Logic - Session 18

Applying our formal semantics

- Let I be the following interpretation:
  UD={a,b}   F:{<a>}   G:{<b>}
- Show that (∀x)Fx is false on I.
  - <b> ∉ I(F)
  - So for arbitrary d, $\langle den_{I,d[b/x]}(x) \rangle \notin I(F)$.
  - So by (2.), d[b/x] doesn't satisfy Fx on I.
  - So not: for every u∈UD, d[u/x] satisfies Fx on I.
  - So by (8.), not: d satisfies (∀x)Fx.
  - So not every variable assignment satisfies (∀x)Fx.
  - So by def. of truth, (∀x)Fx is false on I.

Show: $(\exists x)(Fx \supset (\forall y)Gy)$ is true on I

- $(\exists x)(Fx \supset (\forall y)Gy)$ is true on I iff every d for I satisfies $(\exists x)(Fx \supset (\forall y)Gy)$ on I.
- By (9.), d satisfies $(\exists x)(Fx \supset (\forall y)Gy)$ on I iff for some $u \in UD$, d[u/x] satisfies $(Fx \supset (\forall y)Gy)$ on I.
- By (6.), for some $u \in UD$, d[u/x] satisfies $(Fx \supset (\forall y)Gy)$ on I iff for some $u \in UD$, either d[u/x] doesn't satisfy Fx on I or d[u/x] satisfies $(\forall y)Gy$ on I.
  - Prove the RHS.

- $\langle b \rangle \notin I(F)$
- So for arbitrary d, $\langle den_{I,d[b/x]}(x) \rangle \notin I(F)$.
- So by (2.), d[b/x] doesn't satisfy Fx on I.
- So for some u∈UD, d[u/x] doesn't satisfy Fx on I.
- So for some u∈UD, either d[u/x] doesn't satisfy Fx on I or d[u/x] satisfies (∀y)Gy on I.
- So for some u∈UD, d[u/x] satisfies (Fx⊃(∀y)Gy) on I.
- So d satisfies (∃x)(Fx⊃(∀y)Gy) on I.
- We picked an arbitrary d, so every d for I satisfies (∃x)(Fx⊃(∀y)Gy) on I.
- So (∃x)(Fx⊃(∀y)Gy) is true on I.

Show quantificationally true: $(\forall x)(Rxx \supset (\exists y)Rxy)$

- $(\forall x)(Rxx \supset (\exists y)Rxy)$ is q-true iff it's true on any I.
- $(\forall x)(Rxx \supset (\exists y)Rxy)$ is true on any I iff for any I, every d for I satisfies $(\forall x)(Rxx \supset (\exists y)Rxy)$ on I.
- Pick an arbitrary I and an arbitrary d.
- d satisfies $(\forall x)(Rxx \supset (\exists y)Rxy)$ on I iff for any $u \in UD$, d[u/x] satisfies $(Rxx \supset (\exists y)Rxy)$ on I.
- (For any $u \in UD$ d[u/x] satisfies $(Rxx \supset (\exists y)Rxy)$ on I) iff (for any $u \in UD$, either d[u/x] doesn't satisfy Rxx or d[u/x] satisfies $(\exists y)Rxy$ on I.
- Prove that the right-hand side is true by reductio.

- Suppose RHS is false. Then for some u∈UD, d[u/x] satisfies Rxx on I and doesn't satisfy (∃y)Rxy on I.
- Pick an arbitrary u such that d[u/x] satisfies Rxx on I.
- So by (2.), $<den_{I,d[u/x][u/y]}(y), den_{I,d[u/x][u/y]}(x)> \in I(R)$.
- So $<u,u> \in I(R)$
- So for some v∈UD, $<den_{I,d[u/x][v/y]}(y), den_{I,d[u/x][v/y]}(x)> \in I(R)$.
- By (2.), for some v∈UD, d[u/x][v/y] satisfies Rxy on I.
- By (9.), d[u/x] satisfies (∃y)Rxy on I.
- u was arbitrary, for it's not the case that for some u∈UD, d[u/x] satisfies Rxx on I and doesn't satisfy (∃y)Rxy on I.

- Since it's not the case that for some $u \in UD$, $d[u/x]$ satisfies Rxx on I and doesn't satisfy $(\exists y)Rxy$ on I:
- For any $u \in UD$, either $d[u/x]$ doesn't satisfy Rxx or $d[u/x]$ satisfies $(\exists y)Rxy$ on I.
- We had:
  - (For any $u \in UD$, $d[u/x]$ satisfies $(Rxx \supset (\exists y)Rxy)$ on I) iff (for any $u \in UD$, either $d[u/x]$ doesn't satisfy Rxx or $d[u/x]$ satisfies $(\exists y)Rxy$ on I.
- The RHS is true, so the LHS is too.
- So for any $u \in UD$, $d[u/x]$ satisfies $(Rxx \supset (\exists y)Rxy)$ on I.
- So by (8.), d satisfies $(\forall x)(Rxx \supset (\exists y)Rxy)$ on I.
- d and I were arbitrary, so for any I, for any d, d satisfies $(\forall x)(Rxx \supset (\exists y)Rxy)$.
- So for any I, $(\forall x)(Rxx \supset (\exists y)Rxy)$ is true on I.

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