

CHAPTER II

Definition

Euclid's *Elements* begins with definitions, and at every new starting point within the work, new definitions are introduced. Plato's dialogues, on the other hand, almost all seem to be in search of definitions, which constantly elude the interlocutors. Aristotle develops his arguments from initial *arkhai* of definitions and *homologoumena*, but he often first surveys conflicting opinions on the matter, and lets us know that the starting points are not as settled as his treatment will lead us to believe.

We see then that from the beginning, an adequate definition was central to both pursuits. But we also see a contrast: whereas ancient mathematics seemingly cannot proceed without definitions in place, and whereas the bulk of ancient mathematics consists of what is done with the concepts developed in those definitions, ancient philosophy both marches on in the absence of solid definitions, and a good portion of it never leaves the arena of the search for those definitions.

Ralph Waldo Emerson suggests that the Platonic revolution was based on the new centrality of definition: "At last comes Plato, who needs no barbaric point, or tattoo, or whooping; for he can define...He is the arrival of accuracy....He shall be as a God to me, who can rightly divide and define."¹ I think that Emerson is only partially right. Plato, rather than being the first to define, is the first to acknowledge fully the structural barriers to the definition and precise statement of

¹ "Plato; or, the Philosopher" found in *Representative Men*; Dial Press; John D. Morriss & Co.; Philadelphia: 1906

the essence of anything. I do agree, however, that it is only, as Emerson claims, because Plato made defining – the search for the *ti esti* of the realities behind the words we use – his central quest, that he was able to discover the *essential* limits to grasping the essence of important human concepts.² And Emerson is certainly right that whoever can properly define is ‘like a god’ for Plato, for we see in dialogue after dialogue that among mere mortals both the most philosophical and anti-philosophical fail regularly at defining the words that we all use every day.

I. Definition as Foundation

The following passage from Book VI (510b) of Plato’s *Republic* gives us a picture of the foundational starting points of any investigation as lying in a realm beyond ordinary reasoning. Though it does not appear so to modern readers, of the various kinds of *arkhe*, Plato perhaps has definition most in mind here:

“Thus:--There are two subdivisions, in the lower of which the soul uses the *skhêmai* given by the former division as images; the enquiry can only be hypothetical, and instead of going upwards to a principle descends to the other end. In the higher of the two, the soul passes out of hypotheses, and goes up to a principle which is above hypotheses, making no use of images as in the former case, but proceeding only in and through the *eidei* themselves.

I do not quite understand your meaning, he said.

Then I will try again; you will learn³ more easily after what has been said. I suppose that you know that those who are engaged in geometry, *logismos*, and such matters hypothesize the odd and the even, the *skhêmai*, and three *eide* of angles, and their ‘sisters’ in each pursuit, as if they were

² This analysis, I will suggest was intimately related to the Greek understanding of what we call ‘irrational numbers.’

³ *Mathesis*, related to *mathematke*

known. They make these hypotheses, and do not deem it worthy⁴ to give any *logos* of them either to themselves or others, as if they were evident to all. Starting from these, they go through what remains and end at a *homologoumenon* concerning that which the investigation set in motion.⁵

I entirely know this, he said.

And do you not know also that although they avail themselves of the *eide* that can be seen and make *logoi* about them, they are thinking not of these, but of that which they are like; the rectangle itself and the diameter itself. That which they make or draw⁶, of which there are shadows and images in water, they in turn use as images, but they are really seeking⁷ to see⁸ the things themselves, which can only be seen with thought?

You speak the truth, he said.

Accordingly, this is the *eidōs* I spoke of as ‘intelligible’, although the soul seeking⁹ it is forced to use hypotheses; not reaching the beginning, because she is unable to disembark from and rise above¹⁰ hypothesis, but

⁴ *axiosi*, to deem worthy, is related to the word *axioma*, axioms

⁵ This is a description of the method which Proclus will call synthesis – the primary method of Euclid’s elements. See Chapter 3.

⁶ There seems to be a contrast here between *making* something and thinking about it, which parallels the contrast between descending from hypotheses, and ascending to them. Proclus says this explicitly: constructions are a descent from pure being for the purpose of allowing us to arise through theorems back towards Being. ??

⁷ *Zetousi*: the derived term *zetoumenon* is the partner of *homologoumenon* mentioned in the previous paragraph. The two methods of analysis and synthesis involve reverse relationships between *homologoumena* and *zetoumena*.

⁸ *Idein* - first meaning is ‘to see’, and from that means also ‘to understand’. Related to *eidōs*.

⁹ *Zetesin*: see note 7 above

¹⁰ *anôterô ekbainein**, ‘to disembark higher’ At *Phaedo* 77d Plato speaks of the soul disembarking from the body *ekbainousan*

using as images themselves that of which the things below are copies, they believe the latter to be more distinct¹¹, and therefore esteem them.

I understand, he said, that you are speaking of anything that comes under geometry and its sister *technai*.

Then understand now that by the other section of the intelligible I speak of that which the *logos* by itself grasps through the power of making distinctions¹², making hypotheses not the *archai*, but literally hypotheses, the initial upward step to the very location of the non-hypothetical, which is the *arche* of everything existing. Grasping this, and cleaving to that which cleaves to that place, it comes back down to a conclusion using absolutely no sensible things. Using *eide*, through themselves into themselves, it ends in *eide*.

