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LORNA GIBSON: All right, so last time we started talking about energy absorption, and I wanted to try to finish that up today. And then next week we would talk about sandwich panels and using honeycombs and foams in sandwich panels.

So I think we got as far as the idea of introducing what these energy absorption diagrams are. So let me run through this little sequence again, and then I'll put the notes up on the board.

So the idea is that you have your compressive stress strain curve-- so here's a series of curves for different densities of a foam. And we would do these all at the same strain rate and temperature, so that those aren't variables. And then what we would do is we could turn those into energy absorption diagrams.

So notice here, this is a log plot. So here's that energy absorption here. Here's the peak stress here-- so that's the peak stress up to some level of energy that you've absorbed. And we've normalized both of those by the solid modulus.

So say we look at one density here, say we look at the lowest density-- 0.01. So here's our curve here for the test. If I go up to some stress level that's still in the linear elastic region, then that's going to translate into somewhere along here on the energy absorption curve.

And one of the things we're going to do today-- I'm going to show you how you draw these, and how you can set them up. And you can either get them from experiments-- so this is, say the top curve were experiments, doing it from experiments-- or you can use the models for the foams to do it, as well. So we'll see how you can do from both ways.

So this would be the linear elastic bit here. This vertical part-- where the energy is increasing, but the peak stress isn't increasing-- that corresponds to the plateau here. And then this part here, where the energy is not increasing very much, but the stress increases a lot-- that corresponds to the densification part over there.

So what you would do is you would do tests on foams of different densities, and from the test

data, you could draw an energy absorption curve for each density. So you plot-- there's four different densities here, so it forms a family of these curves. And then what you do is you say-- and I think we talked with this last time-- that the place you want to be is at this shoulder point. You want to be absorbing as much energy as possible at that plateau stress. So you want to be at the point just before it turns around to the densification regime.

So you can mark that little shoulder point for each density. So here's like 0.01, here's 0.03, and so on. And then those points can be connected by a line-- so that heavy line then connects those shoulder points, or those optimum points.

And then what you can do is then repeat this whole process for different strain rates. So this family of lines here is at different strain rates-- so this set that goes this way, corresponds to this line here. And if you do them at different strain rates, where the shoulder appears at a slightly different point-- and so if you mark those shorter points, you can draw these lines here that connect up for one relative density. So this line on the left hand side here, these are all relative densities of 0.01, these ones are all relative densities of 0.03.

So this diagram down here, doesn't really look like this at all-- it doesn't look like the basic energy absorption diagram. But it actually has information for what the optimum would be for a range of densities and a range of strain rates. So typically, these foams are viscoelastic and they have some strain rate sensitivity. So you'd like to get the strain rate sensitivity into it.

And it's not shown here, but you could do the same thing at a constant strain rate and varying the temperature, too. So if you had things at different temperatures, you could do the same kind of idea.

OK-- so are we good with how this works? Because now I'm going to write some notes on the board so you have it in your notes.

So the idea is that you turn your stress strain curve to look something like that, into an energy absorption diagram. And you can plot on log log scales, the energy versus the peak stress. And you get something like that.

And this point here, I'm going to call the shoulder point. And that would use the material in the most efficient way-- you get the most energy absorption without getting higher than that plateau stress.

So we can say at this stress plateau, the energy increases without much increase in the peak

stress.

And then as the foam densifies, then you get an increase in that peak stress, with little increase in the energy absorbed. And so ideally, you want to be at that shoulder point.

So to construct these energy diagrams, you can test the series of foams at different relative densities and constant strain rate and temperature. And then you make that plot of the energy absorbed normalized by the solid modulus, versus the peak stress normalized by the solid modulus for each curve. And typically what people do is they would take the solid modulus at a constant strain rate and temperature, so you don't have to introduce that, as well.

Then you would mark the density for each of those shoulder points, and then you would connect them.

And then you could repeat this whole process for different strain rates. And then you would draw the final diagram at the bottom, where you have this family of lines that describes the shoulder points for different densities and different strain rates.

And you could treat different temperatures in the same way, if you wanted to. You would hold the strain rate constant, and vary the temperature. so it's kind of a nice way of just putting a lot of information in one diagram.

So one of the things about this is that, because we're normalizing by E_s , and if you think of elastomeric foams, elastomeric foams, both the Young's modulus depends on E_s , and the plateau stress depends E_s . So the Young's modulus depends on the stiffness of the solid, and also because the plateau stress is related to elastic buckling, it also depends on the modulus of the solid.

So for elastomeric foams-- it's because you normalized it with respect to E_s -- one of these diagrams will represent all elastomeric foams. So that's rather a nice thing-- so you can have different elastomeric foams, but one of those diagrams is going to represent all of them.

So we can say, elastomeric foams can all be plotted on one plot, or one curve, since both the modulus and the plateau stress are related to E_s .

So if we look at this next figure here-- maybe I'll just wait a minute for people to stop writing. Some people write faster than others.

So if we look at this next plot here, here's a compressive stress and strain. These tests are done for one density, but at different strain rates-- so it's kind of the other version of this. But here's the stress strain curves here, and here's the energy absorption diagram that's derived from those. And then here's a summary diagram that has the different strain rates, and that would have different densities here.

And the idea is this diagram here could represent all elastomeric foams. So this has been put together for polyurethane, but it should be able to represent other sorts of flexible elastomeric foams.

