3.15 Electrical, Optical, and Magnetic Materials and Devices
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Final Exam (6 pages)
Closed book exam. Formulae and data are on the last 4 pages of the exam.
This takes 180 min and there are 180 points total. Be brief in your answers and use sketches.

1. Magnetic materials [36]

a) Explain the shape of a M-H loop for a piece of single-crystal cobalt of macroscopic size (e.g. a few mm diameter), for H applied parallel to the c axis. In your answer explain how the magnetization varies inside the material as a function of H. What happens if H is perpendicular to c? [15]

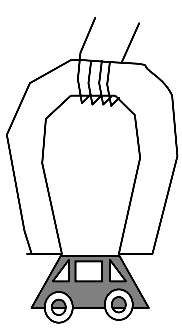
b) What would the B-H loop look like for H parallel to c? [3]

c) The magnetization of a sufficiently small piece of cobalt becomes thermally unstable. For a spherical particle of Co, estimate the size below which this thermal instability occurs. In a thermally unstable particle, what do you expect the coercivity and remanence to be? [9]
Data: Co $K_u = 5 \times 10^5$ J/m^3.

d) What is the physical basis of the coercivity for the following three materials (one sentence each)? [Note: coercivity data for these materials are given in the data sheet p6] [9]
Alnico
$SmCo_5$
amorphous Fe-B-Si alloy

2. Magnetic devices [36]

We want to build an electromagnet that can pick up a car in a scrapyard.



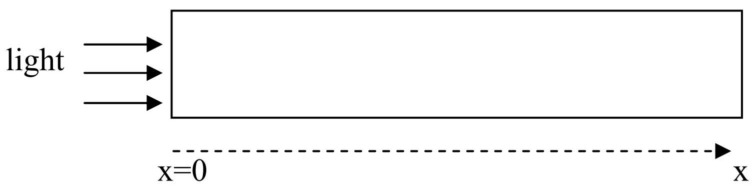
Assume a car has a mass of 2000 kg of which 25% is made of steel (B_s = 1 T, density 2.5 g/cm^3). Assume that the maximum force on a magnetic material of moment M and volume V in a field H is given by μ_oMHV. Suppose the core has a length of 5 m and the gap length is 1 m and there are 10,000 turns of wire around the core. Choose a core material from the list in the data sheet (on p6) and assess the feasibility of building an electromagnet strong enough to do the job.

Hint: Start by calculating how much field you would need to pick up the car.

3. Carriers [36]

a) In a pn junction, where is drift, diffusion and R&G occuring when the junction is
(i) at equilibrium
(ii) in reverse bias [12]

b) We have a piece of p-type Si as follows:

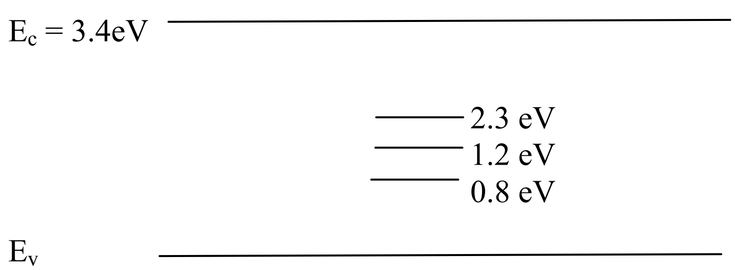


Assume that the light is all absorbed very near the surface. Show how you would derive an expression for the electron density as a function of distance, n(x), explaining your reasoning. You do not have to solve the equation but show where it comes from and which terms it contains. Illustrate with a sketch of n vs. x. [18]

c) Explain briefly what happens to n(x) after the light is turned off. (however, you do not need to derive the equation relating n to time) [6]

4. Optics [36]

Erbium (Er) at concentrations of ~ 1% in a GaN semiconductor has the following energy levels:



a) If you made it into a LED, what colors of light can this Er-doped GaN produce? Draw a sketch of light intensity vs photon energy. What factors influence how bright each color is and the spectral width of the peaks? [16]

b) If the GaN were amorphous instead of crystalline, how would this affect your answer? [4]

c) We now want to make the Er-doped crystalline material into a laser. It turns out that the transition from 0.8 eV level to the valence band is the slowest. How would you pump it, and what color light would the laser make?

