3.032 Problem Set 1
Fall 2007
Due: Start of Lecture, 09.14.07

1. The I35 bridge in Minneapolis collapsed in Summer 2007. The failure apparently occurred at a pin in the gusset plate of the truss supporting the bridge span, at a time when the loads on the bridge span were unequal due to construction crews working on the westbound side. We'll treat a simplified section of this truss (Fig. 1a) to consider whether this unbalanced load could have generated enough force for the truss to fail. We will examine the truss section denoted by the dotted lines, and replace the gusset plate with a pin joint.

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Figure 1a: Bridge truss with gusset plates at pins.

On the day of the bridge failure, construction crews occupied the westbound lanes (right side of Fig. 1b). Let **F** be the forces acting on the truss members by the regular traffic and **2F** be the forces acting on the truss by the heavier construction equipment. Taking into account the average mass of a car to be 1362 kg, we can estimate **F** to be ~13.35 kN.

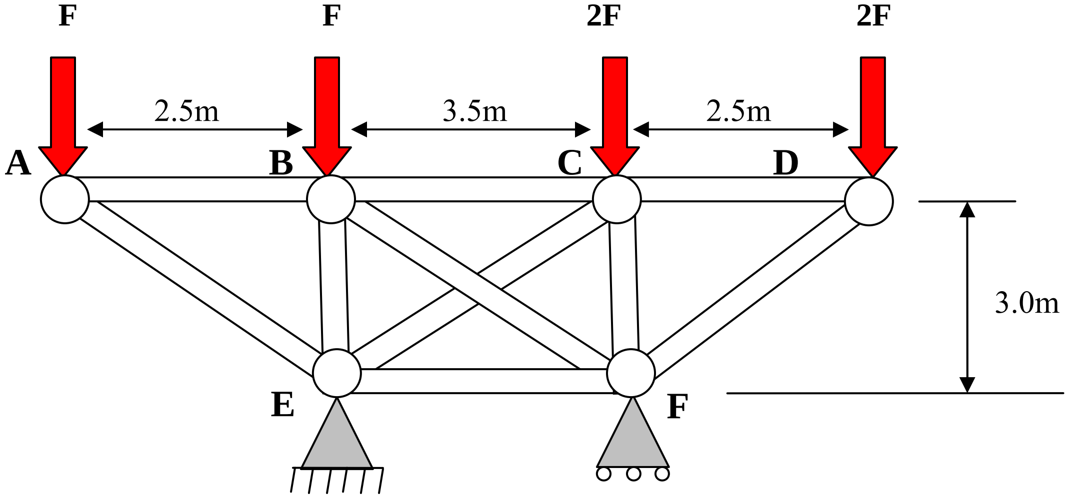
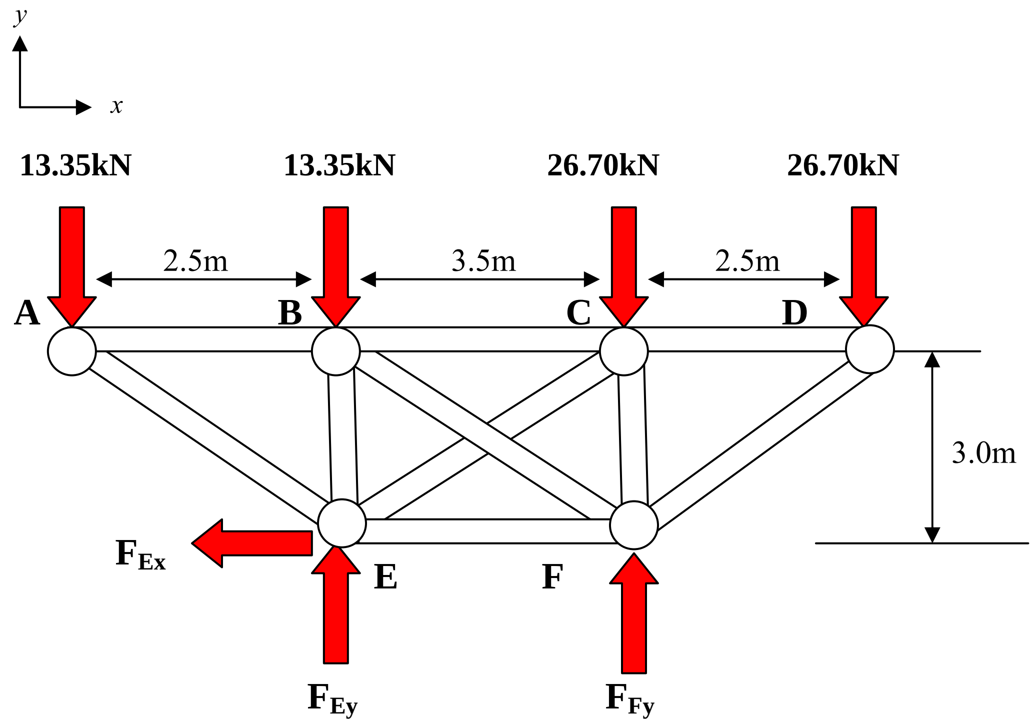