Sorry?

AUDIENCE: In those diagrams, then, the intersection of this strain rate line and your relative density line should be the shoulder position?

LORNA GIBSON: Yeah, exactly. And so this line here is for 0.01, and that one's for 0.03. So 0.02 is going to be-- you'd have to interpolate somewhere in between there. So we've just put on certain values, because we're not going to put on a million values. We just put on certain ones, and then you can estimate where other densities would appear on there. OK?

So here's another example here-- these are curves for two different foams. So here's a polyurethane a polyethylene, so these are both elastomers. And one is the dashed line, and one is the solid line. And you can kind of see how the lines mesh up.

So here's a density of 0.01 for the polyurethane. Here's 0.05 for the polyurethane. And here's 0.06 for the polyethylene. And you can see how the 0.06 and 0.05-- they're not quite on top of each other, but they're pretty close to coming on top of each other. And then 0.1, 0.12-- and so you get a family of them for the different densities.

And you can also do this for materials that have a yield point, so polymethacrylimid has a yield point, so you do exactly the same kind of thing. So here's the energy absorption diagram that's been developed from the stress strain curves. But now this curve here, or this set of curves here, is really just valid for one ratio of σ_{ys} to E_s -- so the solid yield strength of the solid modulus. So it's valid for whatever it was for that particular type of foam-- the polymethacrylimid.

So I'll just say, if we have foams that are made from a material with a yield point-- and so they have a plastic collapse stress-- then the curve will be valid for foams with the same ratio of [σ_{ys} to E_s]

σ_{ys} / E_s to E_s .

So in that case there, for the polymethacrylimid, that ratio is about equal to 1 over 30. So that plot would probably give not a bad description of other foams, with the same value of σ_{ys} over E_s .

So the idea here is we can generate these diagrams either from data-- the way those ones have been done-- or from the models.

So another way to generate these is to think about the models that we have for the foams, and the foam behavior. So we have an equation that describes the Young's modulus, we have equations that describe the plateau stresses, we have an empirical equation for the densification strain. And we can use those to generate these diagrams. And they're kind of useful, because they show you what's going on on a sort of mechanistic basis.

So this is a diagram here that's been generated for open cell elastomeric foams. Here's our energy absorbed, here's our peak stress down here. This little inset is sort of a schematic of the idealized foam behavior.

So here's the Young's modulus, here's the elastic [collapse] stress, here's the densification over here. So obviously it's kind of a very idealized set-up. But we can generate this diagram-- and I'm going to go through the equations that will let us do that.

So one thing to note is here's the curves for each density. Here is this line that connects the shoulder points. There's a couple of other lines on here that I just wanted to mention something about.

If you think about the densities-- like that's 0.01, this is 0.03, that's 0.1-- if you had a fully dense solid that was made of the same elastomer, you could plot the curve for that, and that is going to show up over here. So this is kind of an upper bound. It can't get any more from that.

And we've also got a dotted line here, which takes into account fluid flow within the cells. So there can be some fluid flow, and because we haven't talked about that, we're not going to go into that. So we can just ignore that dotted line for now.

So let me go through how we can do the modeling. So we're going to divide the stress strain curve up into bits, and I'm going to write equations for the energy absorbed for each bit. So we're going to start with a linear elastic behavior. And I'm going to-- let's see-- yeah, so let me

just note here that this is the densification strain out here. And this strain, ϵ_{naught} , corresponds to the strain at which we first reach the stress plateau.

So here, for the linear elastic part, I'm going to say that the strain is less than that. So the strain is less than absolute naught.

And then I can say, the energy absorbed-- so if you just remember from Hooke's law on linear elasticity, energy under the stress strain curve for the linear elastic part is $1/2$ of σ^2 over E . So I'm going to call σ -- whatever the stress is going to be, the peak stress that we get to. And now we're going to divide by E of the foam-- because these were our foams here.

And I can use our model here to say-- and now I've got to divide that by E_s , because I've normalized here. So because I know from my modeling that the Young's modulus of the foam is just equal to E_s times the relative density squared for the open celled foam, that means I've now got an E_s squared in the denominator, so I've got a σ_p over E_s squared. And then I've got a 1 over the relative density squared term.

So that factor there, that equation there, gives you these first set of lines here. Gives you that bit, and this bit, and that bit. So it gives you those first parts of the energy absorption diagram.

And then if we look at the stress plateau-- so here we're going to say that ϵ_{naught} is less than ϵ is less than the densification strain-- so we're on the plateau somewhere. And now the energy absorbed is just going to be our plateau stress times the amount of strain we've got. So that's $\epsilon - \epsilon_{naught}$. And if I normalized with respect the solid modulus, I can write down that my plateau stress is 0.05 times the relative density squared, and then multiply that times $\epsilon - \epsilon_{naught}$.

So that equation then corresponds to these vertical parts-- so this part here, that part there, this part here. It corresponds to those vertical lines on the figure. Vertical lines on the diagram.

And then the plateau stress is going to end at the densification strain. And at that point, the energy diagram is just going to become vertical.

OK, and then the last part-- I'll try to rub this off a little better. And then the last part is when we're at the end of this stress plateau, and the strain is equal to that densification strain. And so the amount of energy we absorb here is really going to be the maximum.

