Semantics

Question 1

To prove:

- '$\sim (\exists x)(Fx \,\&\, Gx)$' is true on I.

Proof:

1. '$\sim (\exists x)(Fx\&Gx)$' is true on I iff every variable assignment d for I satisfies '$\sim (\exists x)(Fx \,\&\, Gx)$' (by the definition of truth).
2. Let d_0 be an arbitrary variable assignment for I. d_0 satisfies '$\sim (\exists x)(Fx\&Gx)$' iff d_0 does not satisfy '$(\exists x)(Fx\&Gx)$' (by clause 3 of the definition of satisfaction).
3. d_0 does not satisfy '$(\exists x)(Fx \,\&\, Gx)$' iff there is no $\mathbf{u} \in UD$ such that $d_0[\mathbf{u}/\text{`}x\text{'}]$ satisfies '$(Fx\&Gx)$' (by clause 9 of the definition of satisfaction).
4. There is no $\mathbf{u} \in UD$ such that $d_0[\mathbf{u}/\text{`}x\text{'}]$ satisfies '$(Fx \,\&\, Gx)$' iff there is no $\mathbf{u} \in UD$ such that $d_0[\mathbf{u}/\text{`}x\text{'}]$ satisfies both 'Fx' and 'Gx' (by clause 4 of the definition of satisfaction).
5. There is no $\mathbf{u} \in UD$ such that $d_0[\mathbf{u}/\text{`}x\text{'}]$ satisfies both '$Fx$' and '$Gx$' iff there is no $\mathbf{u} \in UD$ such that $\langle d_0[\mathbf{u}/\text{`}x\text{'}](\text{`}x\text{'})\rangle \in I(\text{`}F\text{'})$ and $\langle d_0[\mathbf{u}/\text{`}x\text{'}](\text{`}x\text{'})\rangle \in I(\text{`}G\text{'})$ (by clause 2 in the definition of satisfaction (and the definition of denotation)).
6. $\langle a\rangle \notin I(\text{`}F\text{'})$ and $\langle b\rangle \notin I(\text{`}G\text{'})$, so there is no $\mathbf{u} \in UD$ such that $\langle d_0[\mathbf{u}/\text{`}x\text{'}](\text{`}x\text{'})\rangle \in I(\text{`}F\text{'})$ and $\langle d_0[\mathbf{u}/\text{`}x\text{'}](\text{`}x\text{'})\rangle \in I(\text{`}G\text{'})$.
7. So d_0 satisfies '$\sim (\exists x)(Fx \,\&\, Gx)$' (by 2-6).
8. d_0 was arbitrary, so every variable assignment d for I satisfies '$\sim (\exists x)(Fx\&Gx)$' (by 7).
9. So '$\sim (\exists x)(Fx \,\&\, Gx)$' is true on I (by 1, 8).

Q.E.D.

Question 2

To prove:

- '$(\forall x)(Fx \equiv Gx)$' is false on I.

Proof:

1. '$(\forall x)(Fx \equiv Gx)$' is false on I iff no variable assignment d for I satisfies '$(\forall x)(Fx \equiv Gx)$' (by the definition of falsehood).

2. Let d_0 be an arbitrary variable assignment for I. d_0 does not satisfy ‘$(\forall x)(Fx \equiv Gx)$’ iff for for some $\mathbf{u} \in UD$, $d_0[\mathbf{u}/‘x’]$ does not satisfy ‘$(Fx \equiv Gx)$’ (by clause 8 of the definition of satisfaction).

3. There is some $\mathbf{u} \in UD$ such that $d_0[\mathbf{u}/‘x’]$ does not satisfy ‘$(Fx \equiv Gx)$’ iff there is some $\mathbf{u} \in UD$ such that either $d_0[\mathbf{u}/‘x’]$ satisfies ‘Fx’ and not ‘Gx’, or $d_0[\mathbf{u}/‘x’]$ satisfies ‘Gx’ and not ‘Fx’ (by clause 7 of the definition of satisfaction).

