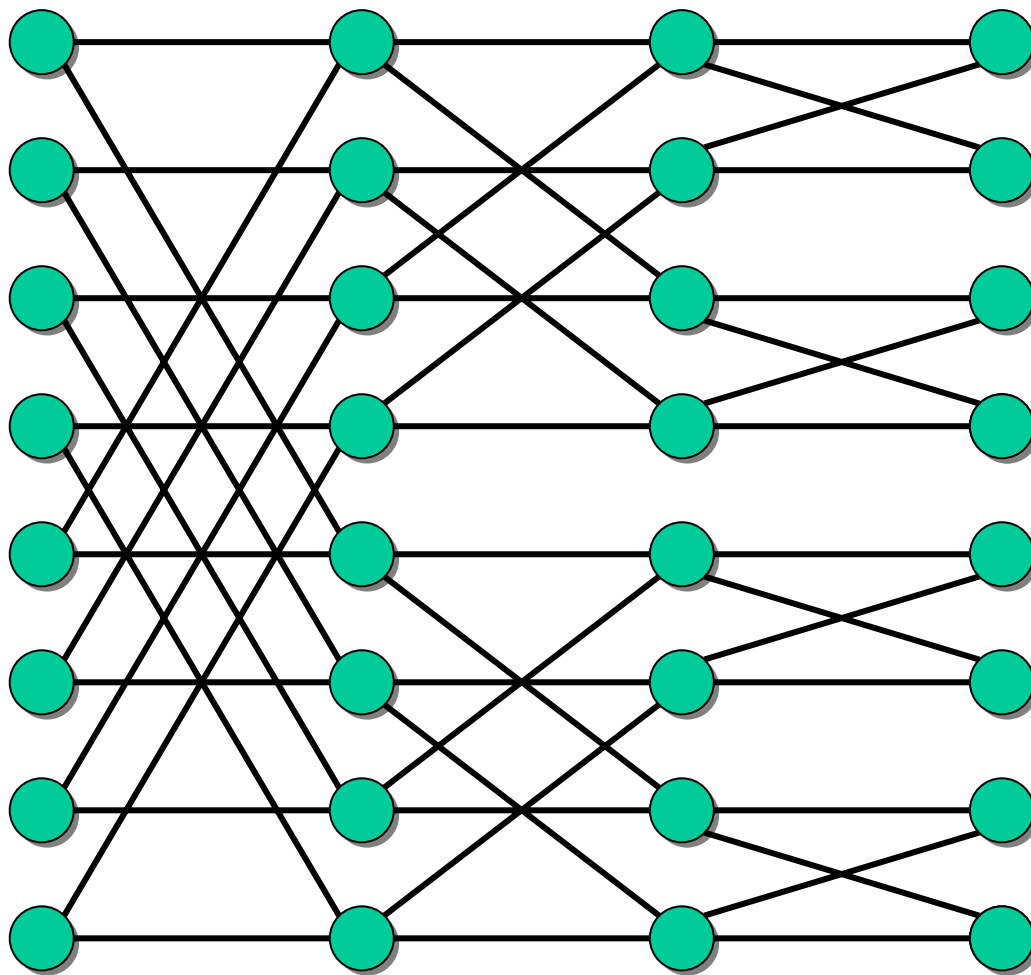
# Decomposing a Butterfly



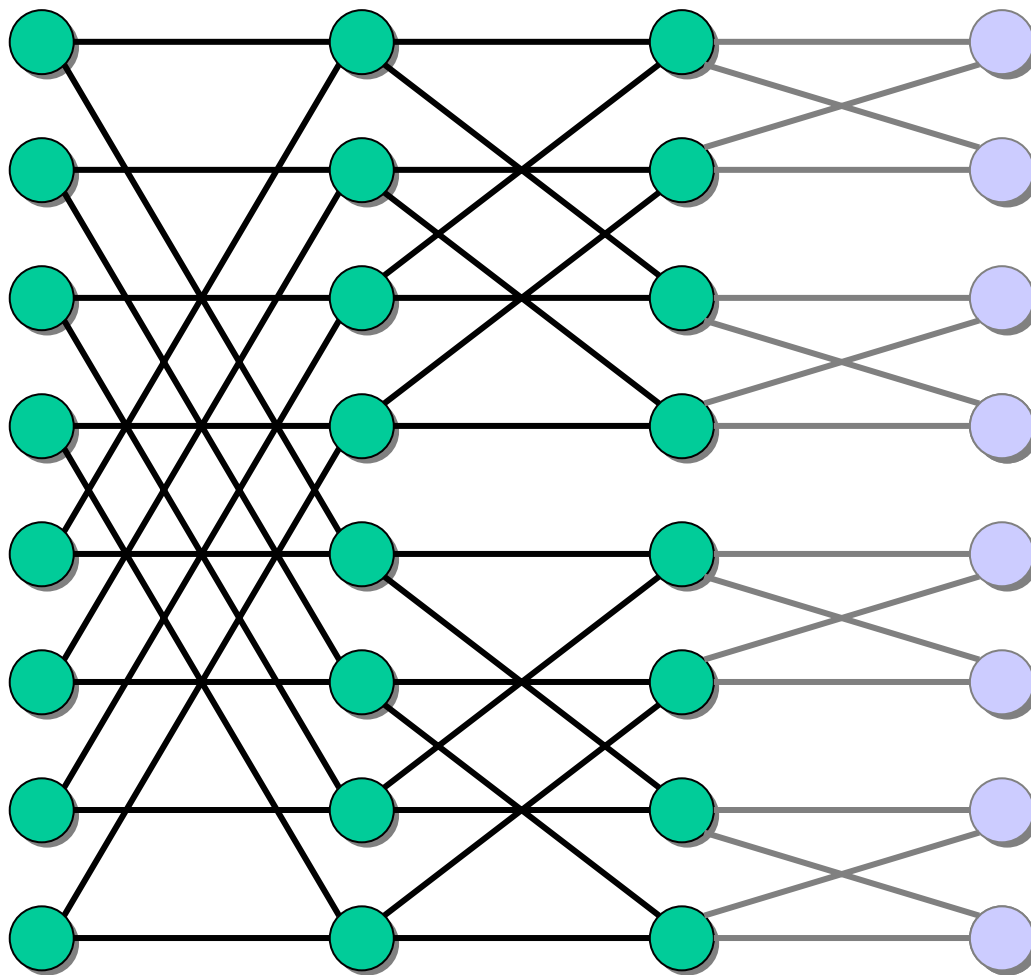
# Decomposing a Butterfly



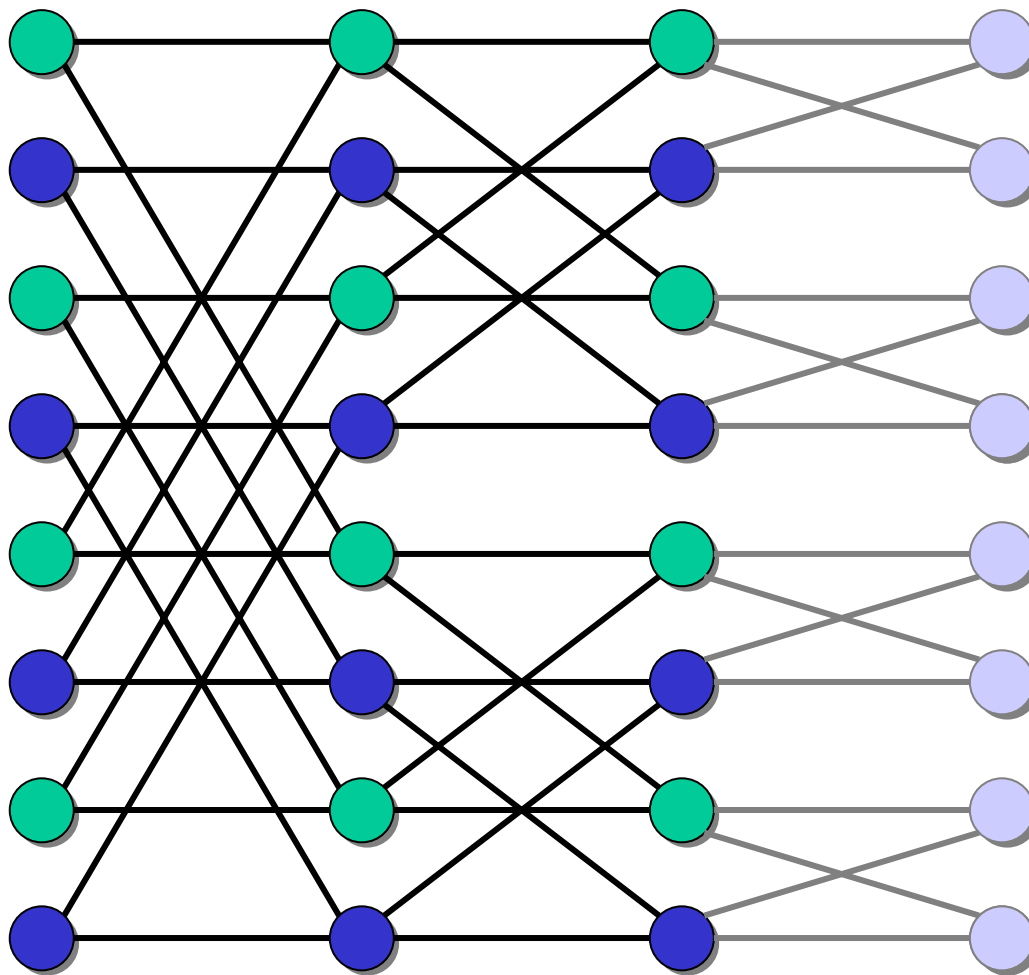
# Decomposing a Butterfly II



# Decomposing a Butterfly II

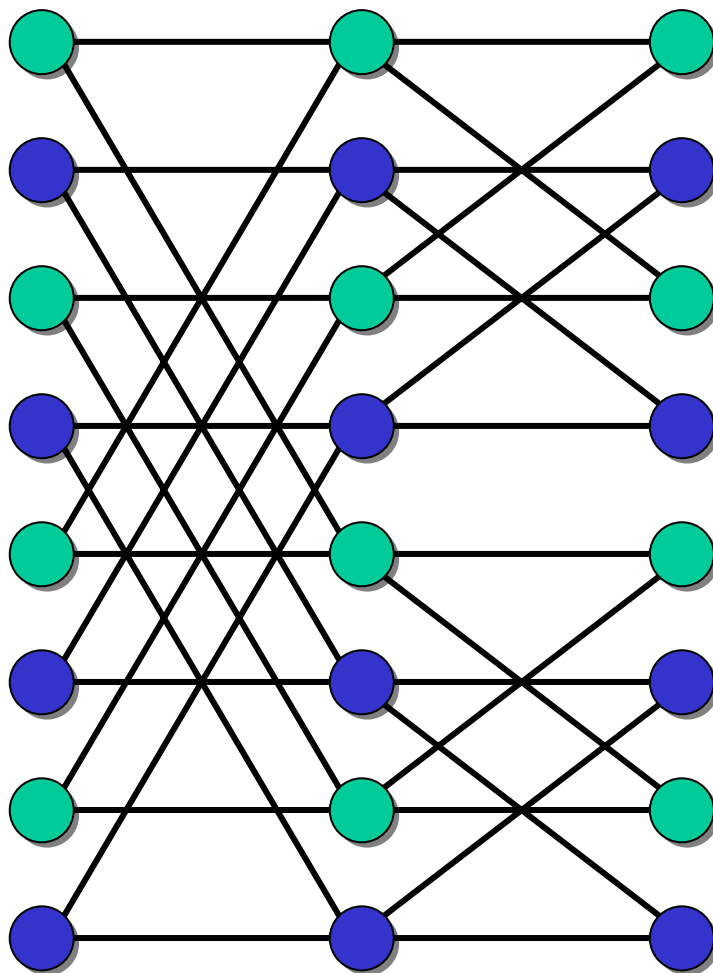