So this is the energy that's going to correspond to that shoulder point that I've been talking about. So I'm going to call that W_{max} , and normalize that with respect to E_s . And that's then going to be our plateau stress times the densification strain. And the densification strain was just $1 - 1.4$ times the relative density.

So here I'm assuming that the densification strain is very much bigger than the strain at which the buckling first occurs, and I'm neglecting that linear elastic part.

So then we could say that the optimum foam is at that shoulder point. And I can say that the peak stress at that point is just equal to the plateau stress.

And what I want to do is get an equation for that solid line up here that connects all those shoulder points. So I want an equation, in terms of the energy, and the peak stress, instead of in terms of the density. So what I'm going to do is solve this for the density, and then plug that back into there.

So I get that the relative density is, then, 20 times the peak stress over the solid modulus, and I take the square root of that. And now I can substitute this up here for the relative density. I think I need another board.

So then I've got-- this is my peak stress, or my plateau stress, over E_s , and this is all just the densification strain. And that's just $1 - 1.4$ times the relative density. But now I'm putting the relative density in terms of the peak stress, or the plateau stress.

If I just simplify that slightly with the constant, it's $1 - 6.26$ times σ_p over E_s to the $1/2$ power.

So that equation there describes the-- oops, no more updates. No updates. Go away.

Ah, so that equation there describes this line here that connects those shoulder points. And that's the line you're the most interested in, because each of those shoulder points is a point where the foam is being used in the most efficient way, or the optimum way.

And then, let's see-- is that going to fit? No-- let me try the other board.

And then the last thing we can do is calculate that line that corresponds to the dense solid.
Yeah?

AUDIENCE: On those ones over there, it says it corresponds to the vertical lines on the diagram. And then later it says then it becomes vertical. Is that referring to two different diagrams?

LORNA GIBSON: So this stress plateau equation here corresponds to these vertical lines here. So for relative density of 0.01, it corresponds to that part. For 0.03 it's this part. And your 0.01 it's that part.

AUDIENCE: But then some of the [INAUDIBLE] it says, then w [? versus ?] σ becomes vertical. Should that--

LORNA GIBSON: So, well the plateau stress ends at the densification strain. So the plateau stress ends here. Oh, let's see-- and then it becomes-- should be horizontal. Sorry.

OK, sorry. Happy? And now I'm going to rub that all off.

OK, did everybody get this? I can rub it off?

OK, so then the last part is what happens when the foam is densified. And if it was fully dense-- and you never really can get to this point-- but if it was fully dense, you would get rid of all the pores, and it would just be a solid. And then the energy absorption curve would be the curve for the solid. So I'll just say, when fully densified, I'm going to say a curve approaches that for the solid.

And for the solid, you would just have that W over E_s is equal to $1/2$ the peak stress squared over E_s .

So this model curve, the curves have the same shape as when you get the diagrams from the experiments. And you can see how the different mechanisms of deformation and failure contribute to the diagram, where the diagram comes from.

And I guess one other point is you can see that the foams are always going to be a lot better than the solid. And remember this is a log log curve, so that a foam that has a density of 3% here, there's a huge difference in the peak stress. So say you wanted to absorb this amount of energy up here, for a foam that was 0.03 dense, the peak stress would be a little less than 10 to the minus 4, normalized by the modulus. And for the solid, it would be 10 to the minus 2-- so it's orders of magnitude better to have the foam rather than the solid.

All right, now let's see what else we have. So we could do a similar thing for closed-cell foams, and you get diagrams that look like this. One of the differences with the closed-cell foams, if

you assume that the faces don't rupture, the plateau stresses and horizontals-- remember we had that gas contribution, and you can take that into account? So I'm not going to go over the details of that.

The next one I wanted to talk about was looking at foams that have a yield point. And again, you can generate a similar kind of diagram, but now, instead of having an elastic failure here, you've got a plastic failure-- you form plastic hinges.

And then again, this diagram is less general than the one for elastomeric foams, so this diagram would be valid for whatever ratio of σ_{ys} over E_s you've the calculation for. So this one here is for 0.01.

So let me just run through the same kind of calculation for the plastic foams. Can I rub this off, and then I can use this board to start here?

OK, so the linear elastic part is just the same as for the elastomeric foams. So you get W over E_s is $1/2$. σ_p over E_s squared times 1 over the relative density squared, so it's just the same thing. And the stress plateau-- you get w over E_s is just the plastic collapse strength times the strain range that you go up to. So if you remember the plastic collapse strength was $0.3 \sigma_{ys}$ times the relative density to the $3/2$ power, and then times the strain range.

And then at the end of the stress plateau, you've got the maximum energy absorbed. So normalize that by E_s . And that's going to be your peak stress over E_s times the densification strain again. So this bit here is the densification strain.

And then you can do a similar thing to figure out the equation of that line that joins the shoulder points.

So the first step is to solve for the relative density there.

So if this part here-- $0.3 \sigma_{ys}$ times the relative density to the $3/2$ power is the plastic collapse stress, then at the densification point, the relative density, you just rewrite that and it comes out to the $2/3$ power, because you turn the power around.

And then you just substitute this up in here.

So you get that the maximum energy absorbed, normalized by the solid modulus, is your peak stress, times 1 minus 1.4 times this thing in brackets to the $2/3$ power-- the 3.3 times the peak

stress over the yield strength of the solid. And then if I just rearrange that, and get the constants, it's 1 minus 3.1 times our ratio of the stresses there.

