3.22 Mechanical Properties of Materials
Spring 2008

Problem Set #5
Due: Friday, May 2 by 5:00 PM

1. The data in the supplementary spreadsheet 5-1.xls is obtained from creep experiments on a polycrystalline oxide with a grain size of 10 μm.

   (a) Make a plot of log (strain rate) versus log (stress) for the three temperatures. Identify on your plot the regions that correspond to diffusional creep and those that correspond to power-law creep. At what stress and temperature does the transition from dislocation climb to a mechanism that does not require dislocations occur?

   (b) Does the mechanism that does not require dislocations correspond to Nabarro-Herring creep or to Coble creep?

   (c) Plot the strain rate versus stress at 1700 K for this material at the grain size of 1 μm, 10 μm and 100 μm. How does the change in grain size affect the transition point between dislocation climb and the mechanism you named in part (b)?

   (d) Using the constitutive equations for Nabarro-Herring and Coble creep, show how the barrier in a deformation mechanism map between the two would change with grain size.

   (e) Schematically draw a deformation mechanism map (σ/G vs. T/T_{m}) for this material with a grain size of 10 μm. Clearly label the mechanism that dominates in the different regimes. Using the information you learned in parts (c) and (d), show on this deformation mechanism map how the boundaries between these regimes would change with a change in grain size to 1 μm and 100 μm.

   (f) How would the creep activation barrier change (higher or lower?) and the deformation mechanism map change in this material if it was an amorphous oxide? Explain. How would the activation barrier (higher or lower?) be different between an amorphous oxide and amorphous metal? Explain.

2. Ice is a hydrogen-bonded crystalline material that creeps appreciably, especially under the high stresses and potentially high temperatures characteristic of polar ice caps and glaciers.

   (a) Consider a portion of a polar ice cap that is under an in-plane equibiaxial compressive stress of 10 MPa at -50°C. What is the maximum shear stress imposed on the ice by this stress state?

   (b) Referring to the deformation mechanism map below, what is the strain rate dγ/dt we would expect this polar ice cap region to exhibit under the conditions defined in (a)? Clearly state any assumptions of ice cap geometry and relevant physical/mechanical properties. (A cool, relatively recent paper on ice mechanics is Elvin et al., *J. Eng. Mech.* Jan (1998); check out the author affiliations.)

   (c) Assuming the applied stress on the ice cap remains constant, and the polar ice cap ambient temperature increases to the highest magnitude currently predicted by climatologists, what is the % difference in creep rate with respect to a current temperature of -50°C?

   (d) Assuming the ice cap is already in stage II creep in 2008 and this worst-case temperature is achieved this year, how long will it take for a polar ice cap of this grain size and thickness of 50 cm to shrink in in-plane dimensions by 50%?
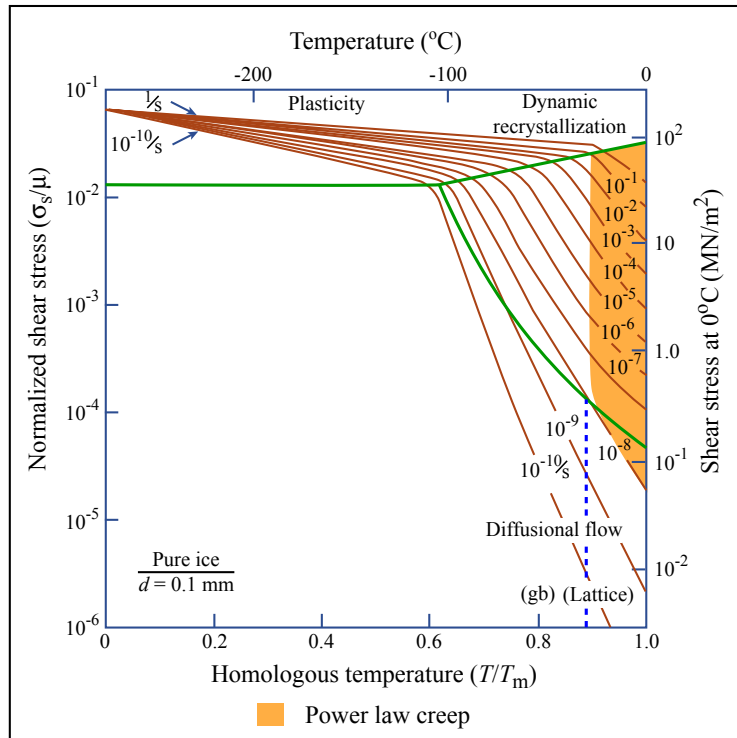
Figure by MIT OpenCourseWare.

Figure 1: Deformation mechanism map of pure ice.

3. What are the units for energy release rate? Explain thoroughly, yet concisely, what is meant by "rate" in this context? Explain how this relates to the material property of toughness. In general, rank the classes of materials (metallic, ceramic, and polymeric) from the highest to lowest energy release rate by using specific, cited examples from literature.

4. Calculation of the stress as a function from distance from an elliptical through-thickness crack tip of width $2a$ and tip radius R was given in class but not derived, as $\sigma = \sigma_o(1 + 2\sqrt{\frac{a}{R}})$. The derivation includes solution of a biharmonic equation of elasticity, the wise selection of an appropriate Airy stress function χ that maintains continuity and strain compatibility across the crack face, and complex variables; this is the same approach used to solve the self-stress around an edge dislocation. You could do it, but it's typically treated in an advanced mechanical behavior class. To put things in perspective, this was solved in 1913 by Inglis, when analytical elastic mechanics was the only game in town.

(a) The solution is a twist on the so-called Eshelby inclusion problem, or the effects of stress due to a spherical hole (or particle) of radius $R = a$ inside a material deformed under uniaxial stress σ_o. It is derived in polar coordinates r (distance from hole center) and θ (in-plane angle around hole center), and has the nice result of being bounded at the maximum to some finite stress. Let's work on part of this to understand why the stress concentration is bounded to be lower than infinity for a circular flaw: The stress distribution σ_{ij} given by this internal defect is:

$$\sigma_{rr} = \frac{1}{2}\sigma_o(1 - \frac{R^2}{r^2})\{1 + (1 - \frac{3R^2}{r^2})cos2\phi\} \tag{1}$$

$$\sigma_{\theta\theta} = \frac{1}{2}\sigma_o(1 + \frac{R^2}{r^2})\{1 + (1 + \frac{3R^4}{r^4})cos2\phi\} \tag{2}$$

$$\sigma_{r\theta} = \frac{-1}{2}\sigma_o(1 + \frac{2R^2}{r^2} - \frac{3R^4}{r^4})sin2\phi \quad (3)$$

Given that you expect the stress to be maximized along the equator of the hole (i.e., orthogonal to the loading axis), use the above expressions to find the magnitude of the maximum stress component in terms of only the applied stress σ. In other words, what is the numerical, maximum stress concentration factor due to this circular flaw? Graphing σ_{ij} may help illustrate this.

(b) Now let us assume the crack is a through-thickness, sharp crack as Griffith considered, and the material is amorphous silica for which others have reported γ_{surf} and E. What is the fracture stress σ_f if this silica initially contained defects of length $2a = 1$ mm?

(c) If your application required a fracture strength of 1 GPa and you could chemically modify any internal flaw surfaces in the above silica via chemical vapor deposition, what would the target surface tension be? Is this reasonably achievable?

5. Please see comments and address new questions related to special topics wiki, which will be posted by Friday 4/25 night (after DMSE social). Also note that this will be the final wiki pset assignment. Next week you will be preparing a short presentation for the class, to be delivered on May 8 (R) or May 13 (T).