# Decomposing a Butterfly II

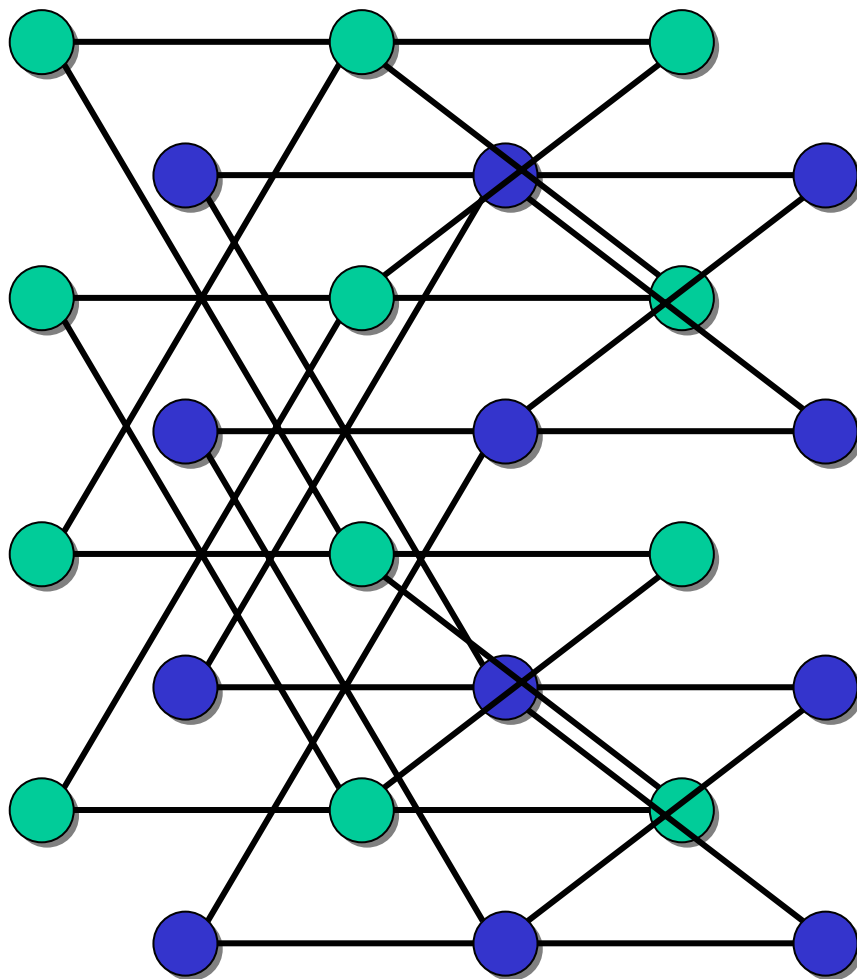




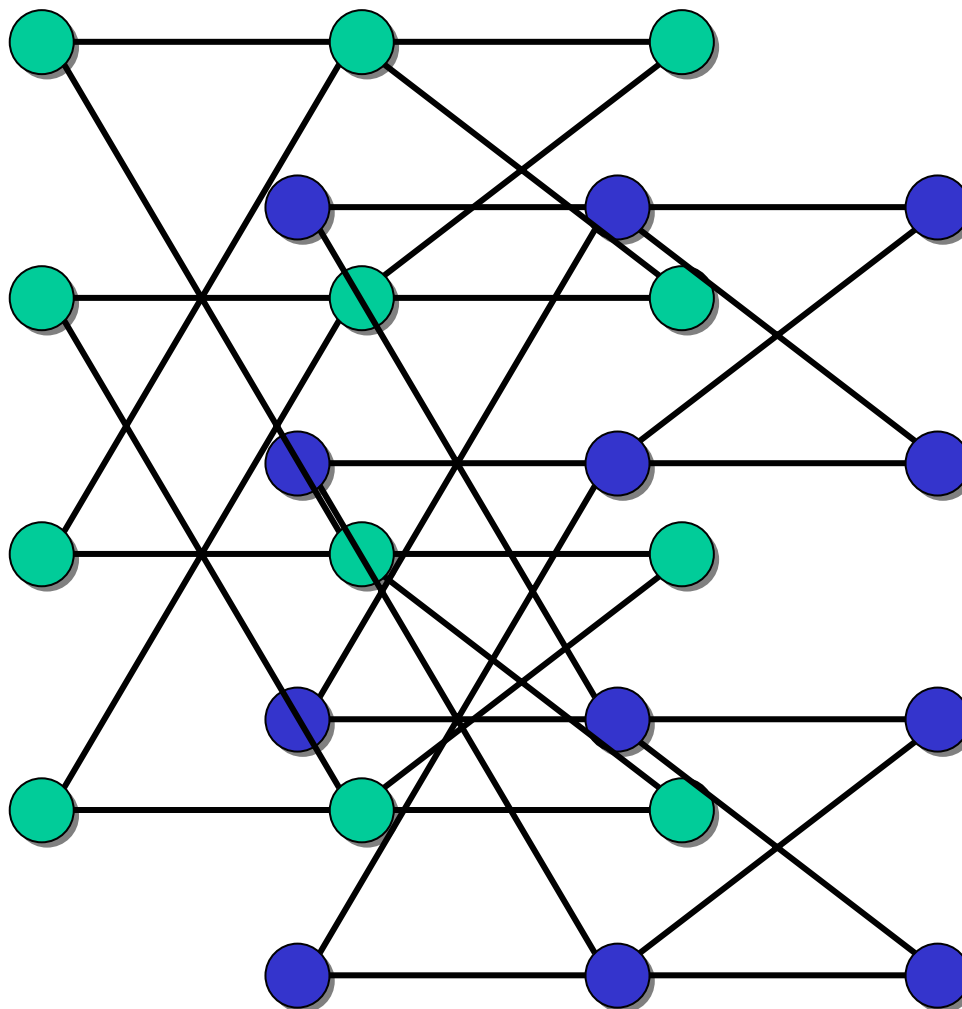
# Decomposing a Butterfly II



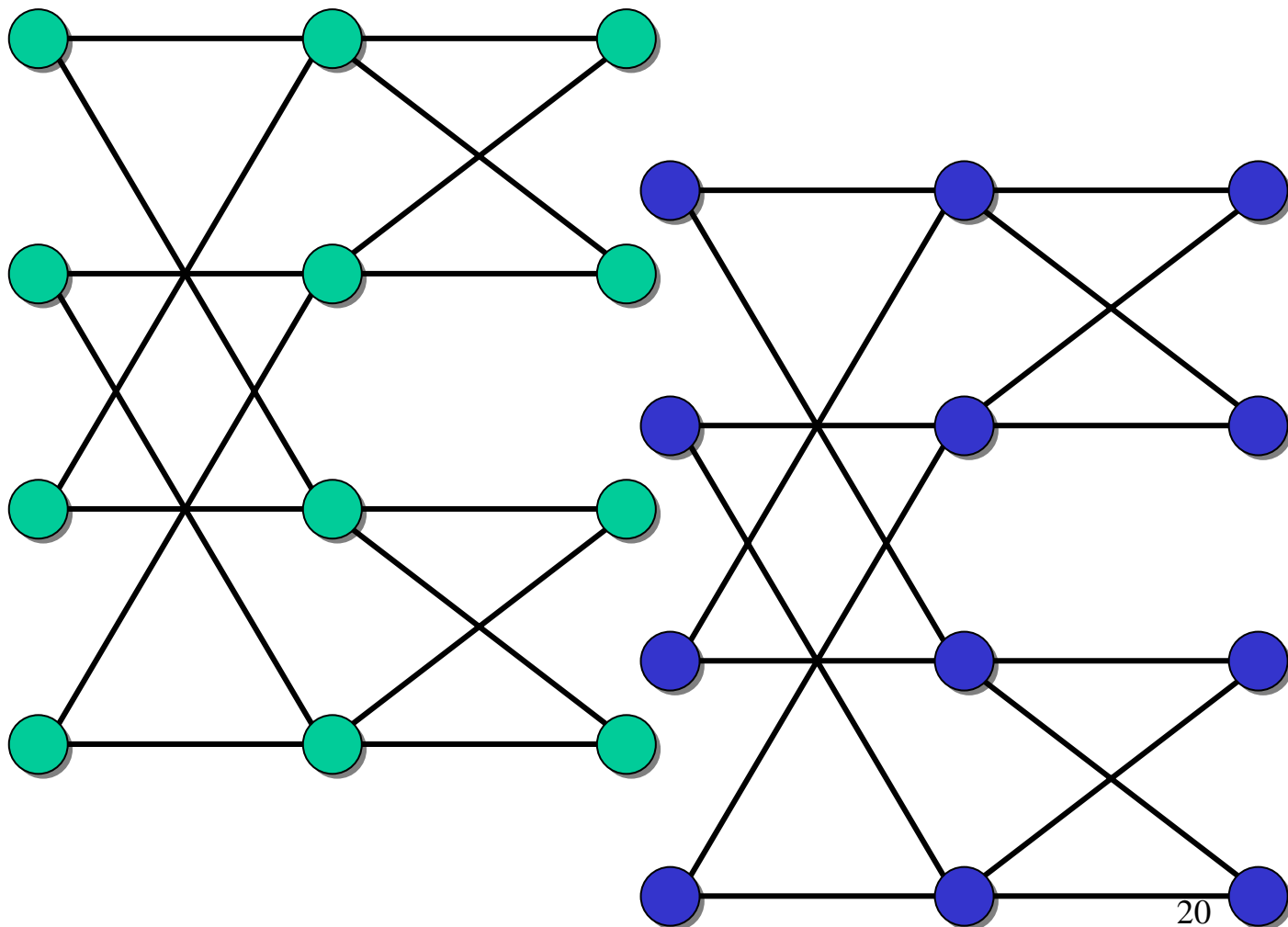
# Decomposing a Butterfly II



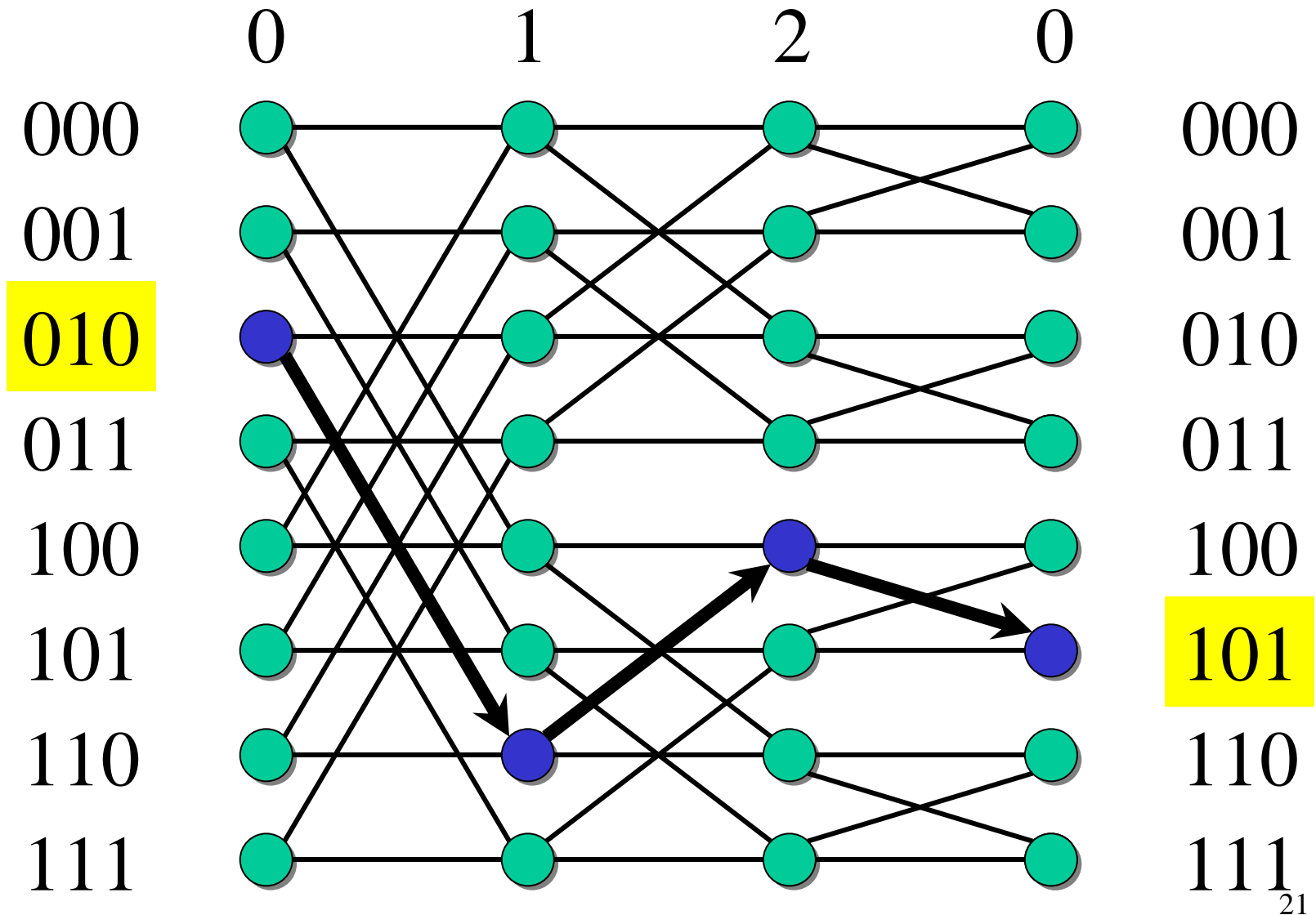
# Decomposing a Butterfly II



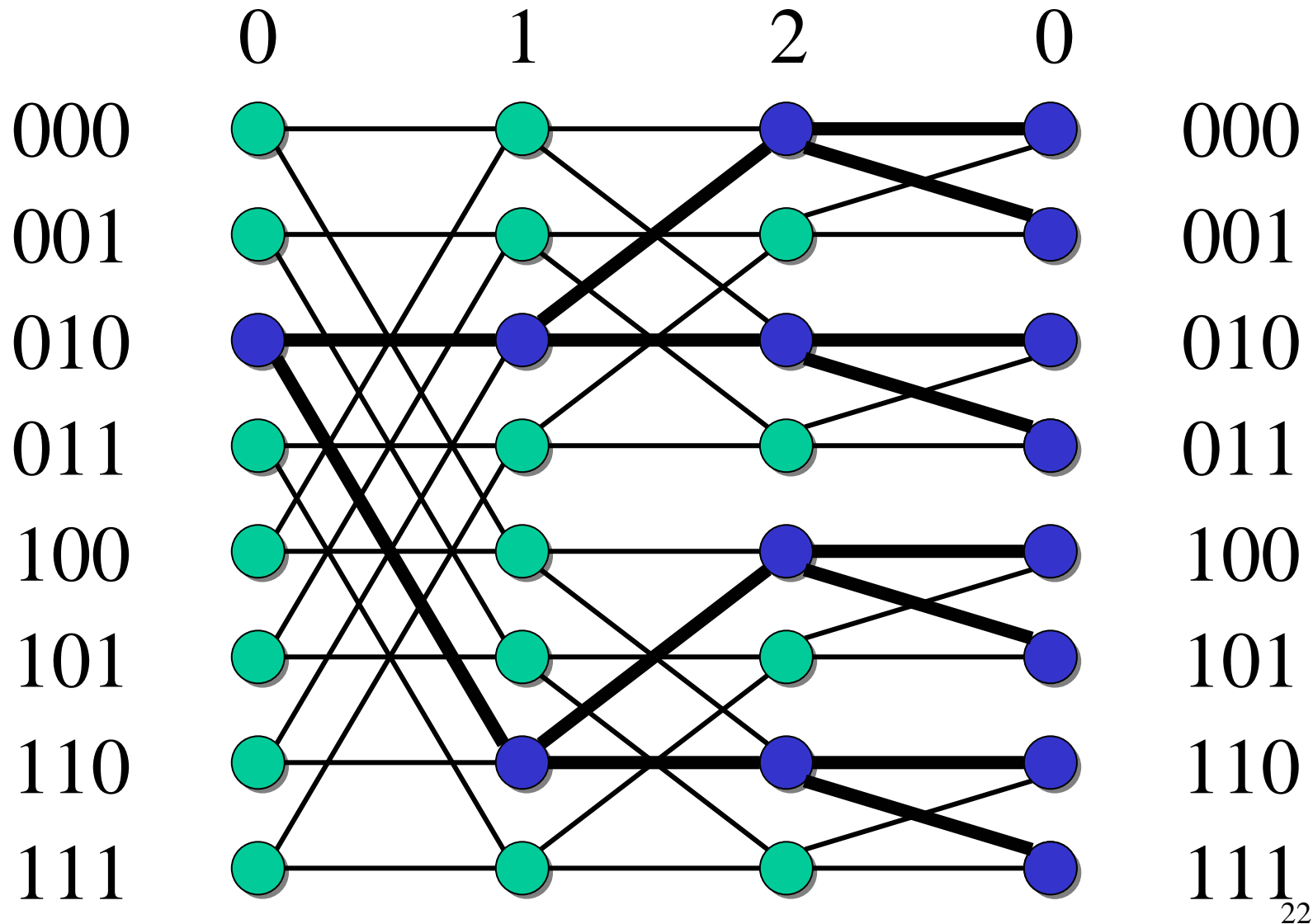