Figure 1b: Schematic of truss section with pin-loaded joints and supports.

(a) Determine the reaction forces at supports E and F. Show any free-body diagrams used in your calculations.

Solution: Here is the free-body diagram of the entire truss



First, we can take the sum of the forces in the x- and y- directions

$$\sum F_x = 0 : F_{Ex} = 0$$

$$\sum F_y = 0 : F_{Ey} + F_{Fy} - 2(13.35) - 2(26.70) = 0$$

$$F_{Ey} + F_{Fy} = 80.1kN$$

To eliminate one of the unknown variables in the above equation, we can take the sum of the moments about joint E

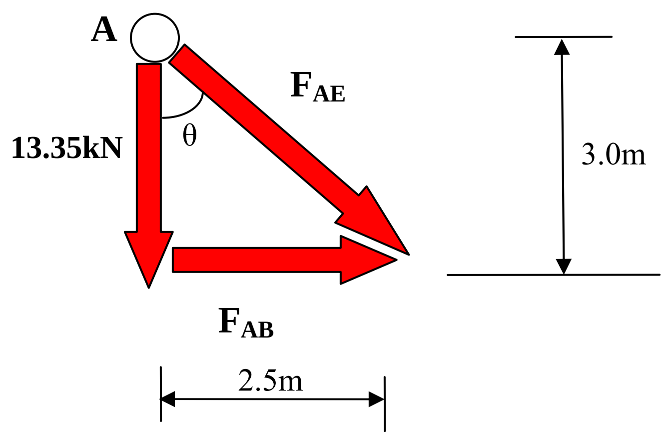
$$M^{+} \circlearrowleft \sum M_E = 0 : 13.35(2.5) - 26.70(3.5) - 26.70(6.0) + F_{Fy}(3.5) = 0$$

$$F_{Fy} = 63.0kN$$

$$F_{Ey} = 80.1kN - 63.0kN = 17.1kN$$

(b) Determine the forces in members AB and AE. Draw all the free-body diagrams you use in your calculations and show whether the forces on the members you calculate are in tension or compression.

Solution: First, using the method of joints, we will construct a force triangle about Joint A to determine the unknown forces in members AB and AE. We arbitrarily assume that both unknown forces are acting away from the joint (i.e. the members are in tension).



The angle, $\angle\theta$ *, can be determined from the truss dimensions as follows*

$$\tan(\theta) = \frac{2.5}{3.0}$$

$$\theta = 39.8°$$

Taking the sum of the forces in the y-directions we get

$$\sum F_y = 0: -13.35 - F_{AE}\cos(39.8^\circ) = 0$$
$$F_{AE} = -17.38kN = 17.38kN(C)$$