If the active region of the laser is 100 microns long, and the laser light has a spectral width that is 2% of the center frequency, what would the output of the laser look like as a function of frequency? [16]

5. Data storage devices [36]

a) Describe briefly the operation of a rewritable optical disk based on phase change material. Identify what materials would be suitable for the data storage layer. [up to 3-4 sentences plus 1-2 figures!] [12]

b) Describe briefly the operation of a rewritable optical disk based on magnetooptical material. Identify what materials would be suitable for the data storage layer. [up to 3-4 sentences plus 1-2 figures!] [12]

c) What limits the data density of each? [6]

d) Why is phase change media now more important than magnetooptical media? [6]

Equations

$g_c(E)\, dE = m_n^* \sqrt{\{2m_n^*(E - E_c)\}} / (\pi^2 \hbar^3)$

$g_v(E)\, dE = m_p^* \sqrt{\{2m_p^*(E_v - E)\}} / (\pi^2 \hbar^3)$

$f(E) = 1/ \{1 + \exp (E - E_f)/kT \}$

$n = n_i \exp (E_f - E_i)/kT, \qquad p = n_i \exp (E_i - E_f)/kT$

$n_i = N_c \exp (E_i - E_c)/kT$ where $N_c = 2\{2\pi m_n^* kT/h^2\}^{3/2}$

$np = n_i^2$ at equilibrium

$n_i^2 = N_c N_v \exp (E_v - E_c)/kT = N_c N_v \exp (-E_g)/kT$

$E_i = (E_v + E_c)/2 + 3/4\, kT \ln (m_p^*/ m_n^*)$

$E_f - E_i = kT \ln (n/ n_i) = - kT \ln (p/ n_i)$

$\sim kT \ln (N_D / n_i)$ ntype or $- kT \ln (N_A / n_i)$ ptype

Drift: thermal velocity $1/2\, mv^2_{thermal} = 3/2\, kT$

drift velocity $v_d = \mu \mathbf{E}$ $\qquad \mathbf{E}$ = field

Current density (electrons) $J = n\, e\, v_d$

Current density (electrons & holes) $J = e\, (n\, \mu_n + p\, \mu_h)\mathbf{E}$

Conductivity $\sigma = J/\mathbf{E} = e\, (n\, \mu_n + p\, \mu_h)$

Diffusion $J = eD_n \nabla n + eD_p \nabla p$

Einstein relation: $D_n/\mu_n = kT/e$

R and G $R = G = rnp = r\, n_i^2$ at equilibrium

$dn/dt = dn/dt_{drift} + dn/dt_{diffn} + dn/dt_{thermal\ RG} + dn/dt_{other\ RG}$

Fick's law $dn/dt_{diffn} = 1/e\, \nabla J_{diffn} = D_n d^2n/dx^2$

so $dn/dt = (1/e)\, \nabla \{J_{drift} + J_{diffn}\} + G - R$

$dn/dt_{thermal} = - n_l/\tau_n$ or $dp/dt_{thermal} = - p_l/\tau_p$

$\tau_n = 1/rN_A$, or $\tau_p = 1/rN_D$ $\qquad \lambda_n = \sqrt{(\tau_n D_n)}$ or $\lambda_p = \sqrt{(\tau_p D_p)}$

If traps dominate $\tau = 1/r_2 N_T$ where $r_2 >> r$

pn junction

$\mathbf{E} = 1/\varepsilon_o\varepsilon_r \int \rho(x)\, dx$ where $\rho = e(p - n + N_D - N_A)$

$\mathbf{E} = -dV/dx$

$eV_o = (E_f - E_i)_{n\text{-type}} - (E_f - E_i)_{p\text{-type}}$

$= kT/e \ln (n_n/n_p)$ or $kT/e \ln (N_A N_D/n_i^2)$

$\mathbf{E} = N_A e\, d_p/\varepsilon_o\varepsilon_r = N_D e\, d_p/\varepsilon_o\varepsilon_r$ at $x = 0$

$V_o = (e /2\varepsilon_o\varepsilon_r)\, (N_D d_n^2 + N_A d_p^2)$

$d_n = \sqrt{\{(2\varepsilon_o\varepsilon_r V_o/e)\, (N_A/(N_D(N_D + N_A))\}}$

$d = d_p + d_n = \sqrt{\{(2\varepsilon_o\varepsilon_r(V_o + V_A)/e)\, (N_D + N_A)/ N_A N_D\}}$

$J = J_o\{\exp eV_A/kT - 1\}$ where $J_o = en_i^2\, \{D_p/N_D\tau_p + D_n/N_A\tau_n\}$