# Decomposing a Butterfly II



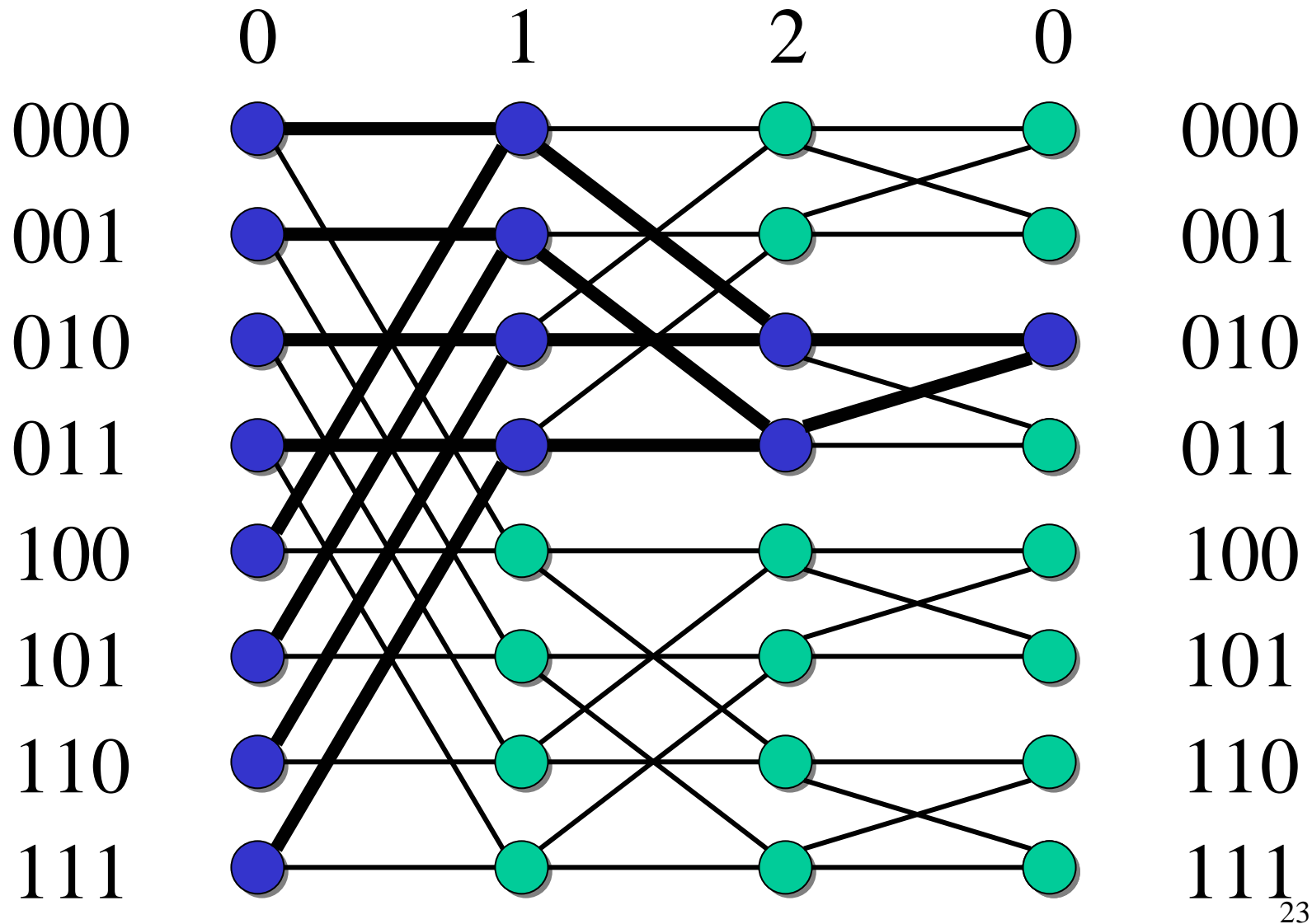
# Routing on a Butterfly



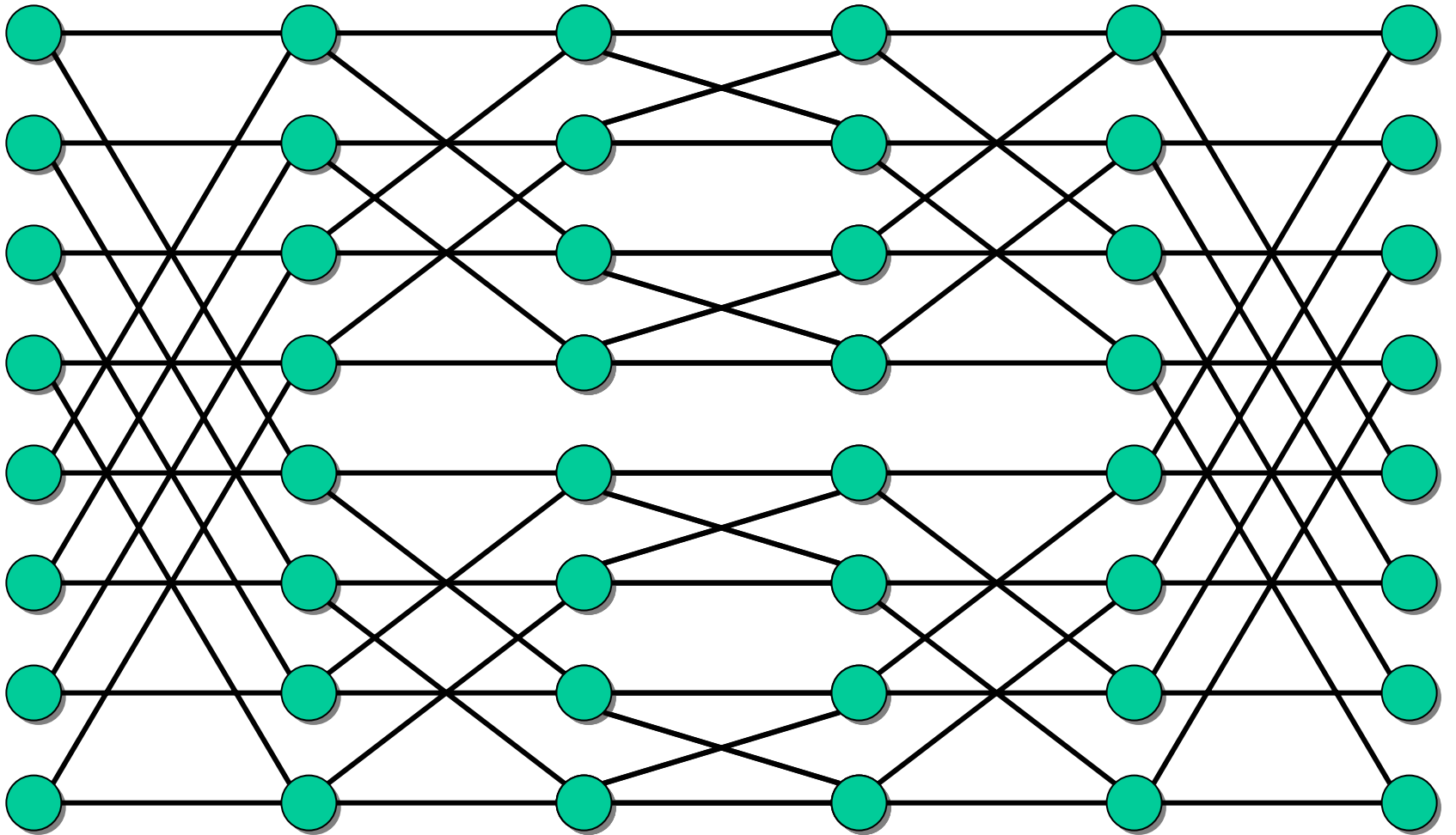
# Tree in Butterfly



# Tree in Butterfly

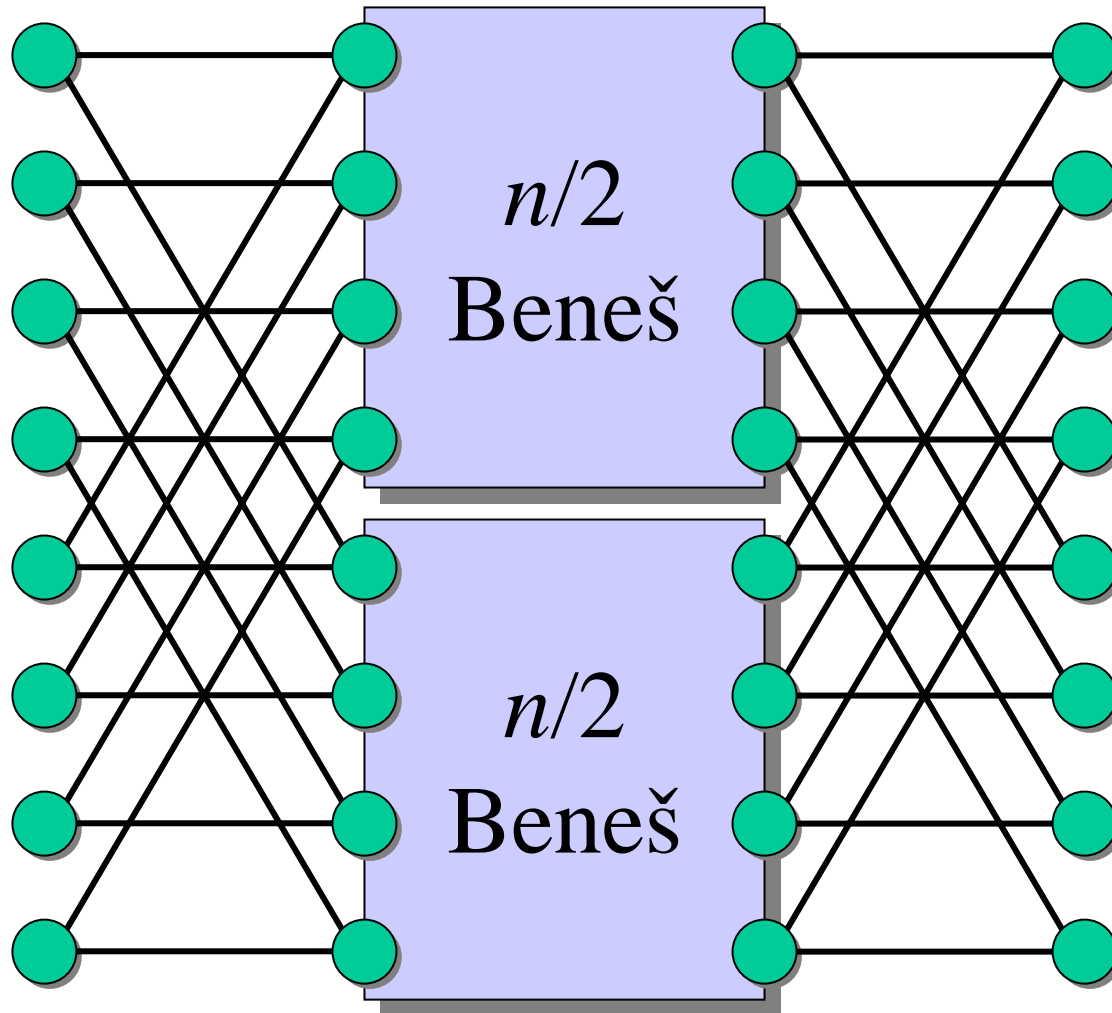


# Beneš Network

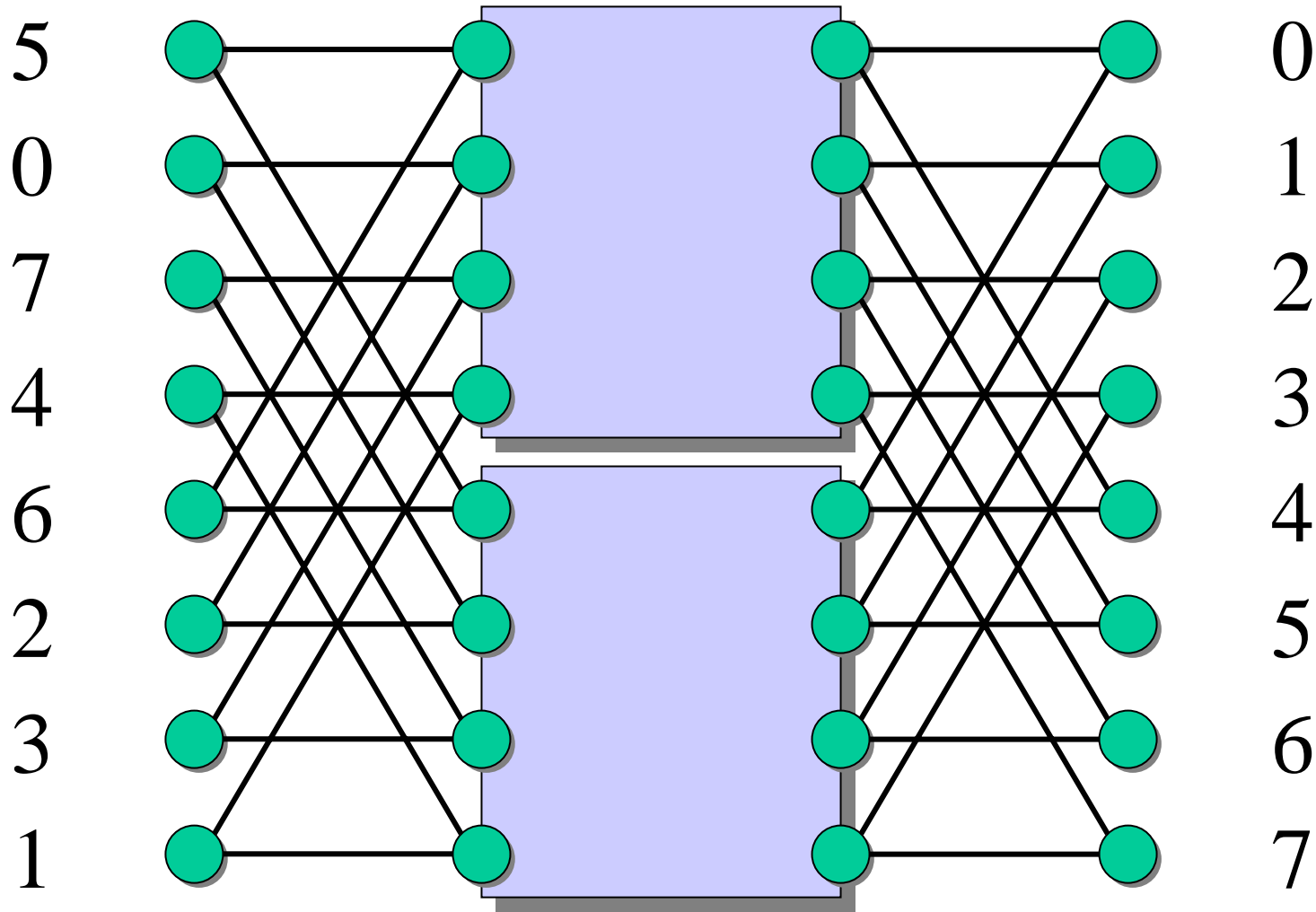




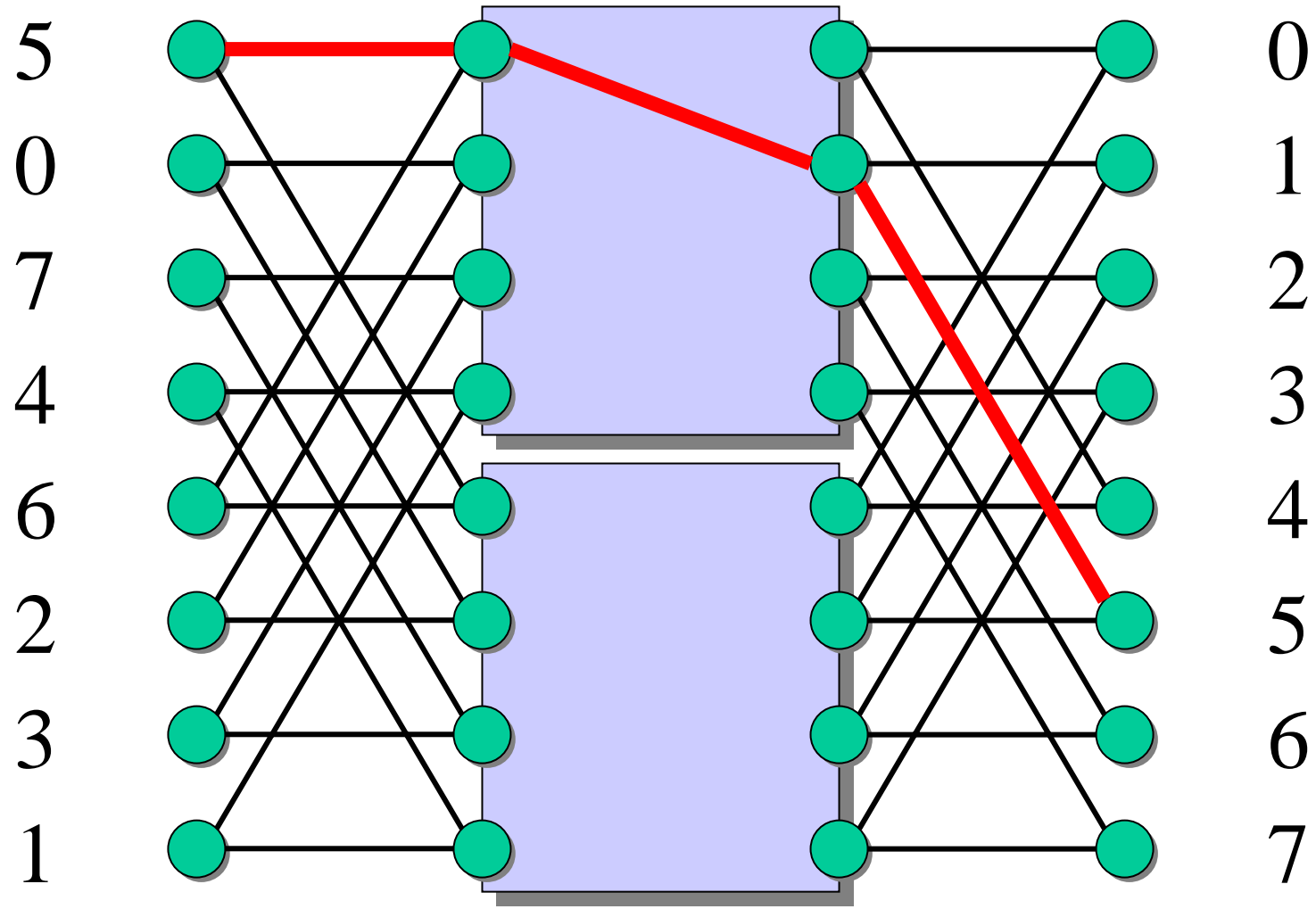
