3.032 Problem Set 6
Fall 2006
Due: 11/22/06 ; no lecture on 11/22/06

1. When engineers decided that, indeed, dislocations must exist in crystalline materials, several new questions emerged.

   (a) For example, experimentally it is known that crystalline materials that have been cold worked (mechanically deformed at temperatures much lower than melting temperatures) exhibit compressive/tensile yield stress σ_y that increases as the dislocation density ρ increases. The more deformation, the more dislocations, the higher the yield stress. However, we learned that the theoretical shear strength of a material with NO dislocations is orders of magnitude greater than the the shear strength of a well-annealed material that has some dislocations ($10^2/\text{cm}^2$). Explain how both of these things can be true.

   SOLUTION: Dislocations make crystals weaker—that is, weaker than they would be if dislocations did not exist.

   In the theoretical *absence of dislocations, deformation would require whole planes of atomic bonds to be broken simultaneously. (Alternatively, other mechanisms of deformation such as twinning or phase transformation would be involved.) A dislocation moving through a material allows deformation to occur with a relatively few number of bonds breaking at any one time.*

   Conversely, any *density of dislocations in a real material increases the chance of intersection and entanglement. Just like any impediment to dislocation motion, entanglement increases the stress required to initiate and continue permanent deformation.*

   (b) Others have shown that, theoretically, dislocation density increases with the square root of strain:

   $$\frac{\rho}{\sqrt{\varepsilon}} = C \tag{1}$$

   where C is a constant particular to the crystalline material. In addition, others have shown that, experimentally, shear strength τ of materials such as single crystal and polycrystalline Cu varies linearly with ρ:

   From these findings for $\varepsilon(\rho)$ and $\tau(\rho)$, find the strain hardening (aka work hardening) exponent n that would be expected solely from cold working of a crystalline material. [Adapted from Hosford, 2005]

   SOLUTION: We note that the slope of log(shear stress) vs. log(dislocation density) is $\frac{1}{2}$, so that the shear stress increases as $\rho^{1/2}$. Since $\rho = C\varepsilon^{1/2}$, the shear stress is proportional to $\varepsilon^{1/4}$ and $n = \frac{1}{4}$.

   (It is noted in the problem that the shear strength scales "linearly" with ρ. Credit was given if you used this information to calculate that $n = \frac{1}{2}$.)
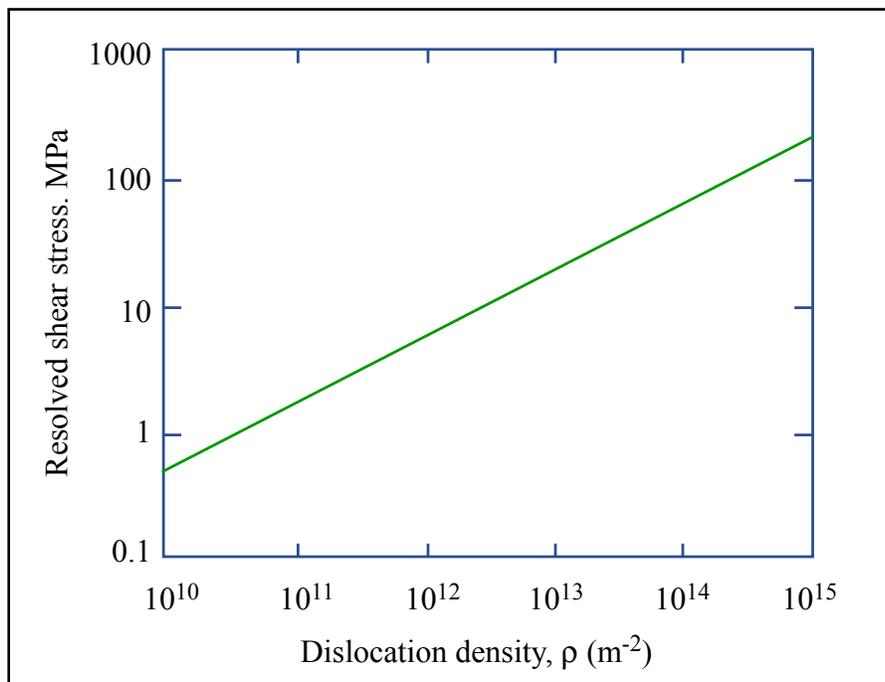
Figure 1: Dependence of resolved shear stress on dislocation density. Image source: Hosford, 2005. Data source: H. Weidersich, *J. Ins. Metals*, 16 (1984).

> *Note that this range of n (0.25–0.5) is a bit higher than is experimentally observed (0–0.4).*

2. Your boss at NewAlloys, Inc. has asked you to investigate a new nickel alloy with several weight percent of other metals to provide strengthening. You decide to perform a Brinell hardness test with a 10 mm-diameter tungsten carbide sphere and 1000 kgf force. You measure an average indentation diameter of 2.85 mm.

   (a) Estimate the tensile strength of this alloy, using linear interpolation if necessary.

   SOLUTION: The Brinell hardness is found to be (Eq. 3.40 in M&C)

   $$HB = \frac{2P}{\pi D(D - \sqrt{D^2 - d^2})} = 154$$

   where D and d are the diameters of the sphere and the impression, respectively. The tensile strength is estimated to be 524 MPa from Table 3.2 in M&C. It is also acceptable to divide the hardness value by three to obtain an estimate of the alloy's flow stress:

   $$\frac{(151\,\text{kgf}/\text{mm}^2)(9.81\,\text{N}/\text{kgf})}{3} = 502\,\text{MPa}$$

   (b) Your colleague has performed additional indentation tests using much probes of much smaller diameter and lower applied forces, and her data features much larger scatter (variation from measurement to measurement) than yours. Describe two possible reasons why this might occur.