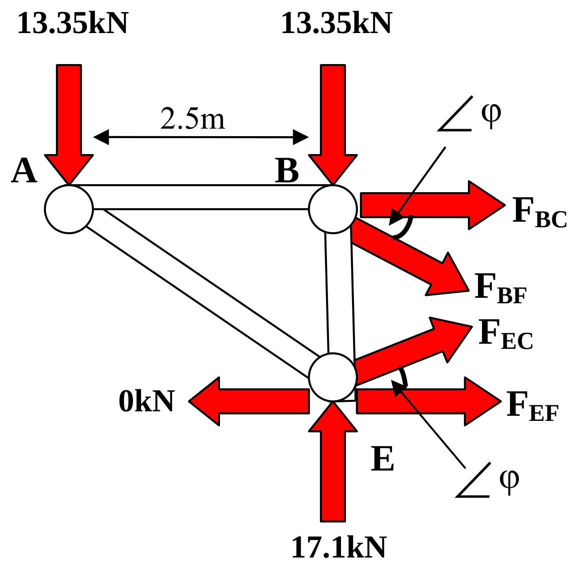
Note: Since we obtained a negative sign for $\mathbf{F_{AE}}$, we know that our original assumption about the direction of force $\mathbf{F_{AE}}$ was incorrect. Therefore, it must point towards joint A (i.e., member AE is in compression).

Then, taking the sum of the forces in the x-directions we get

$$\sum F_x = 0: F_{AB} - 17.38\sin(39.8^\circ) = 0$$
$$F_{AB} = 11.13kN(T)$$

(c) The diagonal members BF and EC are called counters, thin cables that are designed to be loaded only in tension. It is known that the support pins at E and F will fail if any of the counters is under a tensile force greater than 10 kN. From the loading on the truss described above, determine the forces on each of the members BF and EC and state which, if any, of the pins will fail. Show all the free-body diagrams associated with your calculations.

Solution: This problem takes a little bit of intuition in order to solve. If we use the method of sections and take a look at the portion ABE (cut is vertically through the middle of the truss) we get the following



Let us examine the known vertical forces acting on this portion. The total downward force is equal to 26.70kN (13.35x2), which is greater than the one upward force, 17.1kN, being exerted by the pin at E. Therefore, in order for this portion of the truss to be in equilibrium, there must be another upward force to balance out the forces in the y-direction. Looking at the four unknown forces ($\mathbf{F_{BC}}$, $\mathbf{F_{BF}}$, $\mathbf{F_{EC}}$, *and* $\mathbf{F_{EF}}$*) we can see that only the force* $\mathbf{F_{EC}}$ *can provide the upward force necessary for equilibrium and that it must be in tension. Thus, member BF is slack (i.e., does not provide any forces necessary to obtain equilibrium), and so* $\mathbf{F_{BF}} = 0$.

Using the above reasoning, we can now eliminate the force $\mathbf{F_{BF}}$ *from our analysis of portion ABE. To obtain the force in the other counter member, EC, we simply take the sum of the forces in the y-direction using the free-body diagram shown above.*

$$\tan(\varphi) = \frac{3}{3.5}$$

$$\varphi = 40.6^{\circ}$$

$$\sum F_y = 0 : 17.1 - 13.35 - 13.35 + F_{EC}\sin(40.6^{\circ}) = 0$$

$$F_{EC} = 14.75kN(T)$$

Therefore, the pin at E will fail because $\mathbf{F_{EC}}$ *= 14.75kN>10kN. The pin at F will not fail as there are no forces acting on member BF.*

2. The MIT varsity diving team is practicing their back flips on the Z-center's diving boards (Fig. 2). The diving boards have a total length of 3 m, width of 0.5 m, and thickness of 0.1 m. The core of the boards is made of a Douglas fir wood with an estimated Young's modulus E of 0.6GN/m^2. Let us approximate the diving board as a clamped-free beam, neglecting the springboard support at the midspan of the diving board and the weight of the diving board.

Image courtesy flickr user Bret Arnett.

Figure 2: Z-center diving boards.

(a) If the average weight of a student standing on the free-end of the board is about 155 lbs, how much will the board deflect? Note that, in the US, "weight" is commonly stated in units of mass [pounds, or lbs] instead of force, and state your solution in SI MKS units.