# Decomposing a Beneš Network



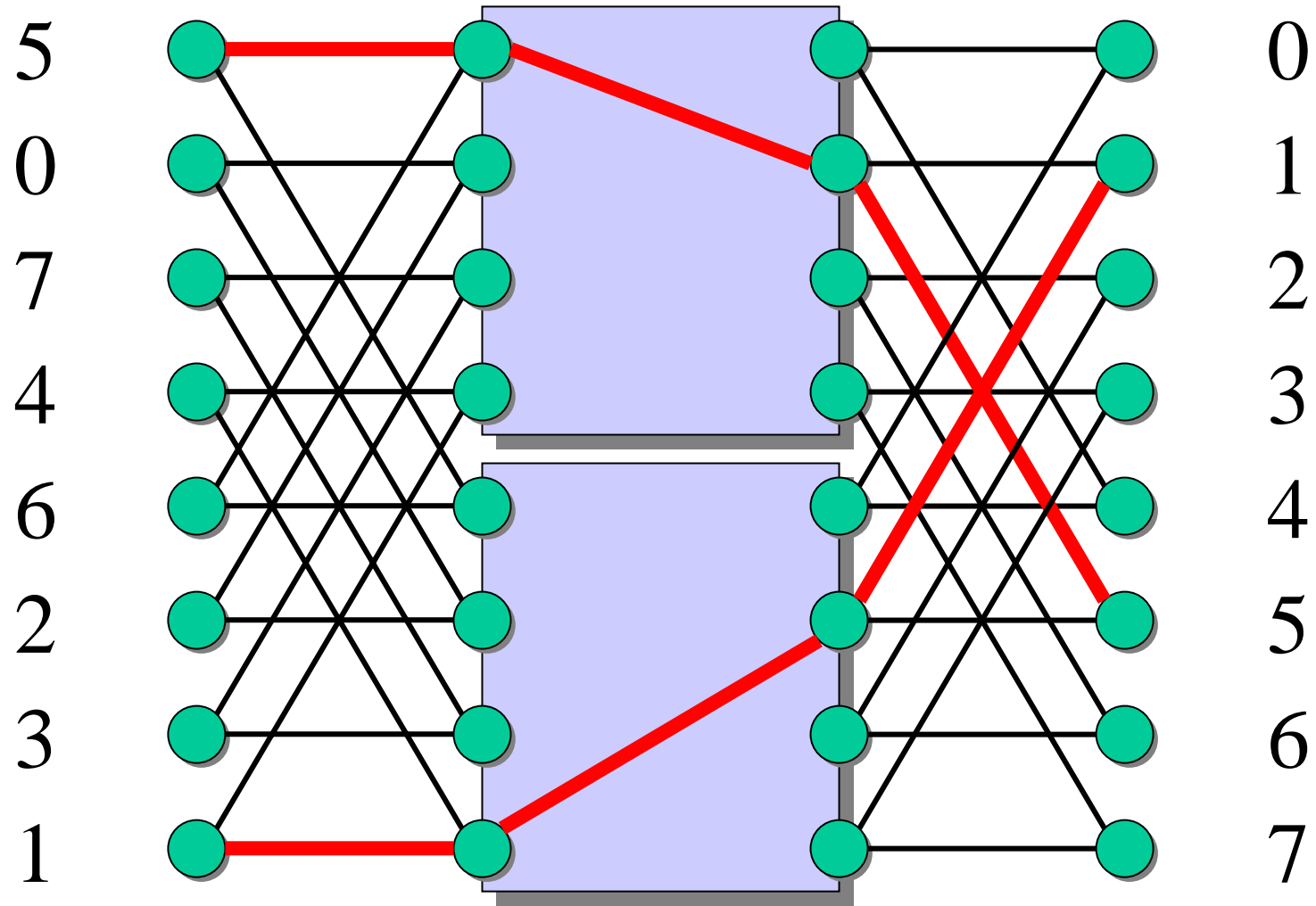
# Routing on a Beneš Network



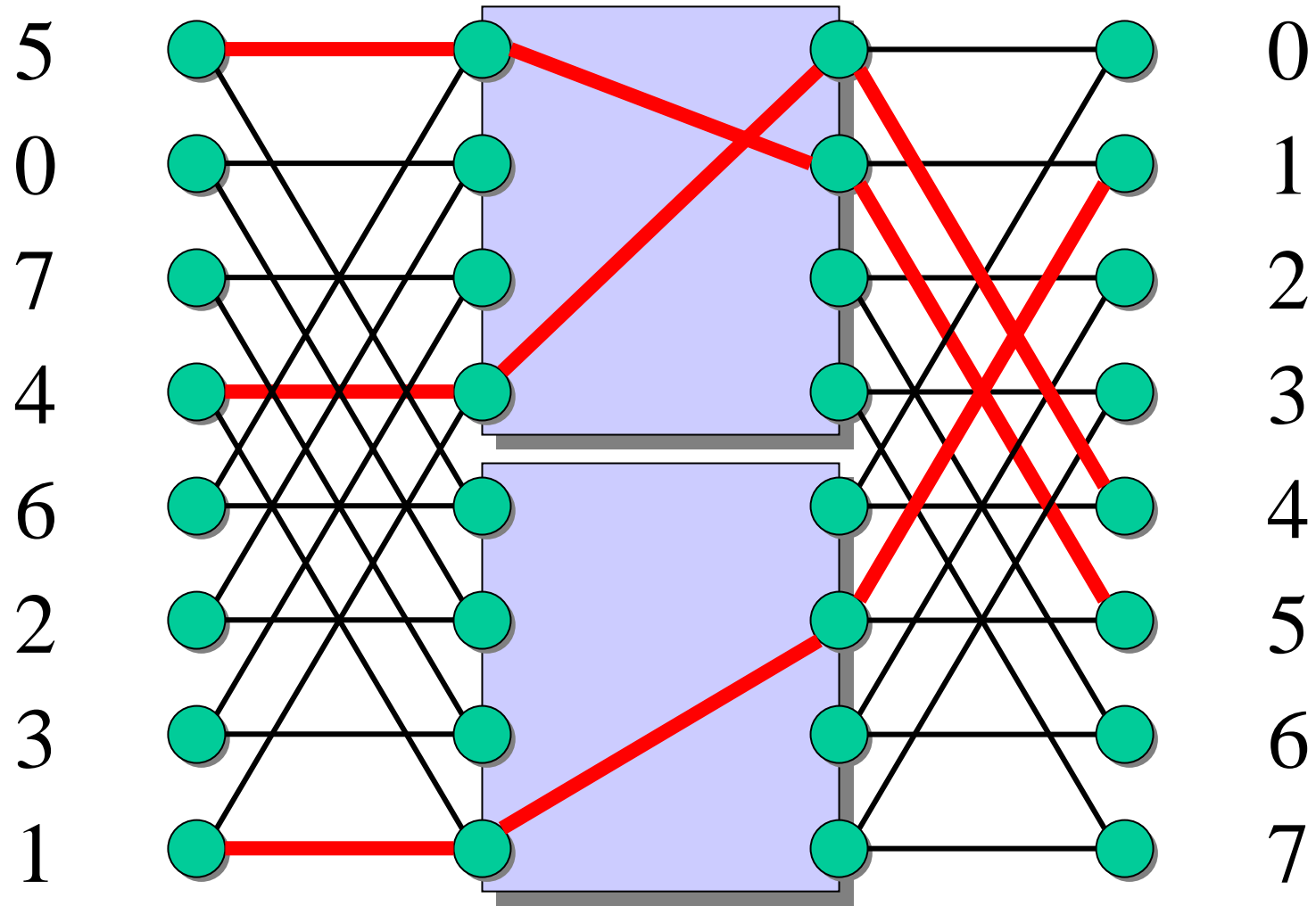
# Routing on a Beneš Network



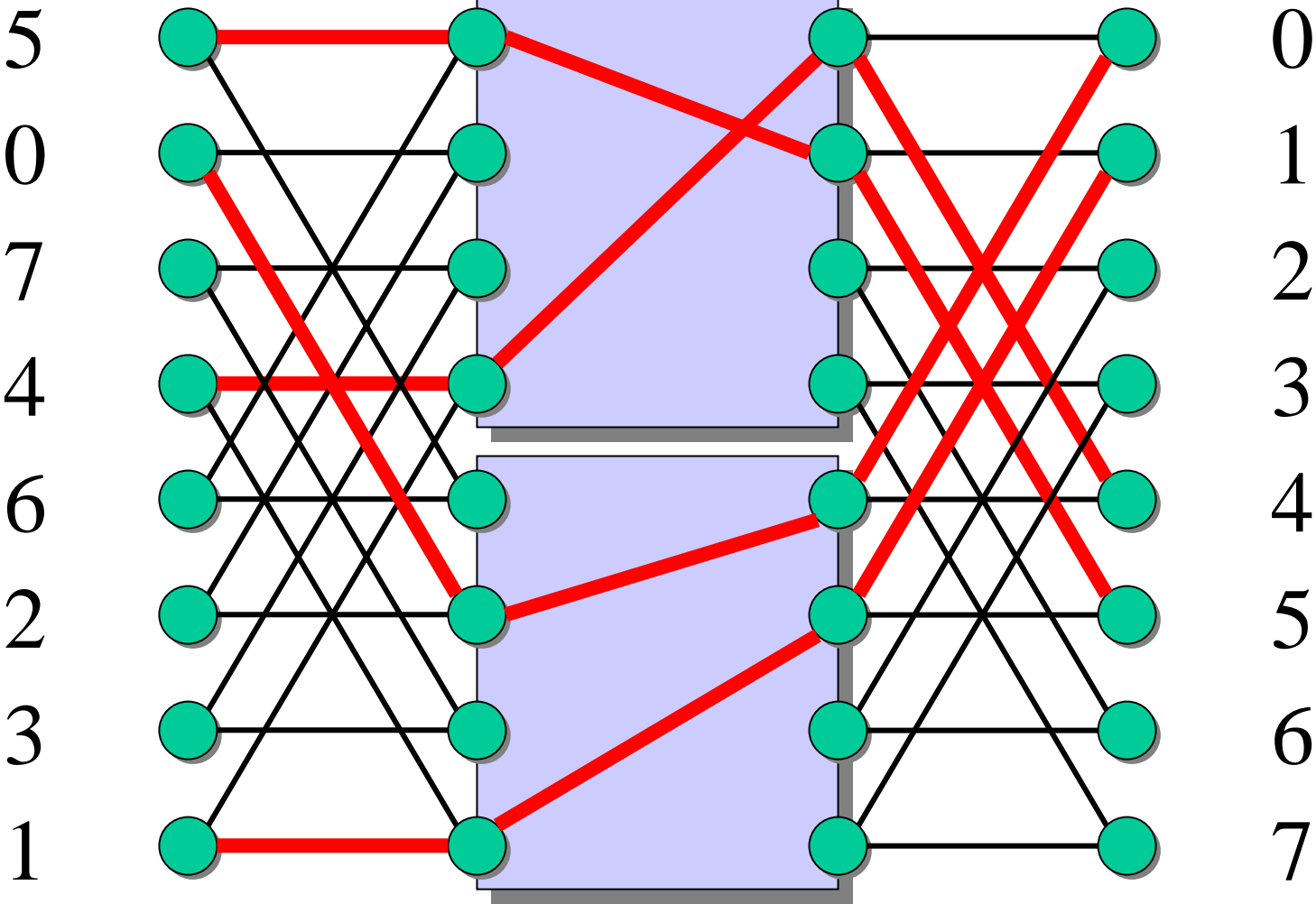
# Routing on a Beneš Network



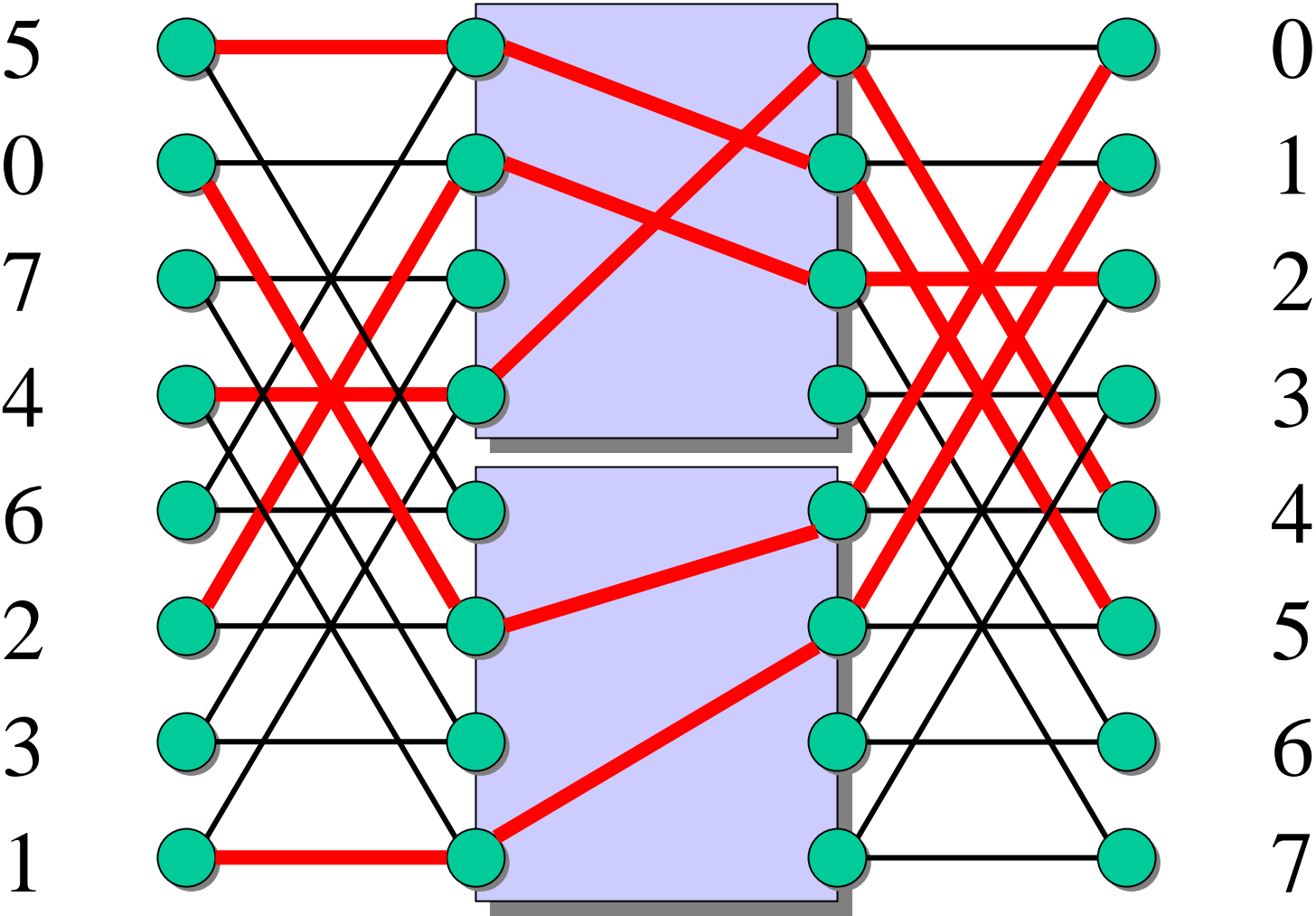