We will investigate this distinction of moving to and from *archai* more closely in the next chapter when we take up the question of analytic and synthetic approaches. The modern reader might be surprised to find this passage in a chapter about definitions. It seems to be concerned with tracing reasoning back to beginnings that are not hypothetical, but which are in some way self-evident truths. We are very aware that some of the basic hypotheses of our sciences are unprovable speculations. Modern thought tends to be skeptical about finding any self-evident foundation for those speculations, but even moderns must admit the *possibility* of a non-hypothetical, self-evident principle. It is natural, however, to think that such a starting point would be a proposition rather than a definition. We tend to think of definitions as mere clarifications of concepts already intuitively

¹¹ *enargesia*: can also mean palpable or bodily

¹² *dialegesthai*: most translators render this ‘dialectic,’ a complex Platonic concept concerning the way philosophy proceeds. I am not so sure that the word should be turned into a static technical term, which is one of the steps in formulating a Platonic ‘doctrine’. I believe that translating it in its common meaning might better capture the understanding of the word that Socrates’ interlocutors held.

understood, but both Plato and Aristotle saw definitions as genuine *archai*: a proper definition captures the *ti esti* of something, and therefore goes behind appearances to a fundamental truth. Aristotle, in the *Posterior Analytics* (70a30 ff), considers definitions to be among the *archai* of any science.¹³ Proclus, probably the most important ancient commentator on Euclid, treats the word ‘hypothesis’ as identical with the word ‘definition.’ (pg. 140) The true starting place of any inquiry is to understand fully the essence of the objects under investigation: in the case of geometry, it is to understand what a point, a line, a straight line, etc., truly are. This meaning of ‘hypothesis’ is perhaps emblematic of the difference between the ancient and modern understanding of the starting point of scientific inquiry. For the ancients, the ideal starting point is a true statement of the nature of those things which can not be reduced to, or explained in terms of, anything else; the *arkhe* which are self-grounded and, seen in the right way, self-evident.

But, while it seems obvious that definitions are in some sense at the ‘beginning’ of any science, it is unclear exactly how they begin a chain of reasoning. We will see, for instance, that Euclid, while giving great care to find definitions which seem to seek the *ti esti* of the basic objects of mathematics, rarely actually employs those definitions in the body of his work. Definitions may indeed be more basic than axioms, but they are apparently less useful. If Euclid rarely refers back to definitions, what role exactly do definitions play in his chain of reasoning? We will take up this question later.

¹³ Proclus, in his commentary on Euclid, seems to be suggesting that it is precisely definitions that are the ‘hypotheses’ of geometry, which are borrowed from a higher science in which they are not hypothetical. He gives the name ‘hypothesis’ precisely to the category of *arkhai* that Euclid name *horai*, [definitions, boundaries]. There is some suggestion that Proclus possessed a copy of Euclid in which ‘hypothesis’ was the name given to definitions. [Proclus, pg. 62] For Aristotle, definitions (*horismos*) and hypotheses (*hypothesis*) are two species of what he calls **posits** (*thesis*). A **hypothesis** asserts one part of a contradiction, e.g., that something is or is not. A **definition** does not assert or deny either part of a contradiction.

II. Philosophy and the Definitions of Mathematics.

The passage from the Republic launches a strong tradition which locates the foundational truths of mathematics outside the scope of mathematics, and more in the province of something like philosophy.

Note for instance Philo of Alexandria, (1st century B.C. – 1st century A.D.): “Geometry ...needs philosophy to express definitions for its subject matter.”¹⁴

This is not far from what Bertrand Russell and Alfred North Whitehead would say 1900 years later: “...a definition is, strictly speaking, no part of the subject in which it occurs. ...”¹⁵ But we note that Philo specifically assigns this question of finding the right definitions to philosophy, whereas Russell and Whitehead do not. If the readings from Aristotle’s *Categories* and *Topics* included in this chapter rightly characterize the ‘ancient view’, and the *Meno* suggests it does, this distinction might have something to do with the difference between the Greek and the prevalent modern conception of what a definition is about. Consider the rest of the Whitehead Russell statement, in contrast to Aristotle:

“A definition is a declaration that a certain newly-introduced symbol or combination of symbols is to mean the same as a certain other combination of symbols of which the meaning is already known...it is not true or false, being an expression of a volition, not a proposition.”

This is in some contrast with Aristotle’s and Plato’s attempts to capture the *essence* of something in a definition. Before we read these treatments of definition, let us consider antiquity’s greatest example of foundational definitions, the definitions found in Book I of Euclid’s *Elements*. We will want to keep these

¹⁴ (Philo of Alexandria *On the Creation* 13-14, 48, 89-128. This opinion is commonly expressed from Aristotle on. Cf. Proclus

¹⁵ *Principia Mathematica*

definitions in mind as antiquity's most precise set of examples, as we consider in this chapter what ancient philosophers thought a definition is.