Transistor BJT gain $\beta = I_C /I_B \sim I_E /I_B = N_{A,E} / N_{D,B}$

$I_E = (eD_p/w)\, (n_i^2/N_{D,B}) \exp(eV_{EB}/kT)$

JFET $V_{SD,\,sat} = (eN_D t^2/8\varepsilon_o\varepsilon_r) - (V_o + V_G)$

Photodiode and Photovoltaic:

$I = I_o + I_G$ $\qquad V = I\, (R_{PV} + R_L)$

$I = I_o (\exp(eV/kT) - 1) + I_G$ $\qquad$ Power = IV

Wavelength $\lambda\, (\mu m) = 1.24/E\, (eV)$

Band structure

Effective mass: $m^* = \hbar^2 (\partial^2 E / \partial k^2)^{-1}$

Momentum of an electron typically $\pi/a \sim 10^{10}$ m^{-1}
Momentum of a photon $= 2\pi/\lambda \sim 10^{7}$ m^{-1}
Uncertainly principle $\Delta x \Delta p \geq \hbar$

Lasers

probability of absorption = B_{13}, stimulated emission = B_{31}, spontaneous emission = A_{31}

$N_3 = N_1 \exp(-h\nu_{31}/kT)$

Planck $\rho(\nu)d\nu = \{8\pi h\nu^3/c^3\}/\{\exp(h\nu/kT) - 1\}\, d\nu$

$B_{13} = B_{31}$

and $A_{31}/B_{31} = 8\pi h\nu^3/c^3$ (Einstein relations)

Cavity modes $\nu = cN/2d$, N an integer.

Optical Properties

Light $c = \nu\lambda$, in a material speed = c/n , n= refractive index

Attenuation (dB/m) $= \{10/L\} \log(P_{in}/P_{out})$ L = fiber length

Snell's law: $n \sin\phi = n' \sin\phi'$

Dispersion coefft. $D_\lambda = -\{\lambda_o/c\}(\partial^2 n/\partial\lambda^2)_{\lambda=\lambda_o}$ ps/km.nm

$\sigma_t = \sigma_\lambda L D_\lambda$

Pockels effect $n = n_o - (1/2)\, r\, n_o^3 \mathbf{E}$ n = refractive index, **E** = electric field, r = Pockels coefft.

Kerr effect $n = n_o + \lambda K \mathbf{E}^2$ K= Kerr coefft.

Magnetism

current i in a wire produces field $H = i/2\pi r$ at radius r

in free space $B = \mu_o H$ $\mu_o = 4\pi\ 10^{-7}$ Henry/m

inside a material $B = \mu_o(H + M)$

or $B = \mu_o \mu_r H$ μ_r = relative permeability

or $M = H(\mu_r - 1)$

or $M = \chi H$ $\chi = (\mu_r - 1)$ = susceptibility

One electron has a moment of 1 μ_B (Bohr magneton) = $9.27\ 10^{-24}$ Am^2

If spins make angle θ, exchange energy = $A(1 - \cos\theta)$ where A is the exchange constant

Anisotropy K

Uniaxial: $E = K_u \sin^2\phi$ E = energy, ϕ = angle between M and easy axis

Cubic: $E = K_1(\cos^2\phi_1\cos^2\phi_2 + \cos^2\phi_2\cos^2\phi_3 + \cos^2\phi_3\cos^2\phi_1)$ + higher order terms

ϕ_i = angle between M and the i axis

Domains

wall width $d = \pi\sqrt{A/2Ka}$ (a = lattice parameter)

wall energy $E_w = \pi\sqrt{2AK/a}$

Thermal instability when $K_{tot}V < 25kT$. (here V is the volume of the particle)

Magnetostatic energy

$E = K_{shape} \sin^2\phi$ ϕ = angle between M and z axis

where $K_{shape} = 0.5(N_x - N_z)M_s^2$ N_i = demagnetizing factor along i axis

The field inside the object along the i axis due to its own magnetization is

$H_d = -N_i M_s$ M_s = saturation magnetization.

Induction: current i_m through n turns of wire: $\oint H.dl = ni_m$

Induced voltage $V = -n'\, d\phi/dt$ where $\phi = B.A$ (A = coil area), n' = number of turns of wire.