# Routing on a Beneš Network



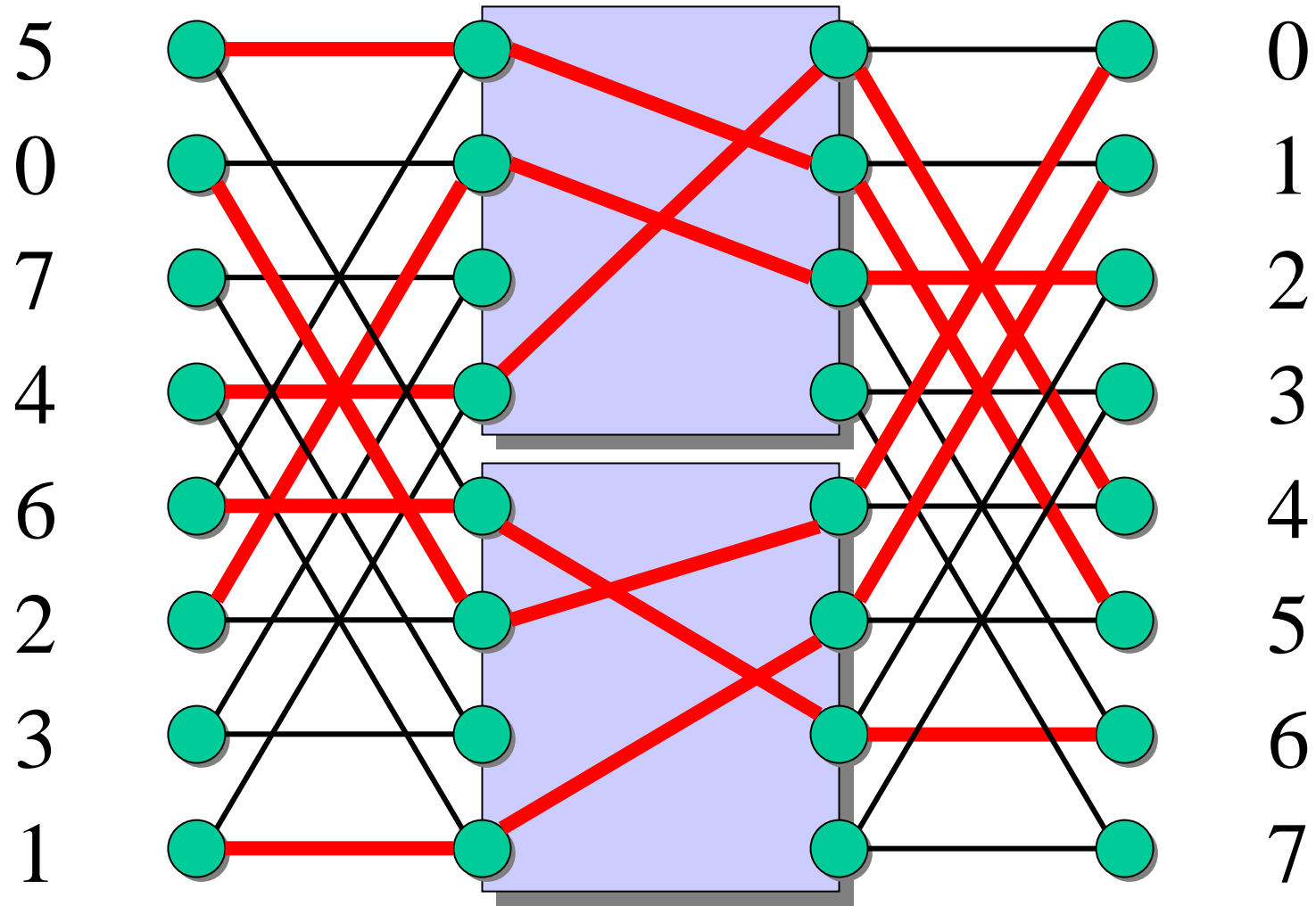
# Routing on a Beneš Network



# Routing on a Beneš Network

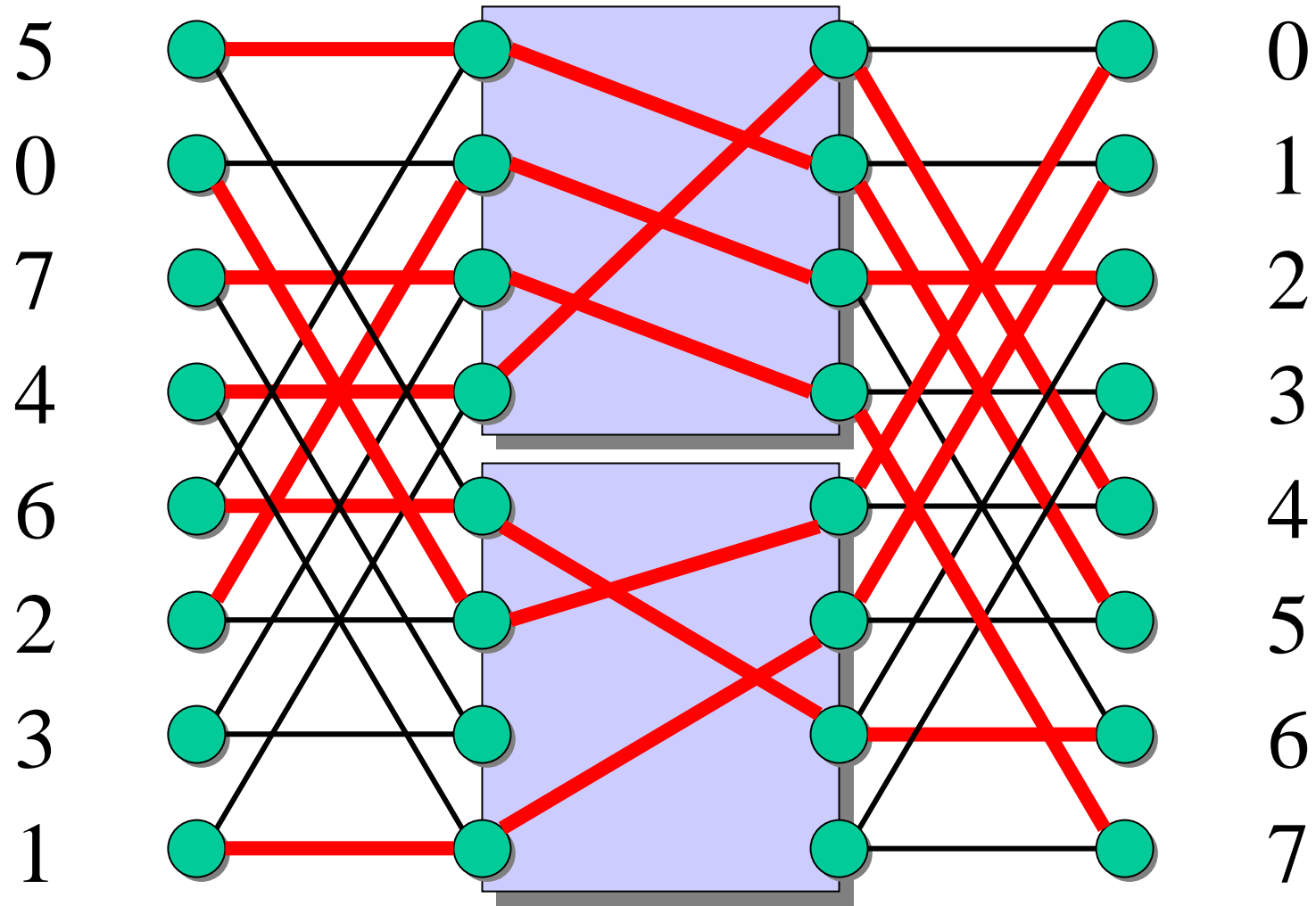


# Routing on a Beneš Network

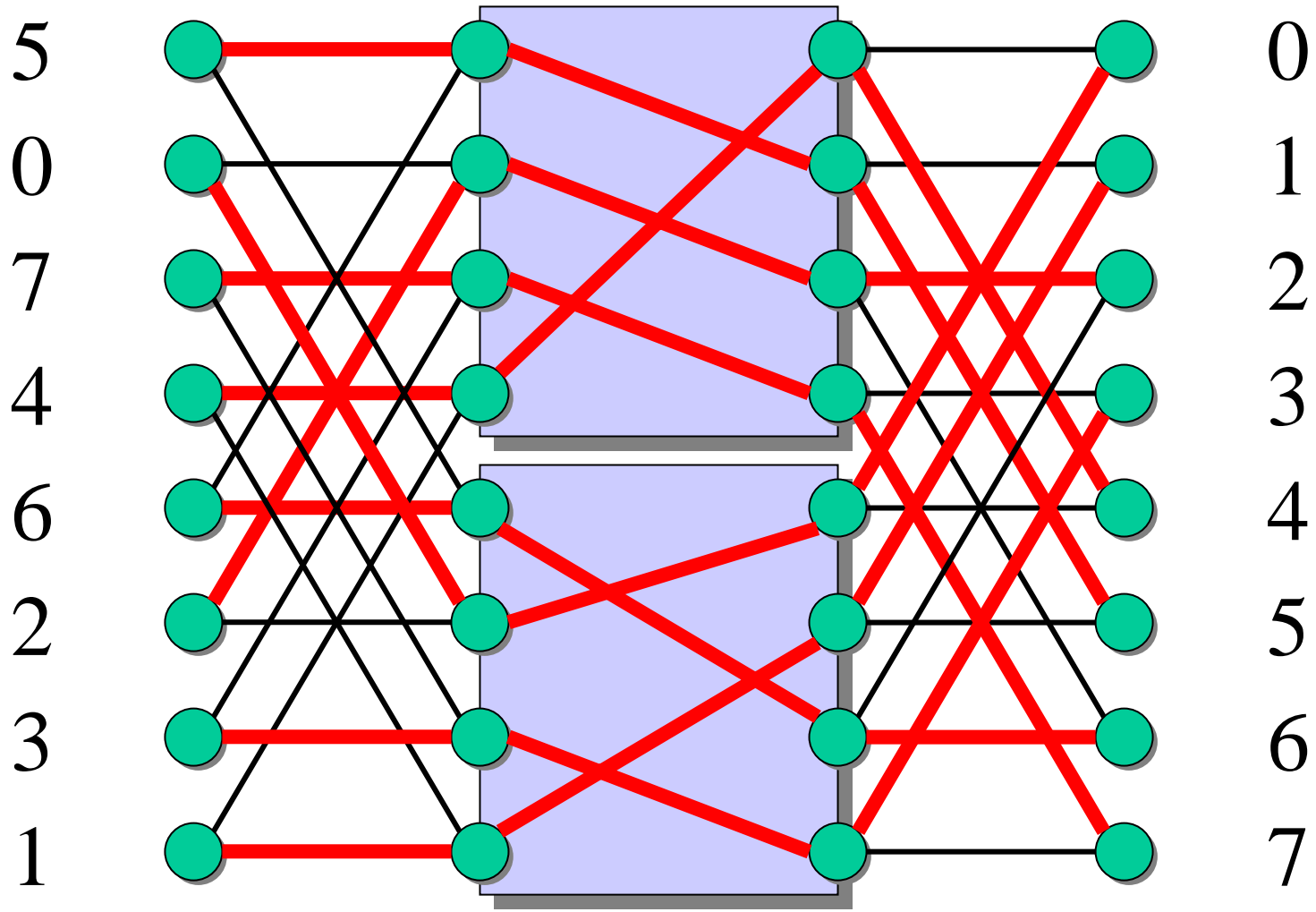




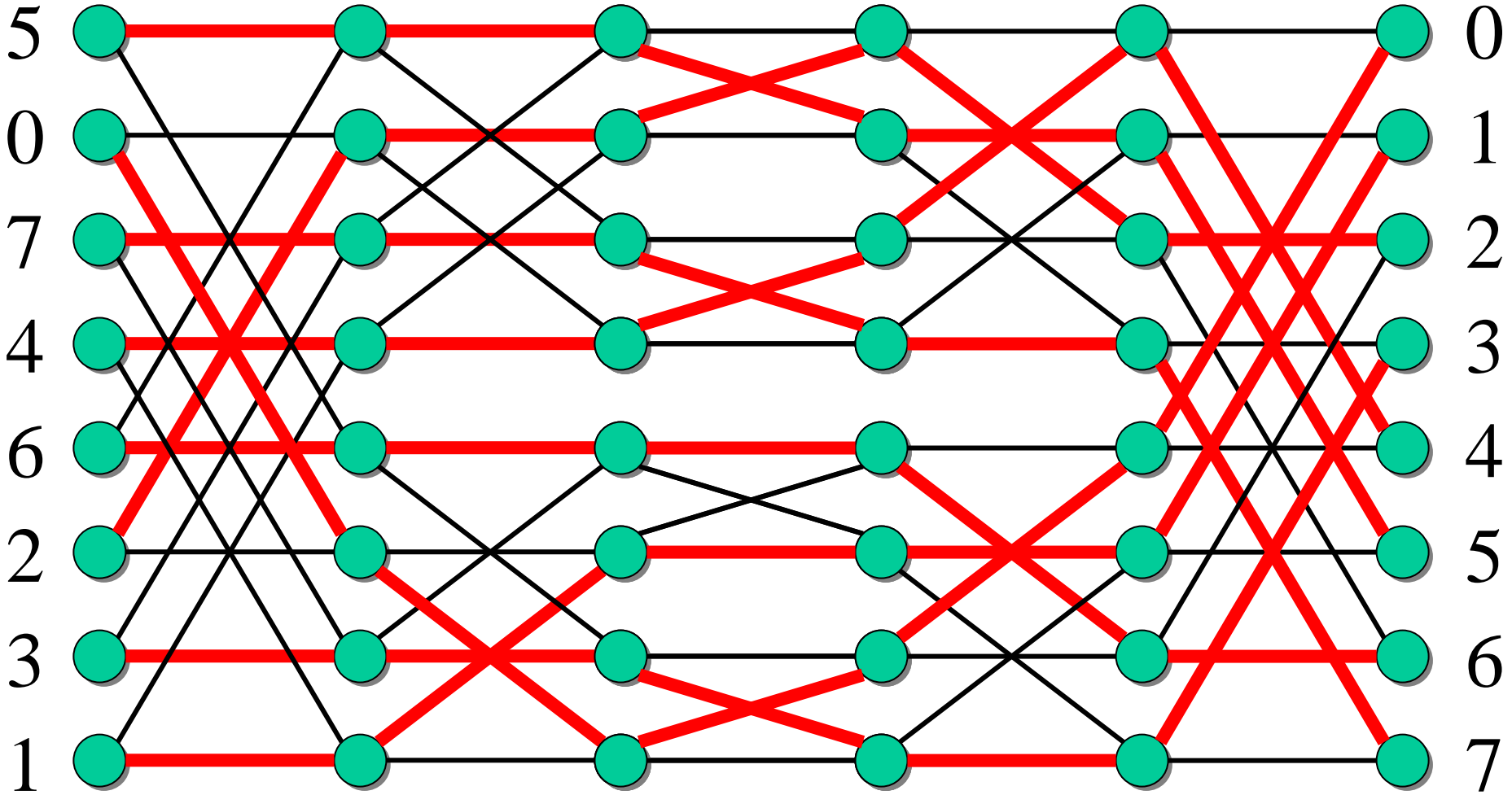
# Routing on a Beneš Network



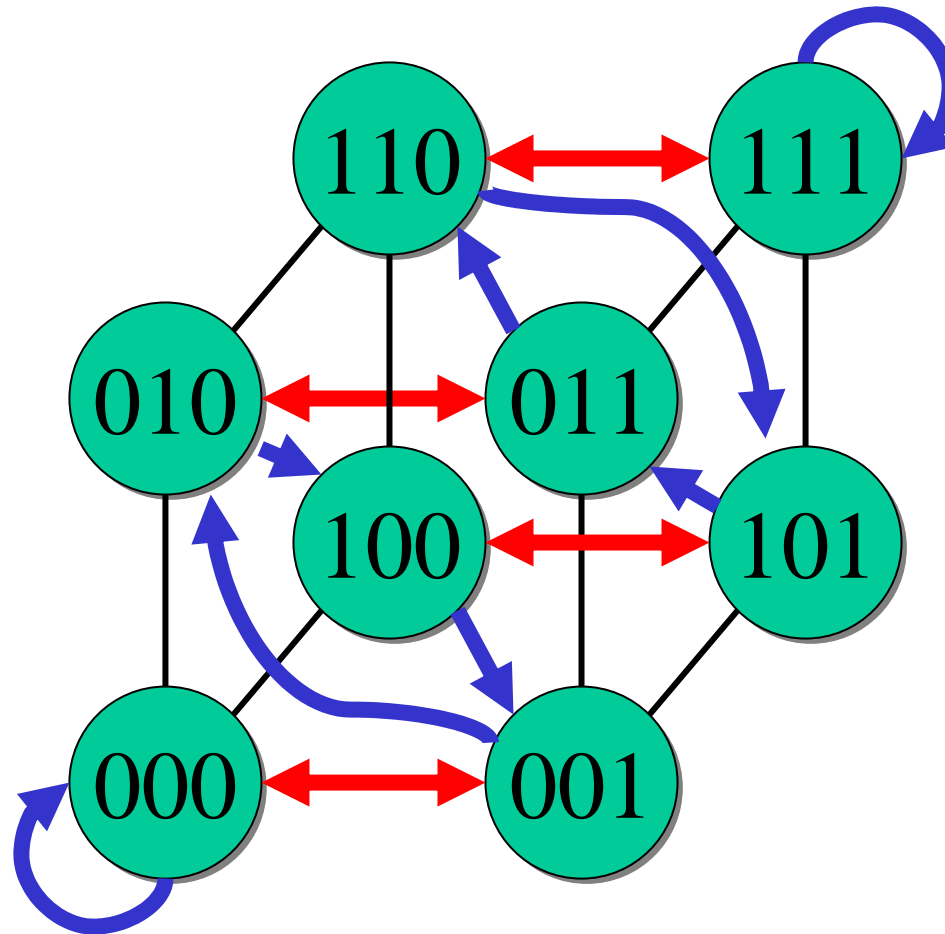