III Definitions in Euclid's *Elements*

These are the fundamental definitions for the whole 13 books of Euclid's *Elements*. I present them here, first without commentary, so that the reader can simply contemplate them. As an exercise, you might want to try to classify these into different types of definitions as you go through them, and then see if your categories match up with Aristotle's. Directly following these, some alternative definitions for the most important terms will be considered, and then the question of the nature of definition itself.

The Elements

Euclid

Book I

Definitions

Definition 1.

A point is that which has no part.

Definition 2.

A line is breadthless length.

Q.2.wv: Must these two be defined negatively (that is, by saying what they are not, or do not have)? Does this affect the capacity of the definition to provide understanding?

Definition 3.

The ends of a line are points.

Definition 4.

A straight line is a line which lies evenly with the points on itself.

Definition 5.

A surface is that which has length and breadth only.

Definition 6.

The edges of a surface are lines.

Definition 7.

A plane surface is a surface which lies evenly with the straight lines on itself.

Definition 8.

A plane angle is the inclination to one another of two lines in a plane which meet one another and do not lie in a straight line.

Definition 9.

And when the lines containing the angle are straight, the angle is called rectilinear.

Definition 10.

When a straight line standing on a straight line makes the adjacent angles equal to one another, each of the equal angles is right, and the straight line standing on the other is called a perpendicular to that on which it stands.

Definition 11.

An obtuse angle is an angle greater than a right angle.

Definition 12.

An acute angle is an angle less than a right angle.

Definition 13.

A boundary is that which is an extremity of anything.

Definition 14.

A *skhêma* is that which is contained by any boundary or boundaries.

Definition 15.

A circle is a plane *skhêma* contained by one line such that all the straight lines falling upon it from one point among those lying within the *skhêma* equal one another.

Definition 16.

And the point is called the center of the circle.

Definition 17.

A diameter of the circle is any straight line drawn through the center and terminated in both directions by the circumference of the circle, and such a straight line also bisects the circle.

Definition 18.

A semicircle is the *skhêma* contained by the diameter and the circumference cut off by it. And the center of the semicircle is the same as that of the circle.

Definition 19.

Rectilinear *skhêmai* are those which are contained by straight lines, trilateral *skhêmai* being those contained by three, quadrilateral those contained by four, and multilateral those contained by more than four straight lines.

Definition 20.

Of trilateral *skhêmai*, an equilateral triangle is that which has its three sides equal, an isosceles triangle that which has two of its sides alone equal, and a scalene triangle that which has its three sides unequal.

Definition 21.

Further, of trilateral *skhêmai*, a right-angled triangle is that which has a right angle, an obtuse-angled triangle that which has an obtuse angle, and an acute-angled triangle that which has its three angles acute.

Definition 22.

Of quadrilateral *skhêmai*, a square is that which is both equilateral and right-angled; an oblong that which is right-angled but not equilateral; a rhombus that which is equilateral but not right-angled; and a rhomboid that which has its opposite sides and angles equal to one another but is neither equilateral nor right-angled. And let quadrilaterals other than these be called trapezia.

Definition 23

Parallel straight lines are straight lines which, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction.

Q.2.2: *Thinking about all these definitions, what do you think would be Euclid's definition of a definition, including the elements he thinks are necessary for a definition?*

Q.2.3 *Are these definitions all of the same type? Do they have the same sort of ingredients?*

Q.2.4 *What in particular, does Euclid mean by his definition of a point; A straight line?*

III] Alternate Definitions

Now that you have contemplated these definitions in themselves, here are some alternative definitions of some of the most controversial terms in Euclid, most from antiquity, and a few from modernity.

A] Definitions of a point:

1. That which has no part
2. A unit that has position (Pythagoreans) [Aristotle, *De Anima* 409a6]
3. Beginning of a line (Plato, according to Aristotle, *Metaphysics* 992a20)
4. The indivisible beginning of all magnitudes (Herundes, reported by an-Nairizi)
5. A point may be a beginning or an end, or division of a line, but it is no part of it. (Aristotle, *De Caelo* 300a14, *Physics* 220a1-21)
6. An extremity which has no dimension or an extremity of a line (Posidonius)
7. The intersection of two lines is a point (Posidonius)
8. The point is the limit of localization...the content of space vanishes; relative position remains (Max Simon, 19th Century)
9. ...the extremest limit of...spatial presentation (Thomas Heath)

10. A point is the extremity of a line, the smallest possible mark (Apollonius, reported by Diogenes Laertius, 7.135)
11. A point is the simplest locus, or the locus of not other locus (Leibniz, *Metaphysical Foundations of Mathematics* 1715)

B| Definitions of a line

1. The path of a point in motion (Aristotle, *De Anima* 409a4, implied)
2. The flowing of a point (Aristotle *De Anima* 409a4)(Proclus, p. 79 and also attributed to Eratosthenes)
3. Magnitude in one direction (Proclus p. 79, also, perhaps Aristotle *Meta.* 1020a11-12)
4. A magnitude divisible one way only (Aristotle *Metaphysics* 1016b25-27)
5. distance (Apollonius)
6. A line is the extremity of a surface or length without breadth, or that which has length alone (Apollonius, *Physics*)
7. One can say that a line is what divides the sunlight from the shadow. (Hero, *Definitions*)

C| Definitions of a straight line:

1. A straight line is whatever has its middle in front of both its ends (Plato, *Parmenides* 137e)
2. ...the limit of a finite plane, such that its centre is in a line with its extremes (Aristotle, *Topics* VI)

3. The shortest distance between two points.¹⁶
4. A homeomeric line in a plane which can be extended as far as one pleases without meeting itself.” Lee Perlman
5. A line in a plane which has the same shape on both sides, OR a line which has the same shape on all sides. OR a line with only one shape (Eidos) (Perlman)
6. A straight line is that which lies equally with respect to its straight points; circular, whose path will be different from the arrangement of its points. [Balbus the surveyor]
7. A line which, when its ends remain fixed, it remains fixed when turned around in the same plane. (Heron of Alexandria)
8. The class of all the points whose places in relation to the two given points are uniquely determined. (Leibniz, *Metaphysical Foundations of Mathematics* 1715)
9. the simplest path through two given points. (Leibniz, *Metaphysical Foundations of Mathematics* 1715)

Q.2.5 Assuming that in each case these authors are trying to define the same ‘thing’ as Euclid, yet disagreeing about how to characterize what that thing is, how might one characterize the common object of definition about which there are contending and disagreeing definitions?

IV] WHAT IS A DEFINITION?

¹⁶ In *On the Sphere and Cylinder* Archimedes lists as the first of his postulates (*lambanomena* – things taken or received): “Of all lines which have the same extremities the straight line is the least” Archimedes thus considers this a property – one so obvious that it is postulated rather than proven – of the straight line, rather than, as we would have it, the very definition of a straight line. **Q.:** *By what criteria would you decide that this is a definition rather than a property, and particularly, what would make it a definition of the ‘essence’ of a straight line?*

Perhaps the question that is the strongest undercurrent in the Platonic dialogues is: What exactly does it mean to define something? That is, what is the definition of a definition? Another way of thinking about this is to ask whether some definitions define better than alternative attempts to define the same thing. If so, then we may decide that certain sorts of definition are inherently more explanatory than others. Consider, for instance, Proclus' comments on definitions (2) and (3) of a line, on page x above:

“The latter definition indicates perfectly the nature of the line, but that which calls it the flowing of a point appears to explain it in terms of its generative cause, and sets before us not the line in general, but the material line.” (pg. 79)

Clearly, both definitions ‘delineate’ the line from other entities. But one Proclus holds to actually state the nature of a line, while the other, he thinks only gives us a kind of procedure for producing a (strictly material) line. Even if this procedure generates lines and only lines, and therefore is perfectly adequate for distinguishing what is and what is not a line, it is held to be an inferior definition because it does not tell us what a line *is*.

The following is one of Aristotle's treatments of ‘definition’, from his *Posterior Analytics*. It is perhaps the oldest treatise on the subject in existence. **Q.: Does he understand the nature of definition the same way that Plato and Euclid do?**

Posterior Analytics

Aristotle

Book. I

Ch. 2

If a thesis assumes one part or the other of an enunciation, i.e. asserts either the existence or the non-existence of a subject, it is a hypothesis; if it does not so assert, it is a definition. Definition is a ‘thesis’ or a ‘laying something down’, since the arithmetician lays it down that to be a unit is to be quantitatively indivisible¹⁷; but it is not a hypothesis, for to define what a unit is is not the same as to affirm its existence.¹⁸

Ch. 8

...a definition is either a primary premise or a conclusion of a demonstration, or else only differs from a demonstration in the order of its terms.¹⁹

Ch. 10

The definition-viz. those which are not expressed as statements that anything is or is not-are not hypotheses²⁰: but it is in the premises of a science that its hypotheses are contained. Definitions require only to be understood, and this is not hypothesis... all hypotheses and illegitimate postulates are either universal or particular, whereas a definition is neither.²¹

Ch. 22

In the case of predicates constituting the essential nature of a thing, [demonstration] clearly terminates, seeing that if definition is possible, or in other

¹⁷ *Q.2.zzz: Is this different from the way you think of a unit? If this is the very definition of a unit, does it explain at all why the Greeks did not think of fractions as numbers?*

¹⁸ Aristotle is here clearly using the term ‘hypothesis’ differently from Proclus.

¹⁹ As in the passage from the *Republic*, it seems we can rise to definitions, *arkhe*, through demonstration.

²⁰ Suggests that some definitions can also be hypotheses.

²¹ Because it does not assert anything about either a whole class or any particulars. But is this the same idea of definition as Plato has? It seems more nominalist. Socrates seeks out definitions that are assertions about a class of beings, and can be tested for adequacy against any individuals that seem to fall into that class. Thus, once we have defined justice, the definition can be refuted ‘empirically’ if we can cite an action that seems obviously to fall into the category justice, yet which would be excluded by the definition. For the Platonists, then, there is not an iron-clad distinction between hypothesis and definition.

words, if essential form is knowable, and an infinite series cannot be traversed, predicates constituting a thing's essential nature must be finite in number.

Now attributes may be essential for two reasons: either because they are elements in the essential nature of their subjects, or because their subjects are elements in their essential nature. An example of the latter is odd as an attribute of number-though it is number's attribute, yet number itself is an element in the definition of odd; of the former, multiplicity or the indivisible, which are elements in the definition of number.

