**Logic I**
**Fall 2009**
**Session 4 Handout**

- Using shortened truth-tables (STTs) to determine the TV of a compound given TVs for the atomic components.

| A | B | C | $\sim$(A $\vee$ B) $\equiv$ (C $\supset$ A) |
|---|---|---|---|
| F | T | T | |

- Building STTs in reverse to determine whether a compound sentence can possibly have a certain TV.

| J | K | J & ( $\sim$K $\vee$ $\sim$J) |
|---|---|---|
| | | |

- P is truth-functionally true iff . . .
  Can be proven by. . .
  Can be refuted by. . .

- P is truth-functionally false iff . . .
  Can by proven by. . .
  Can be refuted by. . .

- Notions and notation to know
  - Curly braces for sets: {A, B, {C}, {C, D}, . . . }
  - Set union. {A, B, C} $\cup${ B, C, D} =
  - Variables for sets of sentences:$\Gamma_n$
  - The empty set: $\emptyset$ or { }.
  - Unit / singleton set

- $\Gamma$ is truth-functionally consistent iff . . .
  Can be established by. . .
  Inconsistency of $\Gamma$ can be established by. . .

- A set $\Gamma$ of SL sentences *truth-functionally entails* a sentence P iff no TVA makes every member of $\Gamma$ true but P is false. In other words, P is true on every TVA that makes all members of $\Gamma$true.
  Notation:$\Gamma_1 \vDash$ P.
  Also, note: $\vDash \Gamma$abbreviates $\emptyset \vDash \Gamma$.

- An argument in SL is a set of SL sentences with one designated as the conclusion and the rest designated as premises. E.g.:

  Argument ($\star$):
  $$\begin{array}{l} A \supset ( B \vee C ) \\ B \equiv C \\ \sim B \\ \hline \sim A \end{array}$$

- $\begin{array}{l} P_1 \\ \vdots \\ P_n \\ \hline C \end{array}$ is truth-functionally valid iff no TVA makes $P_1, \ldots P_n$ true and C false.

- We can connect truth-functional entailment with truth-functional validity:

  In the definition of truth-functional entailment, let $\Gamma$ be $\{P_1, \ldots P_n\}$, and let P be C.

- Thus, $\begin{array}{l} P_1 \\ \vdots \\ P_n \\ \hline C \end{array}$ is valid iff $\{P_1, \ldots P_n\}$ truth-functionally entails C.

- Prove that Argument ($\star$) is valid and that its premises truth-functionally entail its conclusion by means of a truth-table.

| A | B | C | |
|---|---|---|---|
| | | | |

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