# Routing on a Beneš Network



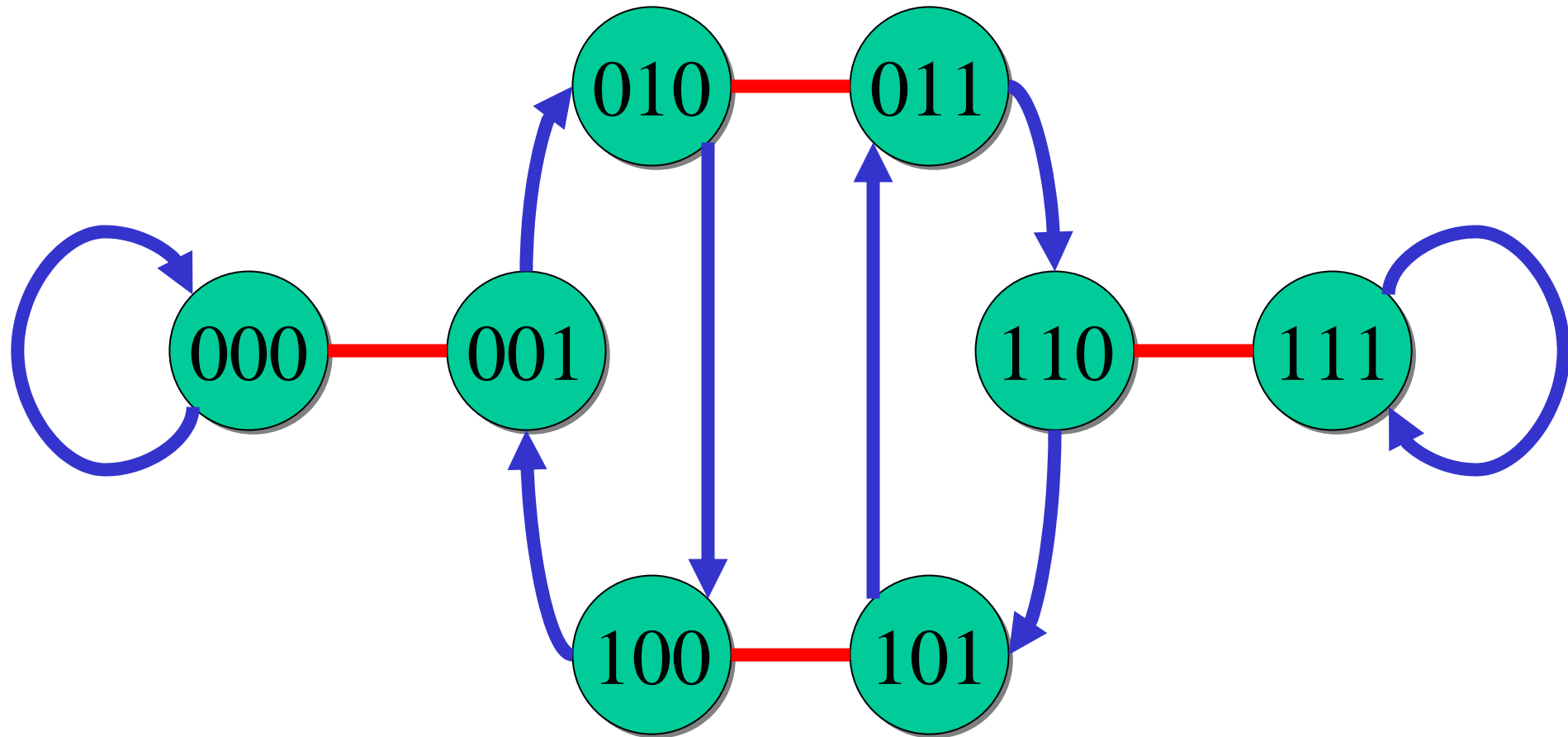
# Routing on a Beneš Network



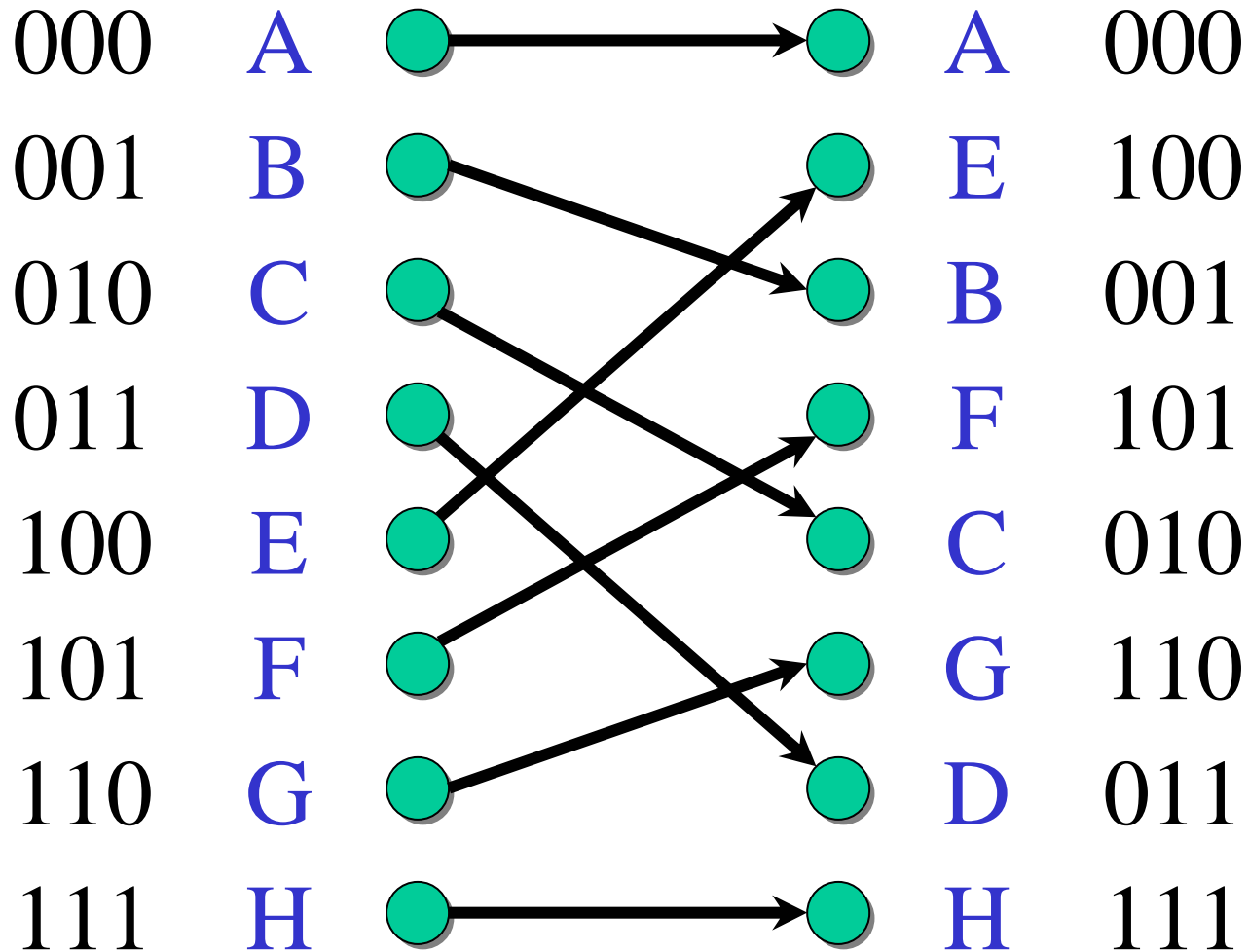
# Shuffle-Exchange Network



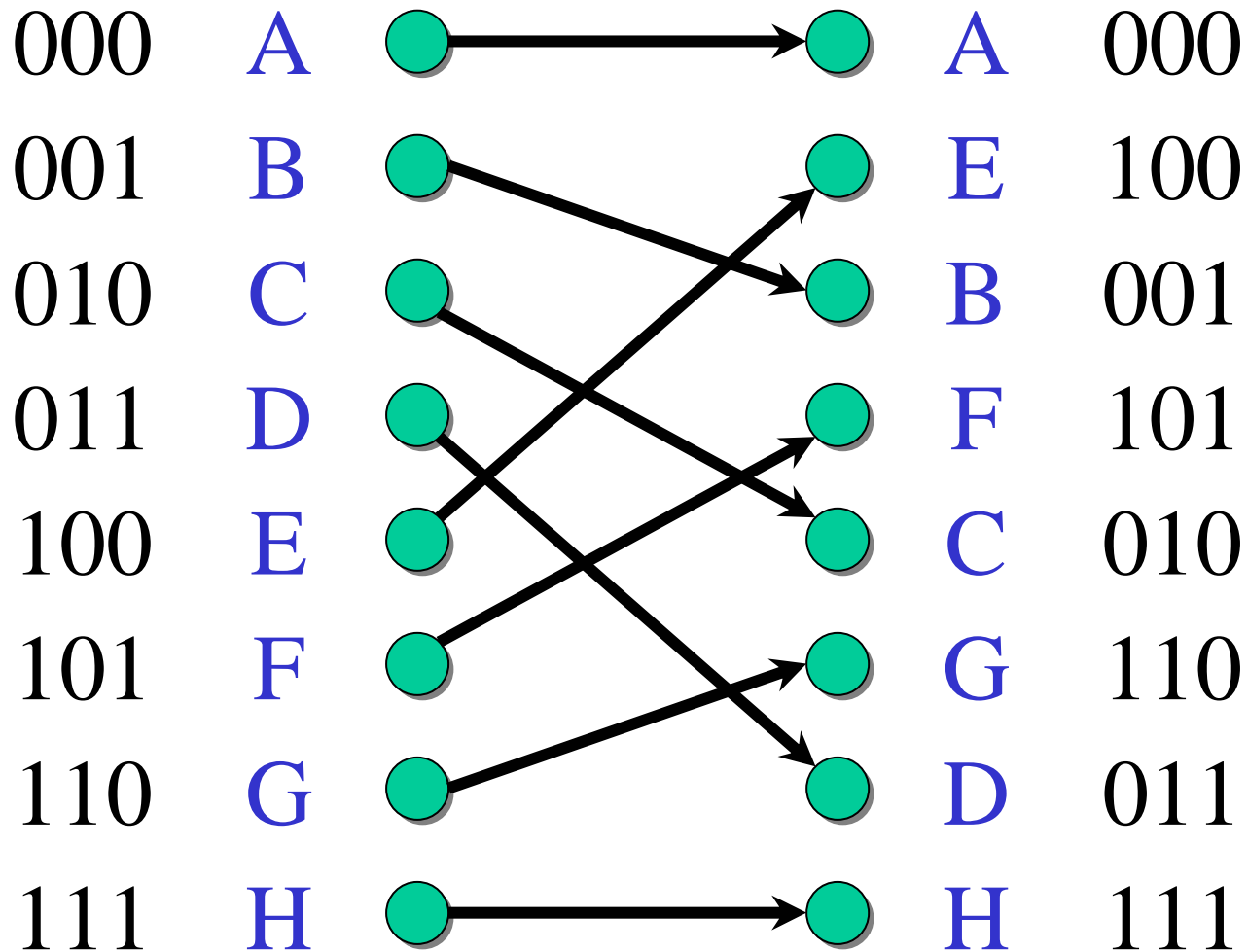
# Shuffle-Exchange Network



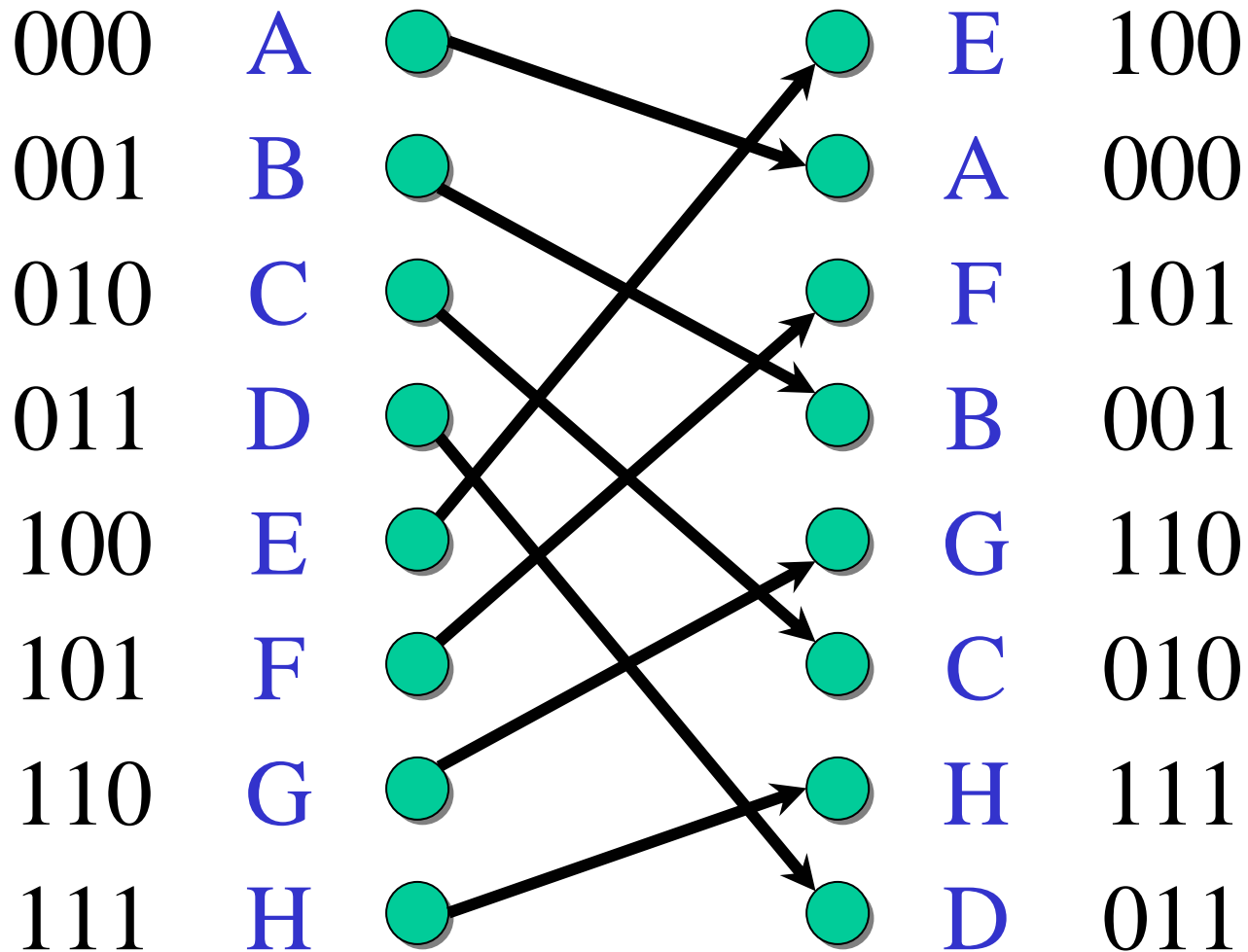