Book II

Ch. 3

Let us now state how essential nature is revealed and in what way it can be reduced to demonstration; what definition is, and what things are definable. And let us first discuss certain difficulties which these questions raise, beginning what we have to say with a point most intimately connected with our immediately preceding remarks, namely the doubt that might be felt as to whether or not it is possible to know the same thing in the same relation, both by definition and by demonstration. It might, I mean, be urged that **definition** is held to concern essential nature and is in every case **universal and affirmative**; whereas, on the other hand, some conclusions are negative and some are not universal; e.g. all in the second *skhêma* are negative, none in the third are universal. And again, not even all affirmative conclusions in the first *skhêma* are definable, e.g. 'every triangle has its angles equal to two right angles'. An argument proving this difference between demonstration and definition is that to have scientific knowledge of the demonstrable is identical with possessing a demonstration of it: hence if demonstration of such conclusions as these is possible, there clearly cannot also be definition of them. If there could, one might know such a

conclusion also in virtue of its definition without possessing the demonstration of it; for there is nothing to stop our having the one without the other.

Induction too will sufficiently convince us of this difference; for never yet by defining anything-essential attribute or accident-did we get knowledge of it. Again, if to define is to acquire knowledge of a substance, at any rate such attributes are not substances.

It is evident, then, that not everything demonstrable can be defined. What then? Can everything definable be demonstrated, or not? There is one of our previous arguments which covers this too. Of a single thing qua single there is a single scientific knowledge. Hence, since to know the demonstrable scientifically is to possess the demonstration of it, an impossible consequence will follow:- possession of its definition without its demonstration will give knowledge of the demonstrable.

Moreover, the basic premises of demonstrations are definitions, and it has already been shown that these will be found indemonstrable; either the basic premises will be demonstrable and will depend on prior premises, and the regress will be endless; or the primary truths will be indemonstrable definitions.

But if the definable and the demonstrable are not wholly the same, may they yet be partially the same? Or is that impossible, because there can be no demonstration of the definable? There can be none, because definition is of the essential nature or being of something, and all demonstrations evidently posit and assume the essential nature-mathematical demonstrations, for example, the nature of unity and the odd, and all the other sciences likewise. Moreover, every demonstration proves a predicate of a subject as attaching or as not attaching to it, but in definition one thing is not predicated of another; we do not, e.g. predicate animal of biped nor biped of animal, nor yet *skhêma* of plane-plane not being *skhêma* nor *skhêma* plane. Again, to prove essential nature is not the same as to prove the *fact* of a connection. Now definition reveals essential nature, demonstration reveals that a given attribute attaches or does not attach to a given

subject; but different things require different demonstrations-unless the one demonstration is related to the other as part to whole. I add this because if all triangles have been proved to possess angles equal to two right angles, then this attribute has been proved to attach to isosceles; for isosceles is a part of which all triangles constitute the whole. But in the case before us the fact and the essential nature are not so related to one another, since the one is not a part of the other.

So it emerges that not all the definable is demonstrable nor all the demonstrable definable; and we may draw the general conclusion that there is **no identical object of which it is possible to possess both a definition and a demonstration.** It follows obviously that definition and demonstration are neither identical nor contained either within the other: if they were, their objects would be related either as identical or as whole and part.

Ch. 4

So much, then, for the first stage of our problem. The next step is to raise the question whether syllogism-i.e. demonstration-of the definable nature is possible or, as our recent argument assumed, impossible.

We might argue it impossible on the following grounds:-(a) syllogism proves an attribute of a subject through the middle term; on the other hand (b) its definable nature is both 'peculiar' to a subject and predicated of it as belonging to its essence. But in that case (1) the subject, its definition, and the middle term connecting them must be reciprocally predicable of one another; for if A is to C, obviously A is 'peculiar' to B and B to C-in fact all three terms are 'peculiar' to one another: and further (2) if A inheres in the essence of all B and B is predicated universally of all C as belonging to C's essence, A also must be predicated of C as belonging to its essence.

If one does not take this relation as thus duplicated-if, that is, A is predicated as being of the essence of B, but B is not of the essence of the subjects of which it is predicated-A will not necessarily be predicated of C as belonging to its essence.

So both premises will predicate essence, and consequently B also will be predicated of C as its essence. Since, therefore, both premises do predicate essence-i.e. definable form-C's definable form will appear in the middle term before the conclusion is drawn.

We may generalize by supposing that it is possible to prove the essential nature of man. Let C be man, A man's essential nature—two-footed animal, or aught else it may be. Then, if we are to syllogize, A must be predicated of all B. But this premise will be mediated by a fresh definition, which consequently will also be the essential nature of man. Therefore the argument assumes what it has to prove, since B too is the essential nature of man. It is, however, the case in which there are **only the two premises-i.e. in which the premises are primary and immediate**-which we ought to investigate, because it best illustrates the point under discussion.