So maybe I'll just put over here-- the curves are less general than for elastomeric foams. So each family of curves would be for a particular ratio of the solid yield strength to the solid modulus.

So you get the idea? It's fairly straightforward.

So I wanted to finish up this topic by giving you a few examples of how you can use these curves. So the next thing is to look at the selection of foams for impact protection. And typically, you're given some information about the objects-- so typically you want to protect some object. It could be a computer, it could be your head, some part of your body.

So typically you know something about the object you want to protect. So you might know its mass, you might know the area of contact, you might say, well, if it's my computer, I want to make sure it doesn't break if I drop it from a height of a meter. Or you could say whatever, [? maybe it's ?] 2 meters. But you pick something. And so there's a certain amount of energy you know that you need to absorb.

And you may know that whatever the component is, or the body part, or whatever-- there's some maximum acceleration you can tolerate. So you might say, well, I want to make sure my computer doesn't break under acceleration of so much. So you're given the acceleration.

And so if you know the mass and the acceleration, and you know the area of contact, you can figure out a force over an area, and that gives you the peak stress. So typically, you know those things in the problem. And typically the problem involves choosing a material, or choosing-- like choosing what kind of material do you want to make the foam out of, and what density of foam do you want to use, what thickness of foam do you want to use?

So I've got a couple of examples just to show you how this works. So let me just write down a couple notes, and then the rest of it I think I'm just going to take from the slides.

So typically, you know what it is you want to protect, and you know something about it. So if you know what it is, typically you know what the mass is. You might know what the contact area would be. Say a maximum drop height, maximum tolerable acceleration.

So say if you're worried about brain injury-- and making a helmet-- you might know what the

maximum tolerable acceleration would be.

So typically, you know what the peak allowable stress is, just from whatever the object itself is. And the variables that you have to play around with-- variables-- are things like the foam material, the foam density, and the foam thickness.

So I have a couple of examples that have different setups here. And we'll just see how the thing works out here.

So the first example, we're told the mass of the packaged object is 1/2 kilogram. We're told the area of contact between the foam and the object is going to be point. 0.01 of 1 meter squared. And we're told that it's supposed to be designed to withstand a drop of 1 meter-- so if the drop height is 1 meter, then the velocity is just the square root of $2gh$.

So g is gravity, so you can work out the velocity on impact would be 4.5 meters per second. And if you have the velocity on impact and the mass, you can figure out the energy to be absorbed. You can say mv^2 screwed over 2, or you could say it's mgh -- either way. So here it's going to work out to 5 joules.

And in this case, we're told that the maximum deceleration is 10g-- so then if that's 10g, the maximum force is the mass times the acceleration. That works out to 50 Newtons. And then that gives us a peak stress, the maximum allowable peak stress of the force over the area it's 5 kiloNewtons per meter squared.

And in this case, we're told that the foam is going to be a flexible polyurethane, and it has a solid modulus of 50 megapascals. And so we can calculate this normalized peak stress. So the normalized peak stress here is 10 to the minus 4.

So in this problem here, we need to figure out what's the foam density, and what's the thickness of the foam to protect the object? And so that last slide is just summarized here. And I realized that when I was going to put this up, the font is going to be kind of small, so I blew it up a little.

So we have figured out that we're at σ_p over E_s -- the peak stress normalized by the solid modulus is 10 to the minus 4. And we know that we're going to use a flexible elastomeric polyurethane, so we pull out our diagram for elastomeric foams. And this sort of hashed band here corresponds to all the different strain rates, and all the different densities. So I haven't plotted each individual line, we've just got this band that represents the whole thing.

So we know that our normalized peak stress is going to be somewhere along this line here. That's from everything that's given, and what we can calculate. And we want to know what density of foam to use, and what thickness.

So the way you approach this is the thickness is going to affect the strain rate. So the strain rate's just going to be the velocity on impact, divided by the thickness-- or it's an approximation for the strain rate. So we don't know if we're at this point down here, or if we're at this point up there, because we don't know where we are in the strain rate end of things.

So the way you solve this is you just guess a thickness. And if you guess a thickness, you can calculate a strain rate. Then if you calculate the strain rate, you know where in this band you are. You can figure out a value of W over E_s . And you can use an iterative process.

And the way this is set up is we've chosen two very different initial thicknesses, and the point of doing that is to show you that it converges very quickly. So the first iteration on this side here, we've chosen the thickness of a meter, which is probably unlikely that we need a meter of foam. And on the side here, we've chosen a millimeter-- 0.001 meters. So probably, we need more than that. So we probably need to be somewhere between those two bounds.

So if the thickness was a meter, then the strain rate turns out to be 4.5 per second. And then we know where we are in this diagram, and we can read off a value of W over E_s . So we know we're on this line here, and for a particular strain rate, we can read off the W over E_s .

So here's the value-- 5.25 times 10 to the minus 5. And if we know E_s , which we do, we can then calculate the actual energy absorbed per unit volume. We get W -- so W is [? 2620 ?] joules per cubic meter. And we can use that value-- because that's an energy per unit volume-- we know the area of contact, and we can use that to get another value of the thickness. So we use that to calculate the next iteration of the thickness.

So U is the total energy in joules, and W is the energy per unit volume. So U , the energy in joules, is going to equal the energy unit volume times the area times the thickness. So we know what the area is, too. We can then calculate a new thickness.