# Perfect Shuffle



# Perfect **Out**-Shuffle

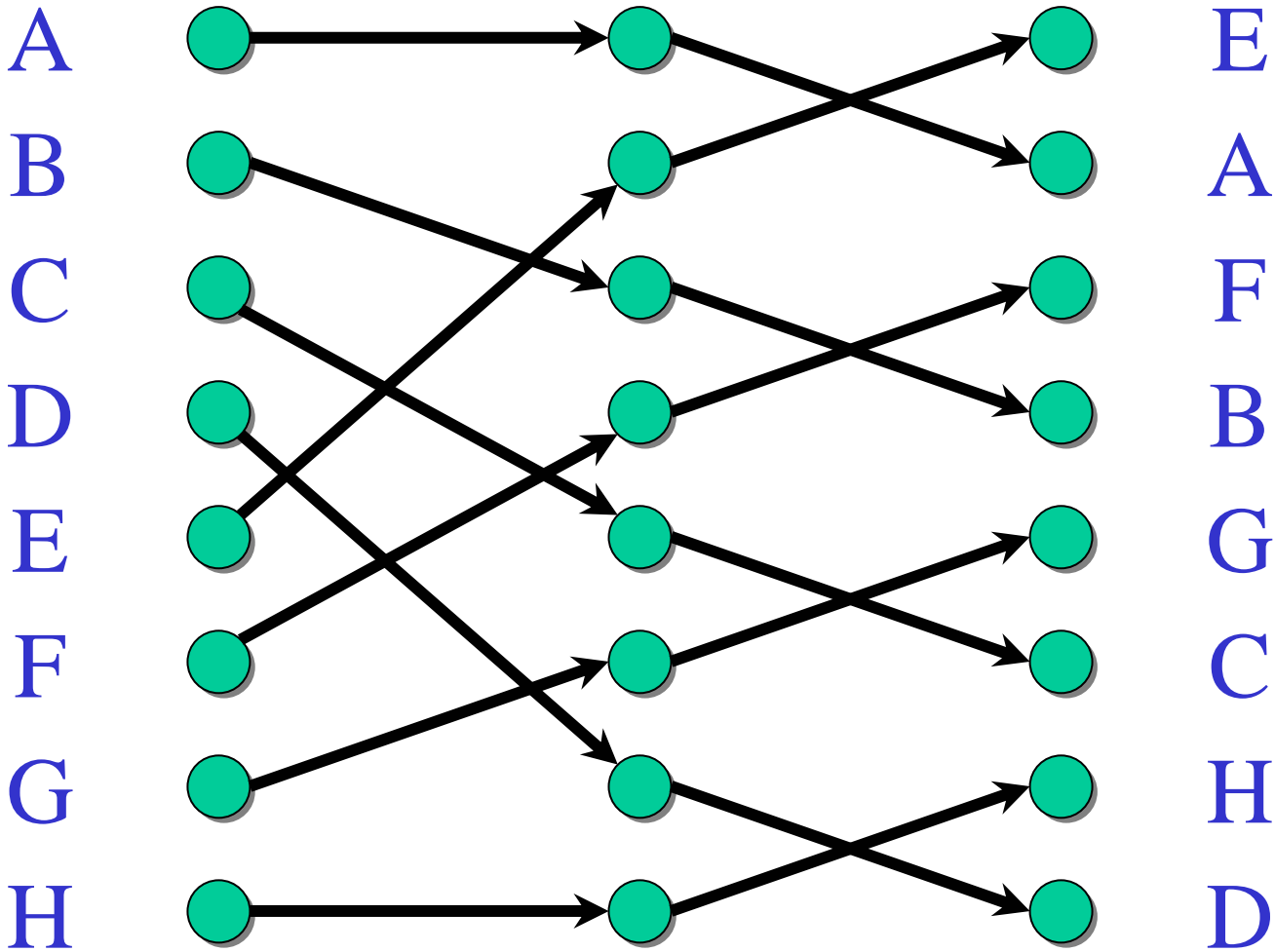


# Perfect **In**-Shuffle

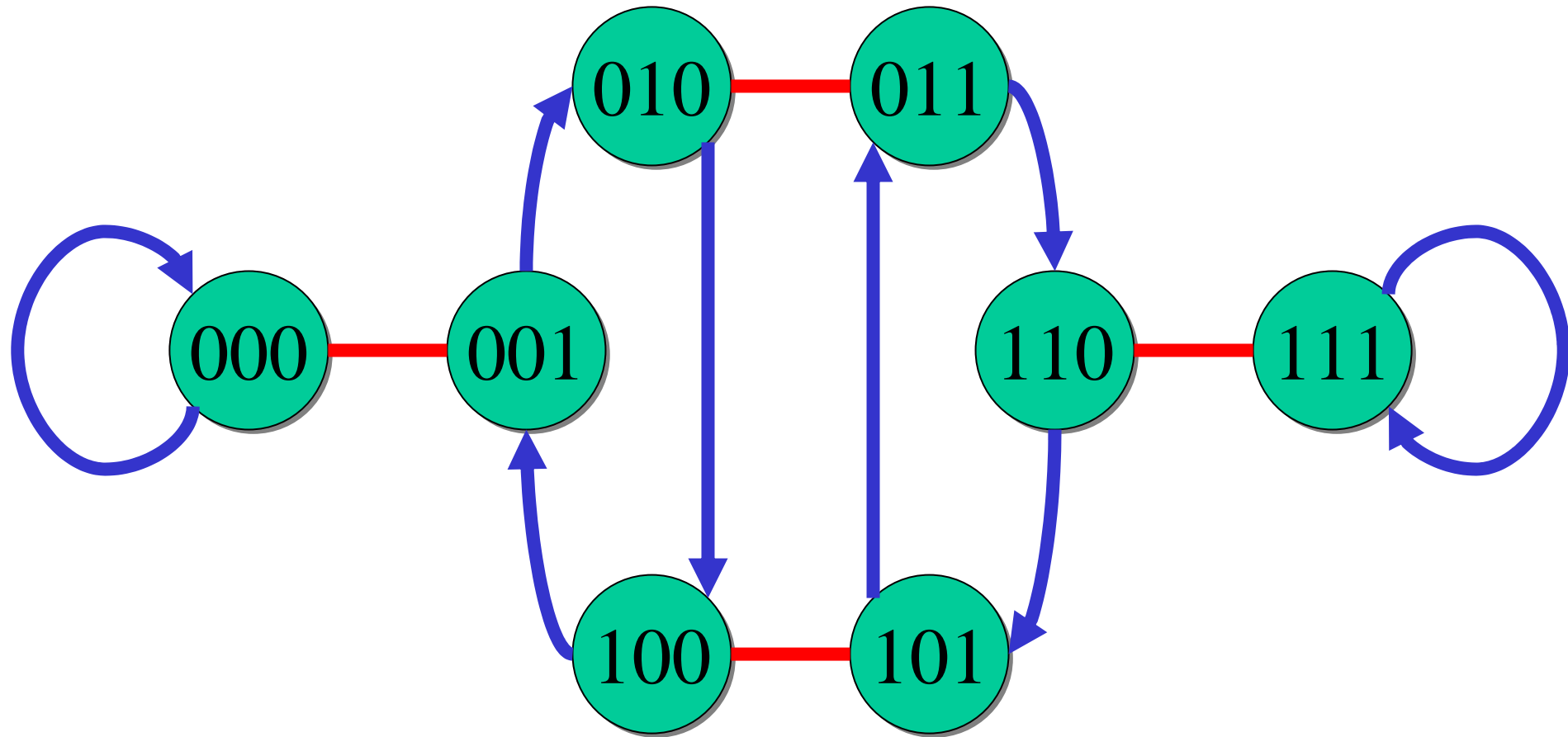




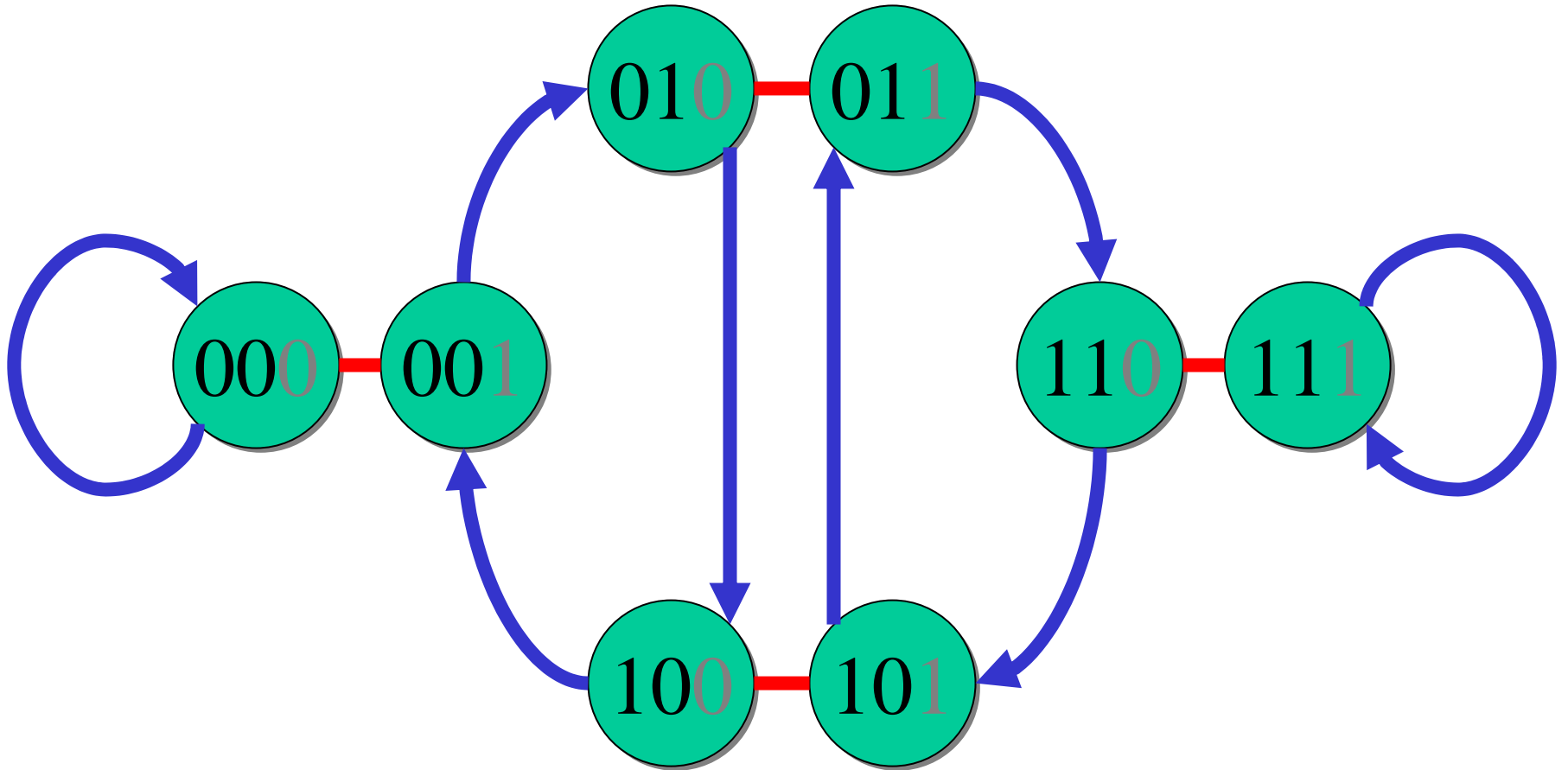
# In-Shuffle = Out-Shuffle + Exchange



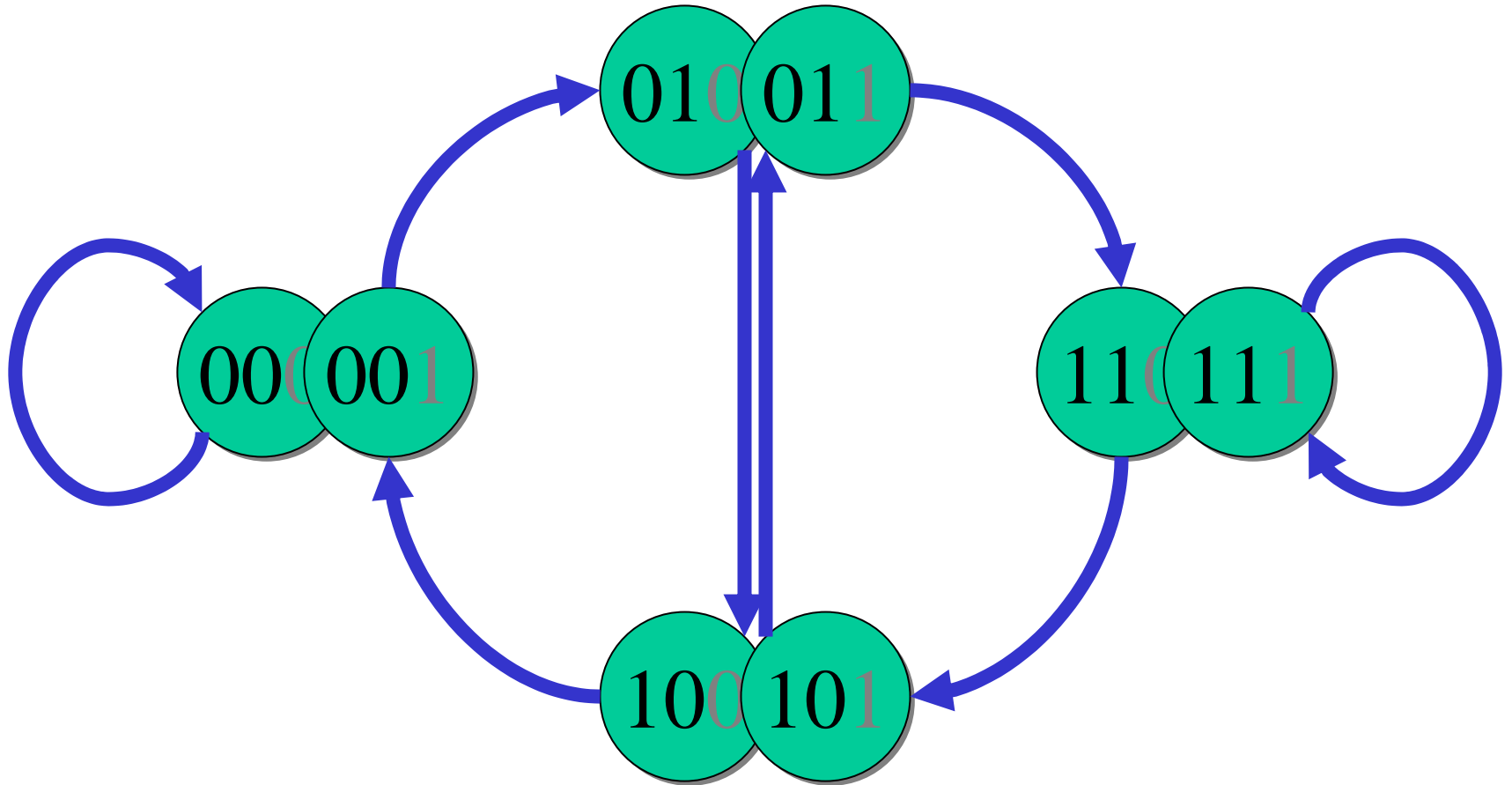
# SE $\rightarrow$ DeBruijn



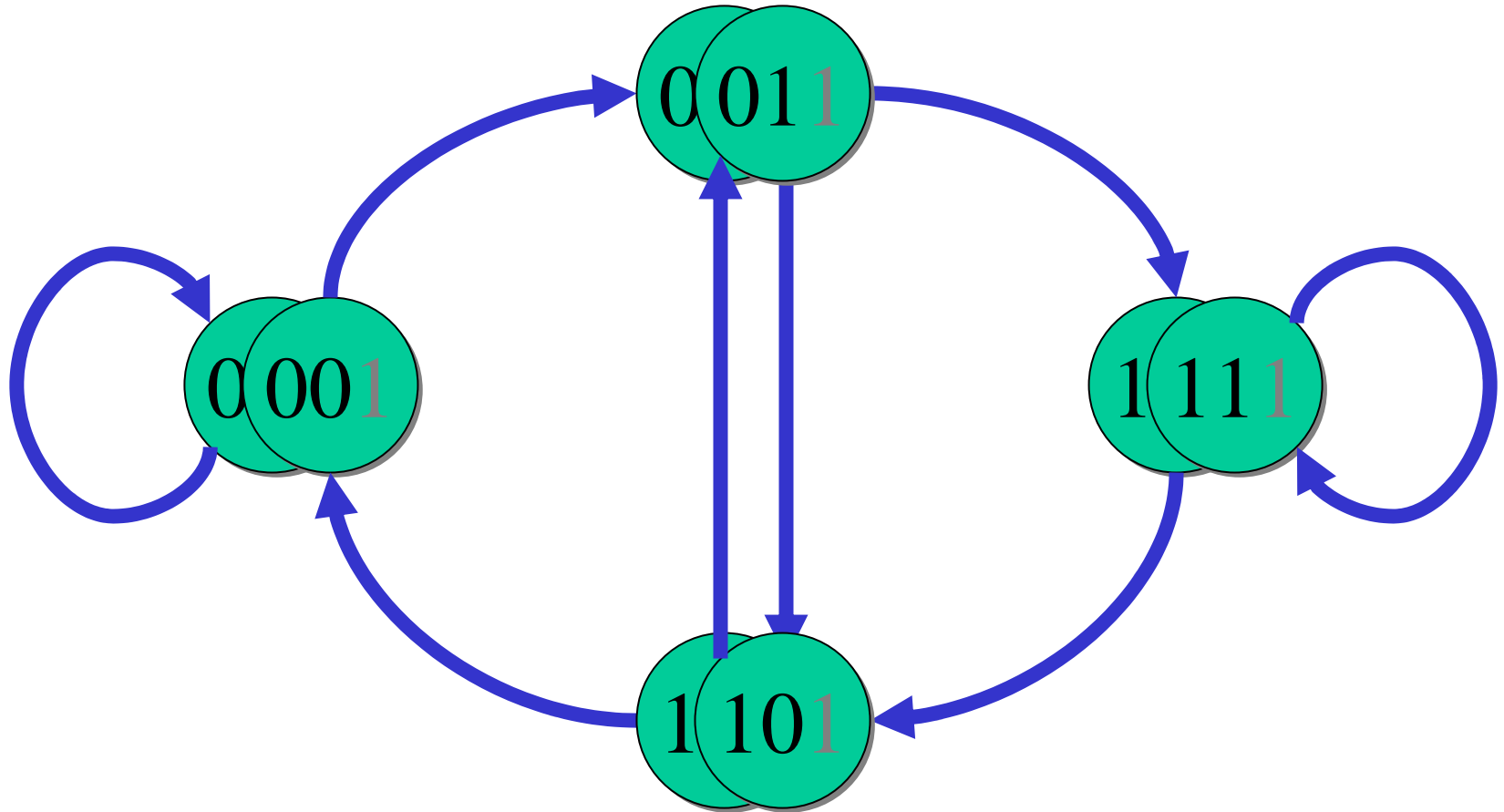