Thus they who prove the essential nature of soul or man or anything else through reciprocating terms beg the question. It would be begging the question, for example, to contend that the soul is that which causes its own life, and that what causes its own life is a self-moving number; for one would have to postulate that the soul is a self-moving number in the sense of being identical with it. For if A is predicable as a mere consequent of B and B of C, A will not on that account be the definable form of C: A will merely be what it was true to say of C. Even if A is predicated of all B inasmuch as B is identical with a species of A, still it will not follow: being an animal is predicated of being a man-since it is true that in all instances to be human is to be animal, just as it is also true that every man is an animal-but not as identical with being man.

We conclude, then, that unless one takes both the premises as predicating essence, one cannot infer that A is the definable form and essence of C: but if one does so take them, in assuming B one will have assumed, before drawing the conclusion, what the definable form of C is; so that there has been no inference, for one has begged the question.

Nor, as was said in my formal logic, is the method of division a process of inference at all, since at no point does the characterization of the subject follow necessarily from the premising of certain other facts: division demonstrates as little as does induction. For in a genuine demonstration the conclusion must not be put as a question nor depend on a concession, but must follow necessarily from its premises, even if the respondent denies it. The definer asks ‘Is man animal or inanimate?’ and then assumes-he has not inferred-that man is animal. Next, when presented with an exhaustive division of animal into terrestrial and aquatic, he **assumes** that man is terrestrial. Moreover, that man is the complete formula, terrestrial-animal, does not follow necessarily from the premises: this too is an assumption, and equally an assumption whether the division comprises many differentiae or few. (Indeed as this method of division is used by those who proceed by it, even truths that can be inferred actually fail to appear as such.) For why should not the whole of this formula be true of man, and yet not exhibit his **essential nature** or **definable form**? Again, what guarantee is there against an inessential addition, or against the omission of the final or of an intermediate determinant of the substantial being?

The champion of division might here urge that though these lapses do occur, yet we can solve that difficulty if all the attributes we assume are constituents of the definable form, and if, postulating the genus, we produce by division the requisite uninterrupted sequence of terms, and omit nothing; and that indeed we cannot fail to fulfill these conditions if what is to be divided falls whole into the division at each stage, and none of it is omitted; and that this-the dividendum-must without further question be (ultimately) incapable of fresh specific division. Nevertheless, we reply, division does not involve inference; if it gives knowledge,

it gives it in another way. **USE THIS AS A FOOTNOTE FOR Theaetetus.** Nor is there any absurdity in this: induction, perhaps, is not demonstration any more than is division, yet it does make evident some truth. Yet **to state a definition reached by division is not to state a conclusion**: as, when conclusions are drawn without their appropriate middles, the alleged necessity by which the inference follows from the premises is open to a question as to the reason for it, so definitions reached by division invite the same question.

Thus to the question ‘What is the essential nature of man?’ the divider replies ‘Animal, mortal, footed, biped, wingless’; and when at each step he is asked ‘Why?’, he will say, and, as he thinks, proves by division, that all animal is mortal or immortal: but **such a formula taken in its entirety is not definition**; so that even if division does demonstrate its formula, definition at any rate does not turn out to be a conclusion of inference.

7

How then by definition shall we prove substance or essential nature? We cannot show it as a fresh fact necessarily following from the assumption of premises admitted to be facts—the method of demonstration: we may not proceed as by induction to establish a universal on the evidence of groups of particulars which offer no exception, because **induction proves not what the essential nature of a thing is but that it has or has not some attribute**. Therefore, since presumably one cannot prove essential nature by an appeal to sense perception or by pointing with the finger, what other method remains?

To put it another way: how shall we by definition prove essential nature? He who knows what human-or any other-nature is, must know also that man exists; for **no one knows the nature of what does not exist—one can know the meaning of the phrase or name ‘goat-stag’ but not what the essential nature of a goat-**

stag is. But further, if definition can prove what is the essential nature of a thing, can it also prove that it exists? And how will it prove them both by the same process, since definition exhibits one single thing and demonstration another single thing, and what human nature is and the fact that man exists are not the same thing? Then too we hold that **it is by demonstration that the being of everything must be proved-unless indeed to be were its essence; and, since being is not a genus, it is not the essence of anything.** Hence the being of anything as fact is matter for demonstration; and this is the actual procedure of the sciences, for the **geometer assumes the meaning of the word triangle, but that it is possessed of some attribute he proves.** What is it, then, that we shall prove in defining essential nature? Triangle? In that case a man will **know by definition what a thing's nature is without knowing whether it exists.** But that is **impossible.**

Moreover it is clear, if we consider the methods of defining actually in use, that definition does not prove that the thing defined exists: since **even if there does actually exist something which is equidistant from a centre, yet why should the thing named in the definition exist? Why, in other words, should this be the formula defining circle?** One might equally well call it the definition of mountain copper. For definitions do not carry a further guarantee that the thing defined can exist or that it is what they claim to define: one can always ask why. **MOST OF HIS EXAMPLES ARE MATHEMATICAL.**

Since, therefore, **to define is to prove either a thing's essential nature or the meaning of its name,** we may conclude that definition, if it in no sense proves essential nature, is a set of words signifying precisely what a name signifies. But that were a strange consequence; for (1) both what is not substance and what does not exist at all would be definable, since even non-existents can be signified by a name: (2) all sets of words or sentences would be definitions, since any kind of

sentence could be given a name; so that we should all be talking in definitions, and even the Iliad would be a definition: (3) no demonstration can prove that any particular name means any particular thing: neither, therefore, do definitions, in addition to revealing the meaning of a name, also reveal that the name has this meaning. It appears then from these considerations that neither definition and syllogism nor their objects are identical, and further that definition neither demonstrates nor proves anything, and that **knowledge of essential nature is not to be obtained either by definition or by demonstration.**

