Session #17 - Homework Solutions

Problem #1

You are operating an x-ray tube with a chromium (Cr) target by applying an acceleration potential (V) of 60 kV. Draw a schematic of the x-ray spectrum emitted by this tube; label on it three characteristic λs and give the numerical value of two of these.

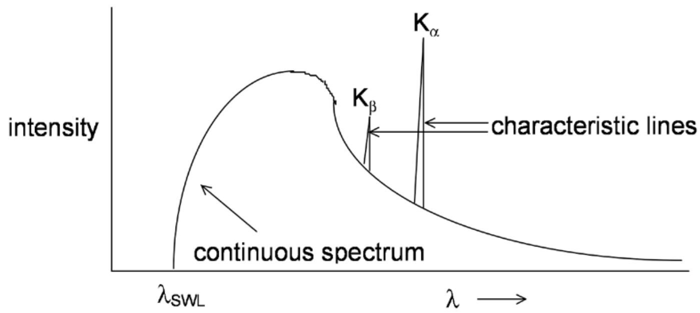
Solution

A characteristic x-ray spectrum of Cr will show λ_{SWL} , K_β , K_α and the continuous spectrum or *Bremsstrahlung*. We may quantify λ_{K_α} and λ_{SWL} .

$$_{24}Cr: \quad \bar{\nu}_{K_\alpha} = \frac{3}{4}R(Z-1)^2 = \frac{3}{4} \times 1.097 \times 10^7 (23)^2 = 4.35 \times 10^9 \text{ m}^{-1}$$

$$\lambda_{K_\alpha} = 2.3 \times 10^{-10} \text{ m}$$

$$\lambda_{SWL} = \frac{hc}{eV} = \frac{1.24 \times 10^{-6} \text{ m}}{6 \times 10^4} = 2.07 \times 10^{-11} \text{ m}$$



Problem #2

(a) An X-ray tube with a silver (Ag) target at a plate voltage of 66 kV. Calculate the value of λ_{SWL} , the shortest wavelength.

(b) Sketch the emission spectrum (intensity vs. wavelength) of the Ag target in part (a). On your sketch, indicate the relative positions of the K_α , K_β , L_α , and L_β lines and λ_{SWL} . It is not necessary to calculate the λ values of the K_β , L_α , and L_β lines.

(c) In one or two sentences explain the origin of the continuous spectrum.

Solution

$$\text{(a)} \quad \lambda_{SWL} = \frac{hc}{eV} = \frac{12400}{66 \times 10^3} = 0.188 \text{ Å}$$

(b) See sketch above in answer to problem #1. The L_α and L_β lines will appear to the right of the analogous K lines (at higher values of λ), the L_α to the right of the L_β.

(c) Incident electrons are deflected by the negative charge of electrons in the target. Any change in velocity (speed or direction or both) is an acceleration. Accelerating a charge emits radiation. The deflected electrons' acceleration is NOT QUANTIZED. Thus, the spectrum is continuous.

Problem #3

Determine the wavelength of λ_{K_α} for molybdenum (Mo).

Solution

Mo: $Z = 42$; $K_\alpha \rightarrow n_i=2;\ n_f=1;\ \sigma=1$

$$\bar{\nu}_{K_\alpha} = R(Z - 1)^2\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

$$\bar{\nu}_{K_\alpha} = 1.097 \times 10^7\left[\frac{1}{m}\right](42-1)^2\left[\frac{1}{1^2} - \frac{1}{2^2}\right]$$

$$\bar{\nu}_{K_\alpha} = 1.38 \times 10^{10}\ m^{-1}$$

$$\lambda_{K_\alpha} = \frac{1}{\bar{\nu}_{K_\alpha}} = 7.25 \times 10^{-11}\ m$$

Problem #4

Identify the element giving rise to K_α with $\lambda = 2.51 \times 10^{-10}\ m$.

Solution

$$\frac{1}{\lambda_{K_\alpha}} = \bar{\nu}_{K_\alpha} = R(Z - 1)^2\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right] = R(Z - 1)^2 \times \frac{3}{4}$$

$$(Z-1) = \sqrt{\frac{4}{3 \times \lambda \times R}} = 22$$

$Z = 23$ (Vanadium)