   SOLUTION: It's possible that her tests are characterizing individual grains or precipitates rather than the bulk average of these effects. The relatively large influence of surface roughness for small indentations may also cause scatter. Finally, the resolution of her instrument (with its smaller forces and indentation depths) may be poorer.

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(c) Sketch a quantitative stress-strain diagram for this alloy out to a strain of 0.4%. State any assumptions you make and include any references you consulted to produce this diagram.

SOLUTION: The key elements of the diagram are a linear region that corresponds to elastic response (the elastic modulus of nickel is approximately 210 GPa, and the alloy will have a similar stiffness as long as the amount of alloying materials is low), and a plastic region that initiates before a strain of approximately $\frac{\sigma_{TS}}{E}$ = 0.25% (the strain that corresponds to our estimate of tensile strength).

3. The extent of strain hardening of materials is important from a processing perspective, when a desired level of permanent deformation is required to make a part.

   (a) Estimate the strain hardening exponent n for the uniaxial tensile test in Example 3.5 of Meyers and Chawla (p. 131-132).

   SOLUTION: One approach is to take the logarithm of both sides of $\sigma = \sigma_0 + K\varepsilon^n$ and find the slope of $\ln(\sigma)$ plotted against $\ln(\varepsilon)$. The resolution of the answer suffers from the size of the figures in Meyers and Chawla, but we might estimate n to lie between 0.5 and 1.0.

   (b) Prove that true strain ε_t is *always* smaller than engineering strain ε_e.

   SOLUTION: We could Taylor expand $\varepsilon_t = \ln(1 + \varepsilon_e)$:

   $$\ln(1 + \varepsilon_e) \approx \varepsilon_e - \frac{\varepsilon_e^2}{2}$$

   which is less than ε_e. We could also show that the two expressions are both zero at $l = l_0$ and that the slope of positive ε_t is less than the slope of positive ε_e thereafter.

   (c) Given this proof, why does the true strain data extend far to the right of the engineering strain data in Figure E3.5.2 of Meyers and Chawla (p. 133)? Think about what is directly measured and what is inferred in a uniaxial tensile test.

   SOLUTION: At the point of necking, the definition of true strain changes from $\varepsilon_t = \ln(l/l_0)$ to $\varepsilon_t = \ln(A_0/A)$ (Eqs. 3.16 and 3.17 in M&C). True strain calculated by cross-sectional area can be larger than engineering strain (which is still calculated at $\varepsilon_e = \Delta l/l_0$).

4. As the chief materials scientist at Strengthened Metalworks, Inc. you are developing a model to predict the amount of precipitate strengthening (aka precipitate hardening) in a material which includes precipitates shaped as rods rather than spheres. You first attempt is a 2-D model in which a dislocation cuts through a rectangular precipitate with side lengths L and $2r$, and random orientation θ, as shown in Fig. 2.
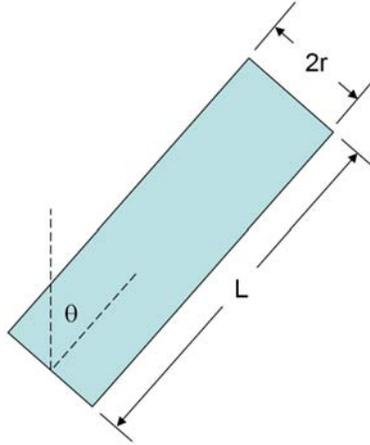
Figure 2: 2-D model of an alloy containing precipitates that will induce strengthening.

Develop an expression for the shear stress τ required for a dislocation to move through this 2-D material. Your model should consider details such as:

- The chance of a dislocation encountering a precipitate (assume a volume fraction f for the precipitated second phase).

- The distance a dislocation must travel through the precipitate as a function of θ, and the average distance for the case where θ is random (as it is here).

- The increase in surface area when a dislocation with Burgers vector $\mathbf{b}$ cuts through a precipitate (assume the energetic cost of creating a new surface is γ [J/m]). The actual type of dislocation is not important.

- The limits of the expression when L and $2r$ are approximately equal, and also when one is much larger than the other.

Your answer will likely be in the form of $\tau = F(r, L, \gamma, f, \mathbf{b})$. Feel free to make any assumptions necessary, but be sure to clearly state and justify them.

SOLUTION: Many people went straight for the full 3-D model, so we follow their bold example. Here is one approach, after Meyers and Chawla:

Chance of encountering a dislocation*: Assume that the dislocations are arrayed in a cubic arrangement. The precipitate volume is*

$$V f = N \pi r^2 L$$

where V is the total volume and N is the number of dislocations. The average distance d between dislocations is

$$d = \left(\frac{V}{N}\right)^{1/3} = \left(\frac{\pi r^2 L}{f}\right)^{1/3}$$

Cross-sectional distance through the precipitate*: Without getting into secants and tangents, we might assume an average angle of 45° so that the average distance l = 2 $\sqrt{2}$r.*

Calculation of required shear stress*: The shear stress required for precipitate cutting is calculated by setting equal the work done by the dislocation (force times distance) to the energy required to create additional surface area:*

$$\tau b d \cdot (2b) = \pi r b \gamma$$

$$\tau = \frac{\pi \gamma r}{2bd}$$

The distance 2b corresponds to two slip steps, one at the new area where the dislocation enters the precipitate and one where is exits. Other solutions are possible; credit was given primarily for clarity of explanation.