10

Since **definition** is said to be the **statement of a thing's nature**, obviously **one kind** of definition will be a **statement of the meaning of the name**, or of an equivalent **nominal formula**. A definition in this sense tells you, e.g. the meaning of the phrase 'triangular character'. When we are aware that triangle exists, we inquire the reason why it exists. But it is difficult thus to learn the definition of things the existence of which we do not genuinely know-the cause of this difficulty being, as we said before, that we only know accidentally whether or not the thing exists. Moreover, a statement may be a unity in either of two ways, by conjunction, like the Iliad, or because it exhibits a single predicate as inhering not accidentally in a single subject.

That then is one way of **defining definition**. **Another kind** of definition is a **formula exhibiting the cause of a thing's existence**. Thus the former signifies without proving, but the latter will clearly be a **quasi-demonstration of essential nature**, differing from demonstration in the arrangement of its terms. For there is a **difference between stating why it thunders, and stating what is the essential nature of thunder**; since the first statement will be 'Because fire is quenched in the clouds', while the statement of what the nature of thunder is will be 'The noise of fire being quenched in the clouds'. Thus the same statement takes a different

form: in one form it is continuous demonstration, in the other definition. Again, **thunder can be defined as noise in the clouds, which is the conclusion of the demonstration embodying essential nature. On the other hand the definition of immediates is an indemonstrable positing of essential nature.**

We conclude then that **definition is (a) an indemonstrable statement of essential nature, or (b) a syllogism of essential nature differing from demonstration in grammatical form, or (c) the conclusion of a demonstration giving essential nature.**

Our discussion has therefore made plain (1) in what sense and of what things the essential nature is demonstrable, and in what sense and of what things it is not; (2) what are the various meanings of the term definition, and in what sense and of what things it proves the essential nature, and in what sense and of what things it does not; (3) what is the relation of definition to demonstration, and how far the same thing is both definable and demonstrable and how far it is not.

13

We have already explained how essential nature is set out in the terms of a demonstration, and the sense in which it is or is not demonstrable or definable; so let us now discuss the method to be adopted in tracing the elements predicated as constituting the definable form.

Now of the attributes which inhere always in each several thing there are some which are wider in extent than it but not wider than its genus (by attributes of wider extent mean all such as are universal attributes of each several subject, but in their application are not confined to that subject). while an attribute may inhere in every triad, yet also in a subject not a triad-as being inheres in triad but also in subjects not numbers at all-odd on the other hand is an attribute inhering in every triad and of wider application (inhering as it does also in pentad), but which

does not extend beyond the genus of triad; for pentad is a number, but nothing outside number is odd. It is such attributes which we have to select, up to the exact point at which they are severally of wider extent than the subject but collectively coextensive with it; for this synthesis must be the substance of the thing. For example every triad possesses the attributes number, odd, and prime in both senses, i.e. not only as possessing no divisors, but also as not being a sum of numbers. This, then, is precisely what triad is, viz. a number, odd, and prime in the former and also the latter sense of the term: for these attributes taken severally apply, the first two to all odd numbers, the last to the dyad also as well as to the triad, but, taken collectively, to no other subject. Now since we have shown above' that attributes predicated as belonging to the essential nature are necessary and that universals are necessary, and since the attributes which we select as inhering in triad, or in any other subject whose attributes we select in this way, are predicated as belonging to its essential nature, triad will thus possess these attributes necessarily. Further, that the synthesis of them constitutes the substance of triad is shown by the following argument. If it is not identical with the being of triad, it must be related to triad as a genus named or nameless. It will then be of wider extent than triad-assuming that wider potential extent is the character of a genus. If on the other hand this synthesis is applicable to no subject other than the individual triads, it will be identical with the being of triad, because we make the further assumption that the substance of each subject is the predication of elements in its essential nature down to the last differentia characterizing the individuals. It follows that any other synthesis thus exhibited will likewise be identical with the being of the subject.

The author of a hand-book on a subject that is a generic whole should divide the genus into its first infimae species-number e.g. into triad and dyad-and then endeavour to seize their definitions by the method we have described-the definition, for example, of straight line or circle or right angle. After that, having

established what the category is to which the subaltern genus belongs—quantity or quality, for instance—he should examine the properties ‘peculiar’ to the species, working through the proximate common differentiae. He should proceed thus because the attributes of the genera compounded of the infimae species will be clearly given by the definitions of the species; since the basic element of them all is the definition, i.e. the simple infirma species, and the attributes inhere essentially in the simple infimae species, in the genera only in *arête* of these.