If a current i runs through a wire length l in a B field: Force $F = Bil$

Anisotropic magnetoresistance $R = R_o + \Delta R \cos^2 \theta$; θ = angle between M and current

PHYSICAL CONSTANTS, CONVERSIONS, AND USEFUL COMBINATIONS

Physical Constants

Avogadro constant	$N_A = 6.022 \times 10^{23}$ particles/mole
Boltzmann constant	$k = 8.617 \times 10^{-5}$ eV/K $= 1.38 \times 10^{-23}$ J/K
Elementary charge	$e = 1.602 \times 10^{-19}$ coulomb
Planck constant	$h = 4.136 \times 10^{-15}$ eV $\cdot$s $= 6.626 \times 10^{-34}$ joule $\cdot$s
Speed of light	$c = 2.998 \times 10^{10}$ cm/s
Permittivity (free space)	$\varepsilon_0 = 8.85 \times 10^{-14}$ farad/cm
Electron mass	$m = 9.1095 \times 10^{-31}$ kg
Coulomb constant	$k_c = 8.988 \times 10^9$ newton-m^2/(coulomb)^2
Atomic mass unit	$u = 1.6606 \times 10^{-27}$ kg

Useful Combinations

Thermal energy (300 K)	$kT = 0.0258$ eV $\simeq$ 1 eV/40
Photon energy	$E = 1.24$ eV at $\lambda = \mu$m
Coulomb constant	$k_c e^2$ 1.44 eV $\cdot$ nm
Permittivity (Si)	$\varepsilon = \varepsilon_r \varepsilon_0 = 1.05 \times 10^{-12}$ farad/cm
Permittivity (free space)	$\varepsilon_0 = 55.3e/V \cdot \mu m$

Prefixes

k = kilo = 10^3; M = mega = 10^6; G = giga = 10^9; T = tera = 10^{12}
m = milli = 10^{-3}; μ = micro = 10^{-6}; n = nano = 10^{-9}; p = pica = 10^{-12}

Symbols for Units

Ampere (A), Coulomb (C), Farad (F), Gram (g), Joule (J), Kelvin (K)

Meter (m), Newton (N), Ohm (Ω), Second (s), Siemen (S), Tesla (T)

Volt (V), Watt (W), Weber (Wb)

Conversions

1 nm = 10^{-9} m = 10 Å = 10^{-7} cm; 1 eV = 1.602×10^{-9} Joule = 1.602×10^{-12} erg;
1 eV/particle = 23.06 kcal/mol; 1 newton = 0.102 kg_{force};
10^6 newton/m^2 = 146 psi = 10^7 dyn/cm^2 ; 1 μm = 10^{-4} cm 0.001 inch = 1 mil = 25.4 μm;
1 bar = 10^6 dyn/cm^2 = 10^5 N/m^2; 1 weber/m^2 = 10^4 gauss = 1 tesla;
1 pascal = 1 N/m^2 = 7.5×10^{-3} torr; 1 erg = 10^{-7} joule = 1 dyn-cm

Figure by MIT OCW.

Properties	Si	GaAs	SiO_2	Ge
Atoms/cm^3, molecules/cm^3 x 10^{22}	5.0	4.42	2.27[a]	
Structure	diamond	zincblende	amorphous	
Lattice constant (nm)	0.543	0.565		
Density (g/cm^3)	2.33	5.32	2.27[a]	
Relative dielectric constant, ε_r	11.9	13.1	3.9	
Permittivity, $\varepsilon = \varepsilon_r \varepsilon_o$ (farad/cm) x 10^{-12}	1.05	1.16	0.34	
Expansion coefficient (dL/LdT) x (10^{-6} K)	2.6	6.86	0.5	
Specific Heat (joule/g K)	0.7	0.35	1.0	
Thermal conductivity (watt/cm K)	1.48	0.46	0.014	
Thermal diffusivity (cm^2/sec)	0.9	0.44	0.006	
Energy Gap (eV)	1.12	1.424	~9	0.67
Drift mobility (cm^2/volt-sec)				
Electrons	1500	8500		
Holes	450	400		
Effective density of states (cm^{-3}) x 10^{19}				
Conduction band	2.8	0.047		
Valence band	1.04	0.7		
Intrinsic carrier concentration (cm^{-3})	1.45 x 10^{10}	1.79 x 10^{6}		

Properties of Si, GaAs, SiO2, and Ge at 300 K

Figure by MIT OCW.

Magnetic materials

	T_c/K	B_s/T	Hc / A/m	μ_r
Fe	1043	2.2	4	200,000
Fe-3%Si	1030	2.1	12	40,000
a-FeBSi	630	1.6	1	100,000
Alnico-5	1160	1.4	64,000	1000
$BaO.(Fe_2O_3)_6$	720	0.4	264,000	2000
$SmCo_5$	1000	0.85	600,000	1000
$Nd_2Fe_{14}B$	620	1.1	890,000	2000