Department of Materials Science and Engineering
Massachusetts Institute of Technology

3.044 Materials Processing – Spring 2013

Problem Set 2

Problem 1

When dealing with components that have edges and corners, it is important to remember that these usually heat and cool faster than the rest of the body. Think about ice cubes in your drink - the corners melt first, then the edges, and the cube becomes rounded quickly.

On the last pset, we used an infinite series equation to describe the 1D conduction problem with a fixed surface temperature. In this problem, we will revisit that equation, but use superposition to address the heating of an infinite square prism. Make a plot in the x-y plane within the relevant domain showing reduced temperature contours for values Θ = 0.99, 0.9, 0.75, 0.5, 0.25, 0.1, 0.01. Do this for a few dimensionless times to get a feel for the heat transfer situation. You should use the full series solution.

Repeat the above procedure, but this time for a cube with a fixed surface temperature. Use superposition to make contour plots in the x-y plane of the same dimensionless times and reduced temperature contours in x and y as before, for z = 0.25 and z = 0.5.

Comment on the difference between the 1D, 2D, and 3D solutions.

Problem 2

In class, we will analyze the soak-heating of a billet of steel, prior to rolling. We placed the billet in a furnace at 1000º C to austenitize it, and we needed it to be at least 900º C throughout before we started rolling. The furnace air provided a high heat transfer coefficient of ~ 100 W/m²K.

In class, we treat two cases: a billet of 25 cm thickness, long in the two other dimensions, and a symmetric square bar, 25 x 25 cm in cross-section, long in the third dimension. For this problem, you will analyze the case of a **rectangular** cross section: 25 cm thick, with a width of 1 m. How long does it take to achieve 900º C in the center of this billet? (Hint: you will need to do some guessing-and-checking on this one, iteratively using the graphical solutions from P&G).

Problems 3 – 5

Sometimes in real engineering situations, you can't simplify the problem. In these cases, it is critical to be able to look up solutions and use them. In these three problems, you will use the two recommended reference books:

J. Crank, The Mathematics of Diffusion

Carslaw and Jaeger, Conduction of Heat in Solids

IF NECESSARY, TRANSLATE YOUR ANSWERS INTO THE LANGUAGE OF HEAT TRANSFER. You will also need to understand the dimensionless quantities in these equations, and PROVIDE A TRANSLATION INTO DIMENSIONAL QUANTITIES.

Problem 3

A sheet of material of thickness 2L is very large in the other two dimensions, and is also subjected to heating by passing current through it uniformly. The sheet is sitting in air. Write down the equation for transient conduction (i.e., giving the temperature profile) in the sheet.

Problem 4

The wall of a building, which is subjected to a sinusoidally varying surface temperature due to the cycles of night/day.

Problem 5

A spherical antenna tip on a satellite undergoes uniform internal heating due to transmitting a signal. Write the equation describing the temperature profile (neglect the arm of the antenna, only consider the spherical portion).

Problem 6

An important manufacturing tool you will likely use in 3.042 is a laser cutter. Suppose we are cutting a 10cm thick slab of steel with a 0.1mm diameter laser beam delivering 1000 Watts to the surface. This surface of the slab is in contact with air. The following steps will guide you through producing the final result of a plot of the temperature profile in the slab.

    i.   Draw the geometry representing the problem and identify the boundary conditions.

    ii.   Can you simplify the problem without sacrificing too much accuracy? Do this quantitatively.

    iii.   Rather than reinventing the wheel, find the solution to this problem in a reference book. Plot the temperature profile at steady state as a function of the radial and thickness coordinates. Do you expect melting to occur? Where?

3.044 Materials Processing
Spring 2013