So now our new thickness is 0.19 meters. We can use that to get a new strain rate-- that's 24 per second, and go through the whole thing again. And we end up with a revised energy of 3,300 joules per cubic meter.

Now if we started at the other end, if we started with the first guess was a millimeter, then the strain rate is 4.5×10^{-3} per second. That gives us a different value that we read off here for W over E_s , and a different value for W . And then we use this value here for W to make another guess for the thickness. So that value is 0.14.

And then we go through the whole thing again-- we get a revised strain rate a revised W over E_s , and a revised W . And you can see after just two iterations, these two things are almost exactly the same.

And then on the third iteration, they both would give you a thickness of point 0.15 meters. So even though you can pick wildly wrong first iterations, it converges very quickly, and it's a fairly simple calculation to do.

So you know the thickness that you want is in here, and then you can get the optimum density here. So you know that your σ_p over E_s is along this line of 10^{-4} --we're somewhere in here.

These two strain rates here-- one was 24, one was 32-- so the final value is going to be somewhere around 30. And if you look on this thing here, you can see there's a line that corresponds to 10, there's a line that corresponds to 100. We're going to be right around in there. So the relative density is going to be right around 0.01.

So you can use the diagram to get the thickness and to get the density.

OK, are we good? We're good?

So I've written some notes, and I'll just scan those and I'll put them in the Stellar site.

So here's another example here-- and in this example, it's set up a little bit differently. So in this example here, we're not told the material, but we're told the thickness of the foam. So this time we want to get the material, and we want to get the density of the foam.

So here we've got the specification. We're told the mass is 2 and 1/2 kilograms. The area of contact is 0.025 meters squared. We're told the thickness-- here thickness is 20 millimeters.

We've got a drop height of 1 meter again. And the velocity of impact is then going to be 4.5 meters per second again. And since we know T , we know that the strain rate is going to be around 225 per second.

We can calculate the energy absorbed-- [? MGH-- ?] 25 joules. We can calculate the energy absorbed per unit volume, because we've got the area and the thickness-- so I'm just going to divide that by the area and the thickness. Now we have 5 times 10 to the 4th joules per cubic meter.

And we're told that we're supposed to design it so the package can withstand a deceleration of 100g-- and so that gives you a maximum force. And here we've got a maximum allowable peak stress of 10 of the 5 Newtons per meter squared.

So here we have our curve for the elastomeric foams again-- let's assume it's going to be an elastomeric foam, but we don't know what kind of foam. So the way you solve this problem is that you make a guess for what E_s is. So remember, these diagrams are all normalized by E_s . So to plot a point on there, we need to know what E_s is.

So here, we're going to make a guess, and we're going to assume that E_s is 100 megapascals to start. And if we had that value of E_s , we know W , up here, and we know σ_p there. So we just divide those values of W and σ_p by E_s , and we get these two values here. And we plot those two values on the thing here.

So here's our point A-- and that corresponds to that first guess of 100 mega Pascals for the modulus of the solid. So that's not necessarily the final answer, that's not the right answer-- that's just somewhere to start.

So the thing to notice is, if we have that point there, the strain rate there is probably not quite right. This upper bound here is-- let's see. Got to get closer. Oh, let's see-- that's 10 to the minus 2, that's 10 to the 2. So the strain rate we want to be is closer up to here, it's not quite down there. So we're not at the right strain rate.

But the thing to notice is that if we draw a line of slope 1-- so there's this dash line of slope 1-- when we move up and down that line, we're just changing E_s . Because everything is normalized with respect to E_s -- if that line has a slope of 1, then we're just moving up and down with respect to E_s .

So what we do is we scoot up the line to get to the point that's on the right strain rate. So if this was a strain rate of 100, or around 200, we want to be at point B. And then from point B, we can read off what's the value of W over E_s , and σ_p over E_s . And from that, we can back

out what the solid modulus we want is.

So these are the values that we read off the chart. This gives us an E_s of 28 mega Pascals. And again, you can go to this more detailed diagram here, and if you read off the σ_p over E_s , and the W over E_s , the density you want is about 0.1. So it tells you the modulus of the solid, and from that you can pick a solid, and it tells you the density. Are we good?

OK, so I have one more. So there's a slightly different way you can do it, too. So now I want to talk about bicycle helmets-- you're my bicycle helmet person. So this is another little case study that involves a slightly different way to do this. And this involves a slightly different diagram.

So the idea here is to choose a material for a bicycle helmet. And as you probably all know, the bicycle helmets have a hard shell, and they have some sort of foamy liner. And the foam's usually around 20 millimeters thick. And you want something that's light, because you don't want your helmet to be too heavy-- but you want something that will absorb the energy from the impact.

So I've set this up here-- so we assume the mass of the head is about 3 kilograms. And we assume that it can withstand a deceleration of something like 300g-- and so then you can get a force mass times the acceleration, because you have the force. And I've assumed an area of contact of something like 0.01 meters squared, so that gives you a peak stress.

So this method here is based on the idea that you have these material selection charts for foams. Remember we talked about that earlier. And this is the compressive stress at 25% strain. So the idea is that axis there is meant to represent the plateau stress, and this axis here represents the densification strain. And these dashed lines here correspond to basically a value of W -- and energy absorbed per unit volume.