4. There is some $\mathbf{u} \in UD$ such that $d_0[\mathbf{u}/‘x’]$ satisfies ‘Fx’ and not ‘Gx’ iff there is some $\mathbf{u} \in UD$ such that $\langle d_0[\mathbf{u}/‘x’](‘x’)\rangle \in I(‘F’)$ and $\langle d_0[\mathbf{u}/‘x’](‘x’)\rangle \notin I(‘G’)$ (by clause 2 of the definition of satisfaction (and the definition of denotation)).

5. a is such that $\langle d_0[a/‘x’](‘x’)\rangle \in I(‘F’)$ and $\langle d_0[a/‘x’](‘x’)\rangle \notin I(‘G’)$.

6. So there is some $\mathbf{u} \in UD$ such that $\langle d_0[\mathbf{u}/‘x’](‘x’)\rangle \in I(‘F’)$ and $\langle d_0[\mathbf{u}/‘x’](‘x’)\rangle \notin I(‘G’)$ (by 5).

7. So there is there is some $\mathbf{u} \in UD$ such that either $d_0[\mathbf{u}/‘x’]$ satisfies ‘Fx’ and not ‘Gx’ (by 4, 6).

8. So there is some $\mathbf{u} \in UD$ such that either $d_0[\mathbf{u}/‘x’]$ satisfies ‘Fx’ and not ‘Gx’, or $d_0[\mathbf{u}/‘x’]$ satisfies ‘Gx’ and not ‘Fx’ (by 7).

9. So d_0 does not satisfy ‘$(\forall x)(Fx \equiv Gx)$’ (by 2, 3, 8).

10. d_0 was arbitrary, so no variable assignment d for I satisfies ‘$(\forall x)(Fx \equiv Gx)$’ (by 9).

11. So ‘$(\forall x)(Fx \equiv Gx)$’ is false on I (by 1, 10).

Q.E.D.

Syntax (10.1E)

Question 1

Part (a)

1	$(\forall x)Fx$	A
2	Fa	1, $\forall$E
3	$(\forall y)Fy$	2, $\forall$I

Part (d)

1	$(\exists x)(Fx \,\&\, Gx)$	A
2	$\quad Fa \,\&\, Ga$	A/∃E
3	$\quad Fa$	2, &E
4	$\quad Ga$	2, &E
5	$\quad (\exists y)Fy$	3, ∃I
6	$\quad (\exists w)Gw$	4, ∃I
7	$(\exists y)Fy$	1, 2-5, ∃E
8	$(\exists w)Gw$	1, 2-6, ∃E
9	$(\exists y)Fy \,\&\, (\exists w)Gw$	7, 8, & I

Part (j)

1	$(\forall x)(Fx \supset Lx)$	A
2	$(\exists y)Fy$	A
3	$\quad Fa$	A/∃E
4	$\quad Fa \supset La$	1, ∀E
5	$\quad La$	3, 4, ⊃E
6	$\quad (\exists x)Lx$	5, ∃I
7	$(\exists x)Lx$	2, 3-6, ∃E

Question 2

Part (a)

The mistake is in line 3; this is supposed to be an application of universal elimination, but the sentence to which the rule was applied is not a universally quantified sentence; it is, rather, a conditional. Universal elimination can only be applied to a universally quantified sentence.

Part (b)

The mistake is in line 5. One cannot apply universal introduction to a sentence that contains a constant that is in an open assumption. The sentence on line four contains such a constant — viz., the ‘k’. (The sentence on line 1 is the open assumption containing ‘k’.) So the application of universal introduction to line 4 to get line 5 is disallowed.

Part (c)

The important mistake is the one on line 3: the incorrect application of existential elimination. Existential elimination brings things out from sub-derivations; you can't use existential elimination to go from a sentence on a particular scope line directly to a sentence on the same scope line.

There is also a typo on line 4; the '∃' shouldn't be there. But this is not the important (or, I gather, intended) mistake.

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