Solution: The deflection of a clamped-free beam under a point load at the end of the beam is

$$\delta = \frac{PL^3}{3EI}$$

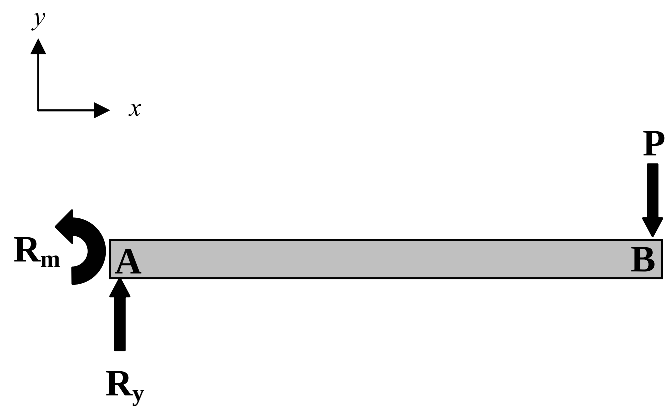
$$I = \frac{1}{12} wh^3 = 4.17 \times 10^{-5} m^4$$

$$P = mg = (155 lbs / 2.2 \frac{lbs}{kg}) * 9.81 m/s^2 = 691N$$

$$\delta = \frac{691 * (3.0)^3}{3 * (0.6 \times 10^9) * (4.17 \times 10^{-5})} = 0.25m$$

(b) Draw a free-body diagram of the diving board, again neglecting the spring board near the midspan for simplicity, and determine all reaction forces and moments for the "typical diver" in (a).

Solution:



To solve for R_y we simply take the sum of the forces in the y-direction and set them equal to zero according to equilibrium conditions

$$\sum F_y = 0 : R_y - 691 = 0$$

$R_y = 691N$ (upward direction)

To solve for R_m, we now take the moment about point A. Note that we arbitrarily assumed R_m to be positive (counter-clockwise in our convention) in our free-body diagram

$$M^+ \circlearrowleft \sum M_A = 0 : R_m - (691)(3) = 0$$

R_m = 2,073 N-m

3. Bundles of protein filaments inside tissue cells are structural elements that can generate contractile force against the materials to which they adhere. Figure 3 shows struts called "stress fibers" that extend from one end of the cell to the other. These fibers are made of crosslinked, filamentous actin, with an average diameter of 2 μm and an effective "stiffness" E of 16 kN/m^2.

If we assume the connections at either end of the stress fiber can be modeled as pins, how much force P would the cell need to generate to buckle the longest of these stress fibers?

Images removed due to copyright restrictions. Please see
http://www.biology.arizona.edu/Cell_bio/tutorials/cytoskeleton/graphics/microfilament.gif