Divisions according to differentiae are a useful accessory to this method. What force they have as proofs we did, indeed, explain above, but that merely towards collecting the essential nature they may be of use we will proceed to show. They might, indeed, seem to be of no use at all, but rather to assume everything at the start and to be no better than an initial assumption made without division. But, in fact, the order in which the attributes are predicated does make a difference—it matters whether we say animal-tame-biped, or biped-animal-tame. For if every definable thing consists of two elements and ‘animal-tame’ forms a unity, and again out of this and the further differentia man (or whatever else is the unity under construction) is constituted, then the elements we assume have necessarily been reached by division. Again, division is the only possible method of avoiding the omission of any element of the essential nature. Thus, if the primary genus is assumed and we then take one of the lower divisions, the dividendum will not fall whole into this division: e.g. it is not all animal which is either whole-winged or split-winged but all winged animal, for it is winged animal to which this differentiation belongs. The primary differentiation of animal is that within which all animal falls. The like is true of every other genus, whether outside animal or a subaltern genus of animal; e.g. the primary differentiation of bird is that within which falls every bird, of fish that within which falls every fish. So, if we proceed in this way, we can be sure that nothing has been omitted: by any other method one is bound to omit something without knowing it.

To define and divide one need not know the whole of existence. Yet some hold it impossible to know the differentiae distinguishing each thing from every single other thing without knowing every single other thing; and one cannot, they say, know each thing without knowing its differentiae, since everything is identical with that from which it does not differ, and other than that from which it differs. Now first of all this is a fallacy: not every differentia precludes identity, since many differentiae inhere in things specifically identical, though not in the substance of these nor essentially. Secondly, when one has taken one's differing pair of opposites and assumed that the two sides exhaust the genus, and that the subject one seeks to define is present in one or other of them, and one has further verified its presence in one of them; then it does not matter whether or not one knows all the other subjects of which the differentiae are also predicated. For it is obvious that when by this process one reaches subjects incapable of further differentiation one will possess the formula defining the substance. Moreover, to postulate that the division exhausts the genus is not illegitimate if the opposites exclude a middle; since if it is the differentia of that genus, anything contained in the genus must lie on one of the two sides.

In establishing a definition by division one should keep three objects in view: (1) the admission only of elements in the definable form, (2) the arrangement of these in the right order, (3) the omission of no such elements. The first is feasible because one can establish genus and differentia through the topic of the genus, just as one can conclude the inherence of an accident through the topic of the accident. The right order will be achieved if the right term is assumed as primary, and this will be ensured if the term selected is predicable of all the others but not all they of it; since there must be one such term. Having assumed this we at once proceed in the same way with the lower terms; for our second term will be the first of the remainder, our third the first of those which follow the second in a 'contiguous' series, since when the higher term is excluded, that term of the remainder which is

‘contiguous’ to it will be primary, and so on. Our procedure makes it clear that no elements in the definable form have been omitted: we have taken the differentia that comes first in the order of division, pointing out that animal, e.g. is divisible exhaustively into A and B, and that the subject accepts one of the two as its predicate. Next we have taken the differentia of the whole thus reached, and shown that the whole we finally reach is not further divisible-i.e. that as soon as we have taken the last differentia to form the concrete totality, this totality admits of no division into species. For it is clear that there is no superfluous addition, since all these terms we have selected are elements in the definable form; and nothing lacking, since any omission would have to be a genus or a differentia. Now the primary term is a genus, and this term taken in conjunction with its differentiae is a genus: moreover the differentiae are all included, because there is now no further differentia; if there were, the final concrete would admit of division into species, which, we said, is not the case.

To resume our account of the right method of investigation: We must start by observing a set of similar-i.e. specifically identical-individuals, and consider what element they have in common. We must then apply the same process to another set of individuals which belong to one species and are generically but not specifically identical with the former set. When we have established what the common element is in all members of this second species, and likewise in members of further species, we should again consider whether the results established possess any identity, and persevere until we reach a single formula, since this will be the definition of the thing. But if we reach not one formula but two or more, evidently the definiendum cannot be one thing but must be more than one. I may illustrate my meaning as follows. If we were inquiring what the essential nature of pride is, we should examine instances of proud men we know of to see what, as such, they have in common; e.g. if Alcibiades was proud, or Achilles and Ajax were proud, we should find on inquiring what they all had in

common, that it was intolerance of insult; it was this which drove Alcibiades to war, Achilles wrath, and Ajax to suicide. We should next examine other cases, Lysander, for example, or Socrates, and then if these have in common indifference alike to good and ill fortune, I take these two results and inquire what common element have equanimity amid the vicissitudes of life and impatience of dishonor. If they have none, there will be two genera of pride. Besides, every definition is always universal and commensurate: the physician does not prescribe what is healthy for a single eye, but for all eyes or for a determinate species of eye. It is also easier by this method to define the single species than the universal, and that is why our procedure should be from the several species to the universal genera—this for the further reason too that equivocation is less readily detected in genera than in infimae species. Indeed, perspicuity is essential in definitions, just as inferential movement is the minimum required in demonstrations; and we shall attain perspicuity if we can collect separately the definition of each species through the group of singulars which we have established e.g. the definition of similarity not unqualified but restricted to colors and to *skhêmai*; the definition of acuteness, but only of sound—and so proceed to the common universal with a careful avoidance of equivocation. We may add that if dialectical disputation must not employ metaphors, clearly metaphors and metaphorical expressions are precluded in definition: otherwise dialectic would involve metaphors.

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