So this is an energy absorbed per unit volume of 0.001 megajoules per cubic meter, 0.01, and so on. So if you know the peak stress that you can tolerate, for the numbers I gave you it works out to be 0.9 mega Pascals-- so here it's just a little less than 1. So that's the peak stress that you can tolerate there.

And you can see that the material that's going to absorb the most energy-- so you're absorbing more energy as you move over this way on the curve on the plot-- so the material that's going to do the best is something like an expanded polystyrene that's 5% dense. So I've

highlighted that in red-- expanded polystyrene that's 5% dense. And then this is the densification strain here. And you know that the lines of the energy absorption are just the stress times the strain.

So you can basically just read off from here that expanded polystyrene that was 5% dense would be a good choice for a bicycle helmet foam. Would you like to add anything about bicycle helmet foams? I think that is exactly what they use, yes. So this is just another way to do this kind of thing.

And I did one more little calculation here-- if you know the thickness of the foam is, say, 20 millimeters, and we've estimated the area of the contact, you can figure out what the energy absorbed is per unit volume, and from that you can back out the energy in terms of joules, and from that you can back out the velocity that's the maximum speed that one would want to get dinged at on your bicycle. And the maximum speed works out about 22 miles an hour. So that's just another example.

Are we good? OK-- yeah, exactly, your head would be hitting the ground at 22 miles an hour which, would be ouchy. Which is why you want to wear your helmet, because your skull will not be happy if that happens. And your brain will not be happy. And you will not be happy. And your mom and dad will be really, really, really unhappy. So you don't want that to happen.

so I have a few minutes left-- and I know I've done this for the people in 3032, but I was going to buy woodpecker talk, because it's about energy absorption. So if you don't want to watch it, if you've already seen it, you can go. But there are some people-- you guys haven't seen it, and you haven't seen it. There's a few people that haven't seen. And it's cute, involves birds. And it involves energy absorption-- so I thought I would do this woodpecker talk.

So Barry, this is all just slides. I don't know if you want to do the lights differently. I'm not going to write anything on the board, I'm just going to talk.

BARRY: Well, if it will help them see the slides better then certainly.

LORNA GIBSON: Yeah, I think maybe we could turn the lights down a little, please.

BARRY: Let's try this one.

LORNA GIBSON: Oh, there we go. That's good. Yeah, now I don't feel like I'm looking in the spotlight so much. That's good.

OK, so you guys know that I like to watch birds. And you know that I work on foams. And if you look at bird books, sometimes the bird books say that woodpeckers can withstand the impact from pecking because they have a special material between their skulls and their brains. And I thought, oh, well I study foams, and I'm interested in woodpeckers-- I should find out what this special material is.

So I started looking into this. And it turns out there is no special material. People have looked at the anatomy of woodpecker brains and skulls, and there is no special material.

But it turns out there were also a group of neurologists in the late 1970s who got interested in why woodpeckers don't get brain injury, and they took high speed video of a woodpecker pecking. And it was kind of amazing what they found out.

So it turns out the woodpeckers can withstand incredibly high decelerations-- much higher than our brains could withstand. And so I kind of got interested in this, and I decided to try to figure out how it works.

So here's an acorn woodpecker. And let's see-- I got a little video here. Wait a minute-- there we go. There we go.

So here's a little acorn woodpecker. These live in California. Anybody from California? Yeah-- have you seen them?

No, OK. Well you have not been looking carefully. So here's our little acorn-- and you can see they peck-- oh? Like around San Francisco.

AUDIENCE: [INAUDIBLE].

[LAUGHS]

LORNA GIBSON: So they're pecking, and when they're pecking-- they don't just go bonk. They do this repeatedly-- they can peck at 10 or 20 times per second. So the question is, why don't they get brain injury?

So, first of all, why do they peck? So as we in that little video, that woodpecker was foraging-- it was trying to get little things out of the bark. And woodpeckers eat insects, and so the bird books say that they can actually hear insects scurrying around under the bark, and they'll peck at the bark and forage to try to get insects.

And there's one other anatomical feature of woodpeckers, which is kind of amazing-- and you can see it in this picture here. So this is the woodpecker tongue, and the tongue is connected to something called the hyoid process. And the hyoid process wraps around their eyeballs.

And then when they peck-- I mean, the idea is they're making a hole in the tree, and they've got to get their tongue into the hole to get the bug. And the end of their tongue has little barbs on it. And when they contract this thing here, their tongue scoots out and gets the little bugs.

So they pick partly to forage, but they also build something called cavity nests. So they'll find a tree that's started to rot, and they'll drill a sort of horizontal hole, and then they'll drill a cup underneath that, and they lay the eggs and have their nest at the bottom of the cup.

And then they also-- especially at this time of year, in fact, today I heard woodpeckers drumming-- so it's one of these mating things, one of these courtship things. So woodpeckers will peck on a hollow branch or a hollow tree, just to make a big loud noise to say, here I am, looking for sex, I'm ready, this is my territory. And so in the spring-- so this time of year you hear woodpeckers drumming.

And I think naturally they do this on hollow trees to try to make a big sound, but they have adapted to metal downspouts, which are also very effective for making this loud noise. And people who've-- I wrote a paper on this-- people who have read my paper sometimes email me and say, oh, I heard you know about woodpecker pecking. How can I get them to stop drumming on my downspout? Because it's kind of annoying, if you're the human inside the house.