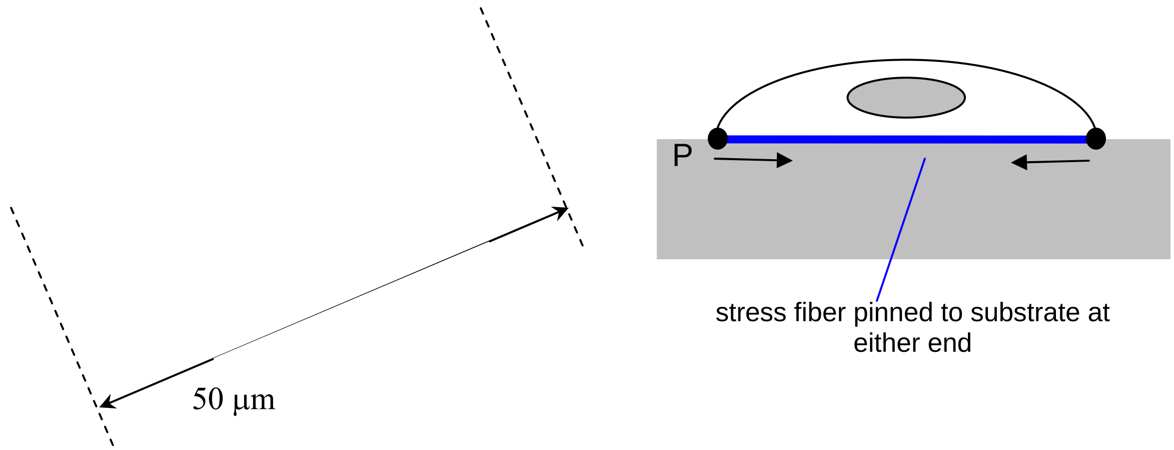


Figure 3: Structural elements of cells under axial end-loads.

Solution:

Here, the force is generated BY the motor proteins on the fiber, rather than an external force, but we'll treat this as an end-loaded beam. The stress fiber is much longer than it is wide, so we will assume the first mode of buckling (n=1) and Euler's solution of critical buckling load:

$P = \pi^2 EI/L_e^2$

Effective length $L_e = L$ *for these supports (50 µm), and* $I = \pi r^4/4 = \pi(1\ \mu m)^4/4 = 7.8 \times 10^{-25}\ m^4$.

$P = [\pi^2 * 16\ \times 10^3\ N/m^4 * 7.8 \times 10^{-25}\ m^4] / (50 \times 10^{-6}\ m)^2 = 4.9 \times 10^{-11}\ N = 50\ pN.$

This is a very small force, on the same order as that required to pull two bound molecules apart! Thus, buckling is plausible if the cell's motor proteins that generate the contractile force can exert at least 50 pN.

4. A U.S. based company was in charge of designing MIT's new physics lab, near the new DMSE headquarters in Bldg 6. They used 30-ft long, wide-flanged structural steel beams for the floor joists to carry the distributed weight of the rooms above. The actual weight distribution is shown in Fig. 4, which is different from what they anticipated, so they want to know where to add an extra support to minimize the shear force and bending moment on this beam. This week, you'll get started on that calculation.

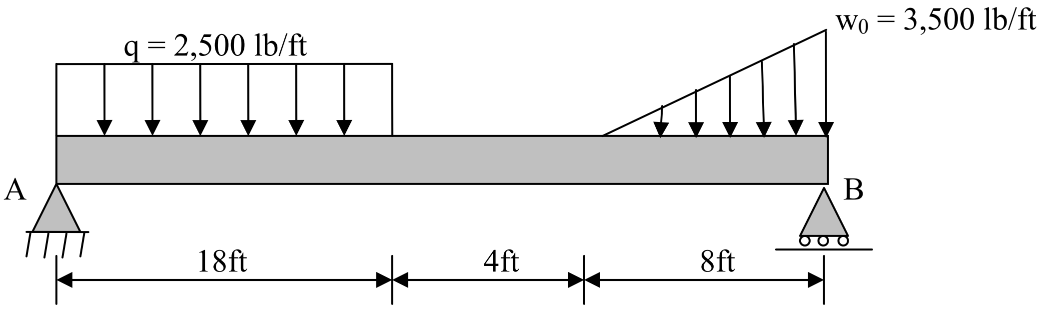


Figure 4: Floor joist beam of new MIT physics building.

(a) Being a US-based engineer, the first step to analyzing the beam is to convert all units from the US architects to SI MKS units. Perform these conversions.

Solution: Use the following conversion factors

$$1lbf = 4.45N$$
$$1ft = .3048m$$

Lengths:

$$18ft = 5.49m$$
$$4ft = 1.22m$$
$$8ft = 2.44m$$

Distributed Loads:

$$q = 2,500\frac{lbf}{ft} * 4.45\frac{N}{lbf} * \frac{1}{.3048}\frac{ft}{m} = 36.5\frac{kN}{m}$$

$$\omega_0 = 3,500\frac{lbf}{ft} * 4.45\frac{N}{lbf} * \frac{1}{.3048}\frac{ft}{m} = 51.1\frac{kN}{m}$$

(b) Draw a free-body diagram of the beam and calculate the reaction forces at A and B.

