Subject 24.241. Logic I. Homework due Thursday, November 17.

I. Let $\mathfrak{A}$ be an interpretation with $|\mathfrak{A}|$ = the set of all animals, $\mathfrak{A}$("D") = the set of all dogs, $\mathfrak{A}$("M") = the set of all mammals, and $\mathfrak{A}$("d") = Davey the dog. Which of the following five formulas are satisfied in $\mathfrak{A}$ by Captain Snuggims, the rabbit?

a) $(Mx \rightarrow Dx)$
b) $(Mx \rightarrow (\exists x)Dx)$
c) $(Dx \rightarrow Mx)$
d) $(\forall x)(Dx \rightarrow Mx)$
e) $(\forall x)(Dd \leftrightarrow Mx)$

II. The following five sentences are all logically valid. Use the search-for-counterexamples method to determine which of them are also tautological:

a) $(\exists x)(Px \rightarrow (\forall x)Px)$
b) $((\exists x)Pd \rightarrow (\forall x)Pd)$
c) $(((\exists x)Px \rightarrow (\forall x)Px) \vee ((\exists x)\neg Px \rightarrow (\exists x)Px))$
d) $(((\exists x)Fx \wedge Gd) \rightarrow (\exists x)(Fx \wedge Gx))$
e) $(((\exists x)Fx \wedge Gd) \rightarrow ((\exists x)Fx \vee Gd))$

III. Give an MPC sentence that is true in some interpretation whose domain has six elements but isn't true in any interpretation whose domain has fewer than six elements.

IV. Give a list of fifteen sentences of the language with two predicates "F" and "G" and no individual constants such that each sentence of the list is complete, each sentence on the list is consistent, and no two sentences on the list are logically equivalent. (A sentence φ is *complete* if every sentence is either implied by φ or inconsistent with φ.)

V. Give a list of sentences of the language with no nonlogical symbols other that the predicate "F" such no two sentenes on the list are logically equivalent, and every sentence is logically equivalent to an item on the list.

VI. Consider a language with two predicates, "F" and "G," and two individual constants "c" and "d."

a) How many sentences would be on the longest list one could give of complete, consistent sentences no two of which are logially equivalent?
b) If you were to give a list of sentences, no two of which were logically equivalent, with the property that every sentence was logically equivalent to a sentence on the list, how many sentences would be on it?