So anyway, they peck for these reasons. And then acorn woodpeckers are special-- acorn woodpeckers are what I think of as the champions of pecking. And they do one more behavior-- so here's from David Sibley's beautiful *Guide to Birds*, here's the acorn woodpecker. They store acorns in what's called a granary, and they look at old tree trunks that are beginning to rot, and they pick holes in the tree trunk and they store the acorns in the holes. So see all those little dark dots on that trunk? Those are all holes that the acorn woodpecker has pecked.

And they'll do this-- they live in social groups, so there might be like 10 or 20 of them living together, and they'll peck like 10,000 holes into their granary. There will be thousands and thousands of these holes.

And here we have the acorn woodpecker with the acorn in its beak, and you can see some of these holes have acorns, and some of them are empty. So here is the acorn woodpecker in action. And here's the little video again-- I won't play the video.

So I often give this talk for non-engineers, so I'm going to explain some things-- there's going to be some writing on the slides that you guys already know. So the impact force depends on the deceleration-- how quickly the brain stops when the beak hits the tree. And I should mention, these videos are from the Cornell Lab of Ornithology-- they have an amazing collection of bird audio, like bird calls, bird songs, and bird photographs and videos. It's incredible, the collection that they've got.

And then I explain what acceleration is-- so I'm going to put acceleration in terms of gravity. And when the beak hits the tree, there's going to be a deceleration on impact. And just as a comparison, human brain injury occurs roughly at about 100g. And so the question is, how much deceleration can the woodpecker brain take?

And that's where the neurologists in the Bay Area come into the picture. They found out that there was a park ranger-- I think maybe at Point Reyes, just north of San Francisco. And he had an acorn woodpecker that, I don't, had an injured wing or something. Anyway, he had this acorn woodpecker that he kept. And they were able to use this acorn woodpecker, and they took high-speed video of the woodpecker pecking.

So from the high-speed video, the video that they took went at something like 2,000 frames a second. So they have a picture of where the head is every 2,000th of a second. So if you know the position at these times, you can get the velocity, and you can get the deceleration.

So they measured the deceleration, and they measured some amazing things. So they measured that on impact, the woodpecker's bill was going something like 15 miles an hour. And the decelerations were up to 1,500g-- so many times more than what we can withstand.

And they also measured the stopping time-- and they thought it was between about 1/500th to 1/1,000th of a second. And that's going to be important later on.

So one of the interesting things about this was how they got this whole thing set up. So I don't know how many of you do UROPs-- but you know, part of the thing in doing UROPs, and doing experiments in the lab is just how do you do the experiments? So you know, the park rangers got the acorn woodpecker, that's all very good. But they have to get the woodpecker to peck in

front of a camera, and they have to get the camera to turn on as it's pecking-- so there's some experimental challenges.

So the important thing, the critical thing, is the date of the paper. This was written in 1979, which meant they probably did the experiments in 1978. And I was a graduate student in 1978. And I can report there were no Apple computers in 1978-- there were no laptops. You couldn't just kind of do your little PowerPoint slides.

And most offices had something called an IBM Selectric typewriter-- I don't know if you've ever seen the old IBM typewriters. But the typewriters, when you typed on the typewriters, they made these noises, and it sounded kind of like a woodpecker pecking. And the ranger had one of these typewriters in his office, and he had discovered that if he typed on the typewriter, the woodpecker thought, oh, there's another woodpecker. I'll start pecking.

And so they used the typewriter as a way to get the woodpeckers to peck. And I think they had some old stump, and I don't know if they put nuts or peanut butter or something into the stump to get the woodpecker to peck at it-- because they needed to peck at a particular spot, so they have the camera all set up.

So anyway, they had this whole arrangement to do this high-speed video of the woodpecker pecking.

OK, so it goes that up to 1,500g, which is kind of amazing. And then, remember also, they do this repeatedly-- they do it at like 10 or 20 times a second.

Then this is just explaining what stress is-- but you already know what stress is, so I'm going to skip over that.

And so the way you can think about this is to think about it in terms of a scaling argument. So imagine there's the brain and there's the skull, and when the head hits the tree, the brain's going to accelerate, and the brain and the skull are both going to decelerate. And you can think of the stress as the force over the area. So the force is the mass times the deceleration over the area. So that value for the woodpecker, you can say, is roughly equal to the same values, but for the human.

So what that argument relies on is the idea that the stress to cause damage in the woodpecker brain is similar to the stress to cause damage in the human brain. And that's not

totally unreasonable. So if you look at bone, for example, like when you measure the strength of the whale bones, it's not going to be that different from what you would measure from human bones. So when you look at a particular type of tissue, and you look at the strength of it in different species, the properties aren't that different from one species to another.

So let's say the brain tissue gets damaged at the same stress in the two species. So we can write this sort of equation down. And the mass is going to depend on the density of the brain tissue times the volume-- and the volume goes as the radius cubed. And the density is going to be the same for the woodpecker, or for the human-- so assume the brain tissue has the same kind of basic stuff.

So the mass goes as the radius cubed, and the area of contact is going to go as the radius squared. And so there's a radius term. So you can say the radius times the deceleration in the woodpecker should be equal to the radius times the deceleration of the human. And obviously, the woodpecker radius is going to be a lot smaller, so the woodpecker deceleration is going to be a lot bigger. So part of the thing is the brain is just a lot smaller.