Solution: In order to do this problem we need first determine the equivalent concentrated load for the triangular-shaped distributed load, ω.

The area under this loading (in kN) is given by the equation for the area of a triangle, A=1/2bh

$$A = \frac{(51.1)(2.44)}{2} = 62.3kN$$

The coordinates for the center of gravity for a triangle is given by

$$\bar{x} = \frac{h}{3}$$

Taking the location of support A as x=0

$$\bar{x} = 9.15 - (\frac{2.44}{3}) = 8.34m$$

Therefore, the equivalent concentrated load is ***W****=62.3kN and it line of action is located at a distance* $\bar{X} = 8.34m$ *to the right of A*

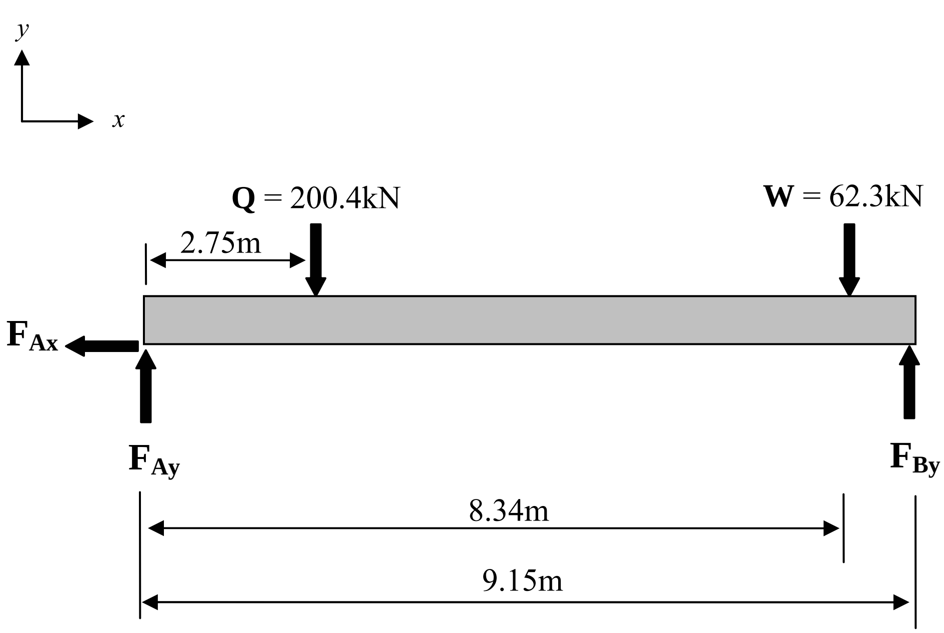
The equivalent concentrated load for the rectangular (uniform) distributed load is simply the area under the rectangle and its line of action is in the center of the rectangle

$$A = (36.5)(5.49) = 200.4kN$$

$$\bar{X} = \frac{5.49}{2} = 2.75m$$

Therefore, the equivalent concentrated load is ***Q****=200.4kN and its line of action is located at a distance* $\bar{X} = 2.75m$ *to the right of A*

Using the determined resultant forces, the free-body diagram can now be drawn as follows



The reaction forces are calculated as follows:

First, we sum up the forces acting on the beam in the x and y directions

$$\sum F_x = 0 : F_{Ax} = 0$$
$$\sum F_y = 0 : F_{Ay} - 200.4 - 62.3 + F_{By} = 0$$
$$F_{Ay} + F_{By} = 262.7kN$$

In order to eliminate one of the unknown variables, we then take the sum of the moments about support A

$$\mathrm{M}^{+} \circlearrowleft \sum M_A = 0 : F_A(0) - (200.4)(2.75) - 62.3(8.34) + F_{By}(9.15) = 0$$

Solving for F_{By} we get

$$F_{By} = 117.0kN$$

Finally, we can solve for F_{Ay}

$$F_{Ay} = 262.7 - 117.0 = 145.7kN$$