# SE $\rightarrow$ DeBruijn



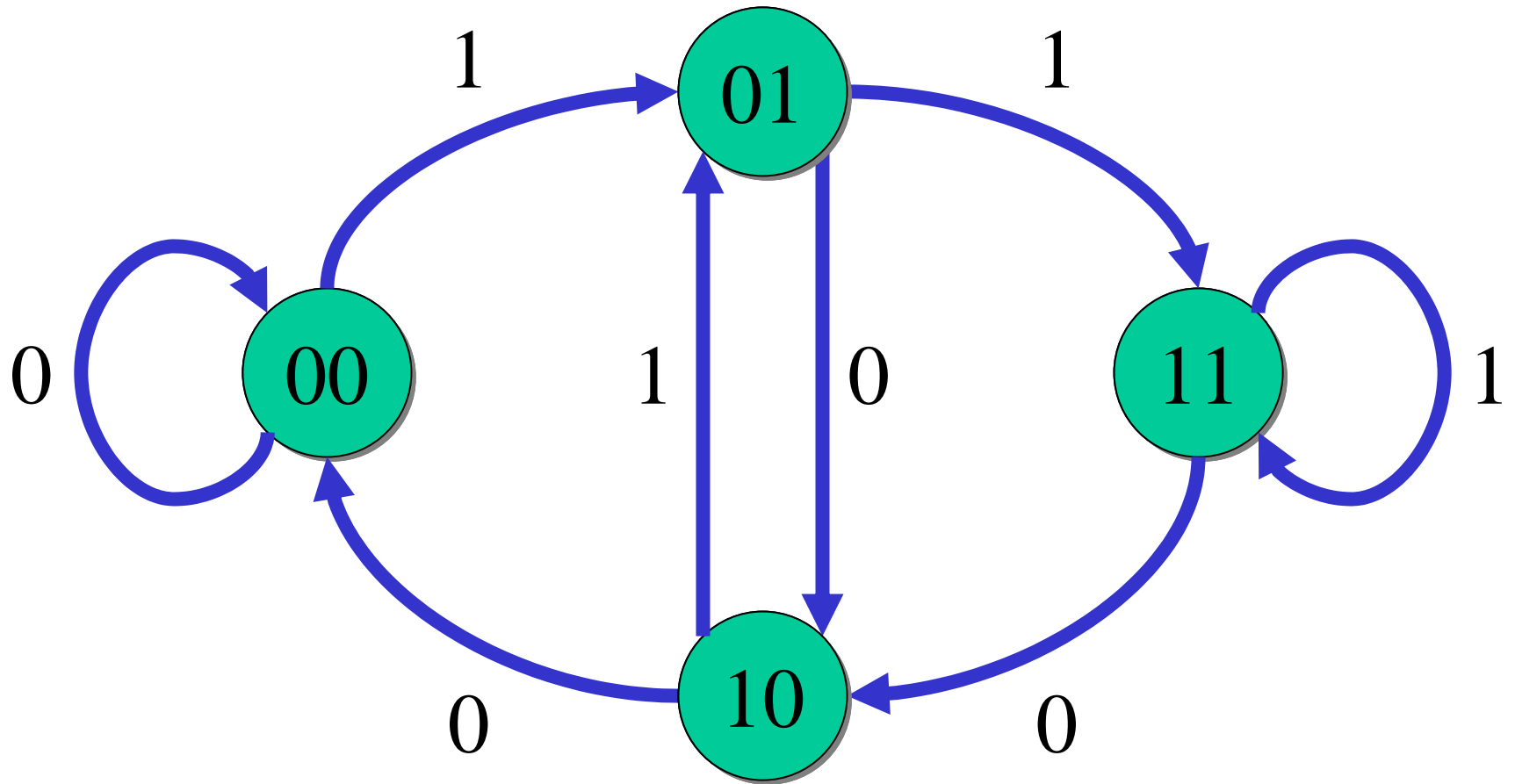
# SE $\rightarrow$ DeBruijn



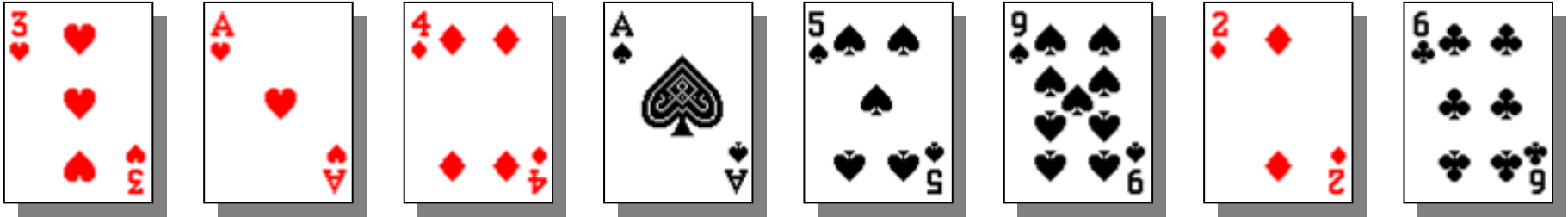
# SE $\rightarrow$ DeBruijn



# DeBruijn Network



# DeBruijn Sequence



0001110100

000

001

011

111

110

101

010

100

# Complex Plane Diagram of SE

