The Cantorian Theory of Size (Part II)

1 Definitions

***A* is infinite:** $|\mathbb{N}| \leq |A|$

***A* is countable:** $|A| \leq |\mathbb{N}|$

2 A Little Lemma

No Countable Difference Principle: $|I| = |I \cup C|$, for I infinite and C countable.

3 Sets with the same size as $\mathbb{R}$

Set	Also known as	Members
$[0, 1)$	unit interval (half closed)	real numbers larger or equal to 0 but smaller than 1
$(0, 1)$	unit interval (open)	real numbers larger than 0 but smaller than 1
$[0, 1]$	unit interval (closed)	real numbers larger or equal to 0 but smaller or equal to 1
$[0, a]$	arbitrarily sized interval	real numbers larger or equal to 0 but smaller or equal to a ($a > 0$)
$[0, 1] \times [0, 1]$	unit square	pairs of real numbers larger or equal to 0 but smaller or equal to 1
$\underbrace{[0, 1] \times \cdots \times [0, 1]}_{n \text{ times}}$	n-dimensional hypercube	n-tuples of real numbers larger or equal to 0 but smaller or equal to 1
$\mathbb{R}$	real line	real numbers
$\wp(\mathbb{N})$	powerset of $\mathbb{N}$	sets of natural numbers

4 Summary of Cardinality Comparisons

$$
\begin{array}{ccccccccc}
|\mathbb{N}| & < & |\wp(\mathbb{N})| & < & |\wp(\wp(\mathbb{N}))| & < & |\wp(\wp(\wp(\mathbb{N})))| & < & \dots \\
\| & & \| & & & & & & \\
|\mathbb{Z}| & & |\mathbb{R}| & & & & & & \\
\| & & \| & & & & & & \\
|\mathbb{Q}| & & |[0,1]| & & & & & & \\
 & & \| & & & & & & \\
 & & |\underbrace{[0,1] \times \ldots \times [0,1]}_{n \text{ times}}| & & & & & &
\end{array}
$$

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