Then there's another factor that comes into it-- these are photographs of an acorn woodpecker skull and a human skull that I got from the Museum of Comparative Zoology at Harvard. So this is looking down on the top, and this is an elevation view.

And if you think of the brain as roughly a hemisphere-- I mean, obviously it's not a perfect hemisphere, but let's say it's roughly a hemisphere-- the orientation of the brain in the skull is slightly different in the birds and in the human. So if you think of it as being this way on, and in the woodpecker it's turned roughly this way on, and the contact area-- think of this as the contact area. The projected area would be a full circle, and in the human, the brain is more this way on. And then the projected contact area is a semicircle.

So there's a factor of 2 difference, just because of the orientation of the brain. And so I've put the factor of 2 that accounts for that. So then I could say the deceleration that the woodpecker can take is twice the ratio of the radii of the human and the woodpecker brain, times the deceleration the human can take. And this was around 100g, remember.

So then I wanted to know what the ratio of the sizes was, and I thought, oh geez-- I'm going to have to mess around with skulls, and try to make some sort of measurements, and this is going to be a drag. And then I found this paper-- I couldn't believe it-- I found this paper called "Brain Size in Birds." And this guy had table after table after table of the mass of the brain in

different birds-- and he had the acorn woodpecker, lucky for me.

So if you just assume that the brain is roughly a hemisphere, if you have the mass, you can work out roughly what the radius is.

So it turns out there's a factor of 8 difference between the radii. And so the human brain is about eight times the size of the woodpecker. And then if you take into account this factor of 2, that means the woodpecker should be able to tolerate a deceleration of 16 times what the human can.

So remember, I said the human can take about 100-- so this would get the woodpecker up to 1,600. But they measured 1,500. And I'm a civil engineer, originally, I like big factors of safety-- that's a little too close for comfort for me. And so it turns out there's one more factor that matters.

People have studied human brain injury pretty extensively, and one of the things they've looked at is how much acceleration you can tolerate without injury, relative to the duration of the impact. So this is from a car crash conference-- so here's the tolerable acceleration, and here's the duration of the impact. And typically, for human head impacts, the deceleration occurs over a few milliseconds-- like over 3 to 10 milliseconds.

So if this is the duration of the typical head impact for a human, here's the range of the tolerable accelerations between about 80g and 160g-- so you know, I said around 100.

So we can take this curve, and now we have this factor of 16, and we can just scale it up by our factor of 16 for the woodpecker. So if I scale it up by the factor of 16, we're there. But the duration of the impacts was more around 1/2 a millisecond, to a millisecond, so I've extrapolated this a little bit. And the duration of them is up in here.

So this is saying the woodpecker can take these sorts of decelerations here, and that adds on another factor of 4. And these were the measured decelerations-- 1,500 was about the biggest, and I think it went to about a few hundred g.

So there's really three factors-- one is the small brain size, so there's this sort of scaling factor. One is the orientation of the brain, that was another factor of 2. And then there's this duration of impact, which is a factor of 4. And so that's how you can get this huge decelerations that they've measured in the woodpecker when they're pecking.

So now I have just a couple more slides. So woodpeckers have various adaptations to pecking, too. So one of the things is they have amazingly stiff tail feathers-- you see the tails here? You can't quite see it because this didn't reproduce quite properly, but the woodpecker is perched on a tree here, and the tail is pressed up against the tree. And if you think about pecking, like imagine if its tail was not pressed up against the tree-- it would be kind of grabbing on with its feet, and it would be trying to peck, and it would be hard to push. You know, you need something to push against.

And so it's got these stiff tail feathers, which make it easier to push against the tree. And so here's the stiff tail feathers here.

It's also got kind of unusual feet for birds-- it's got two toes forward and two toes back. And again, if it's grabbing with its feet, that helps it get some purchase to push against. That's called zygodactyl in the bird world. So it's got these adaptations, the pecking.

And then finally, I just like this slide here from *The New Yorker*, because it's a woodpecker pecking out a woodpecker, so it seemed kind of cute.

And several people helped me with this project. Trey Crisco studies head injury at Brown. He actually does work for the NFL on football brain injuries, and looks at football helmets.

Sharon Swartz is a friend of mine at Brown, who's a biologist. She studies bat flight. And she just thought this was kind of a cool project.

So, in fact, I was walking the dog last week, a few days ago, and I saw bats flying around overhead at night. And it was so great to see the little bats. So I had to immediately email Sharon, and say, bats-- there's bats in my neighborhood.

Andy Biewener runs the Concord Field Station, and studies animal locomotion, and I talked to him about it. Jeremy Trimble was the one who gave me the skull pictures, and Matt Dawson was a student helped me do some of the images.

So that's my woodpecker talk. So that's the end of energy absorption, I just thought that was kind of amusing.

So next time, we'll start talking about sandwich panels. So, let's see-- Monday's a holiday. I will be in Toronto on Monday, seeing my family. In fact, I'm going to see one of my old professors-- I'm going to my old fluid mechanics professor on Monday and have lunch. I'll see my family

on the weekend.

And so we'll meet Wednesday next week, and I'll start the bit on sandwich panels. So I think there's two lectures on engineering sandwich panels, and then there's a lecture on natural sandwich panels-- so bird skulls, for example, are natural sandwich panels. So I'll talk about that.

And then I think the last lecture-- there's only a handful of lectures left-- the last one I think I talk about natural materials, because you know, I like that. So I just do that for fun.