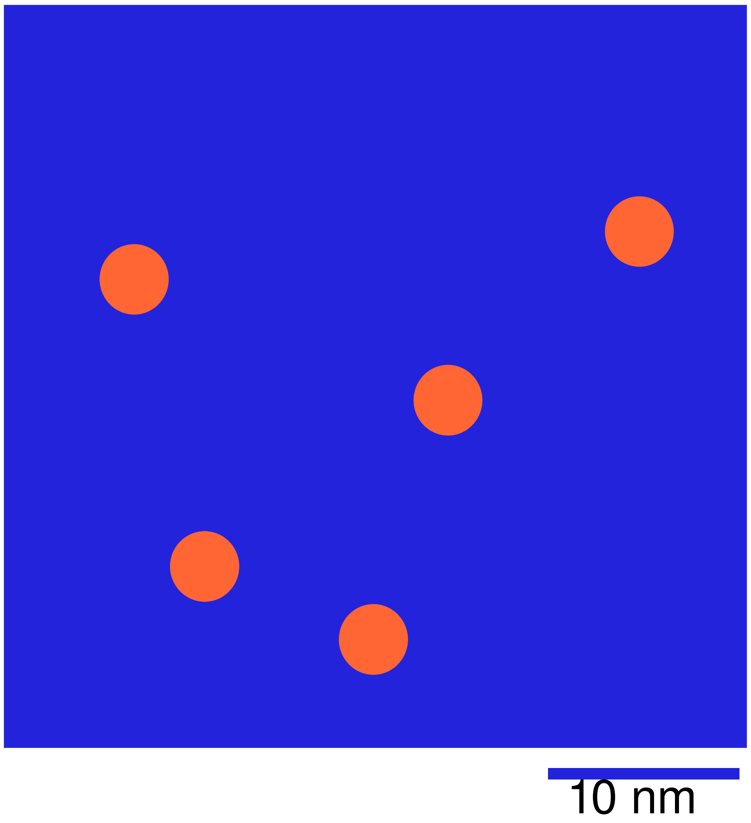
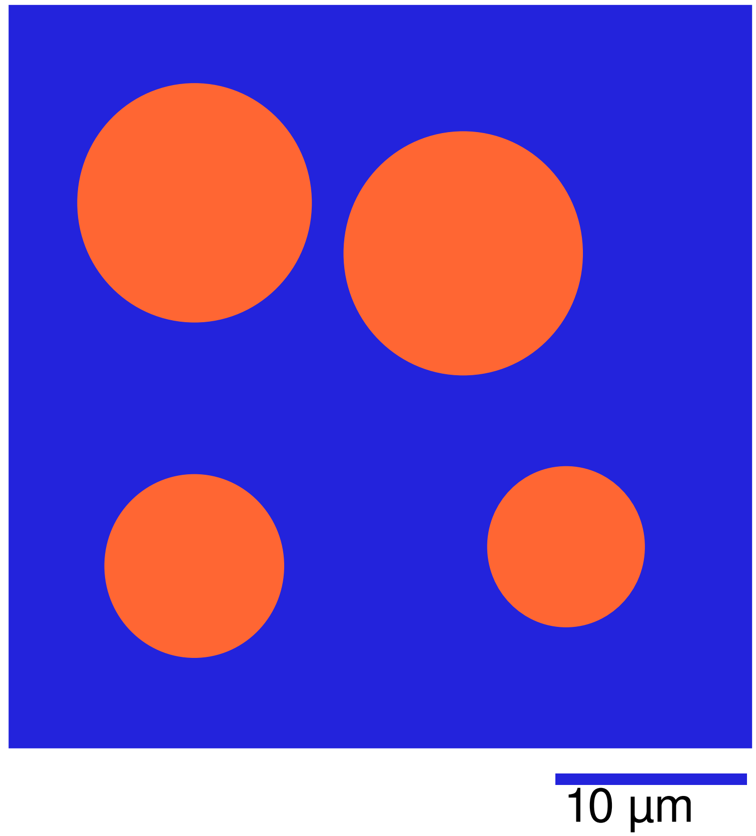
18 November 2009

3.14/3.40 Lecture Summary

We know how small precipitates affect material properties...

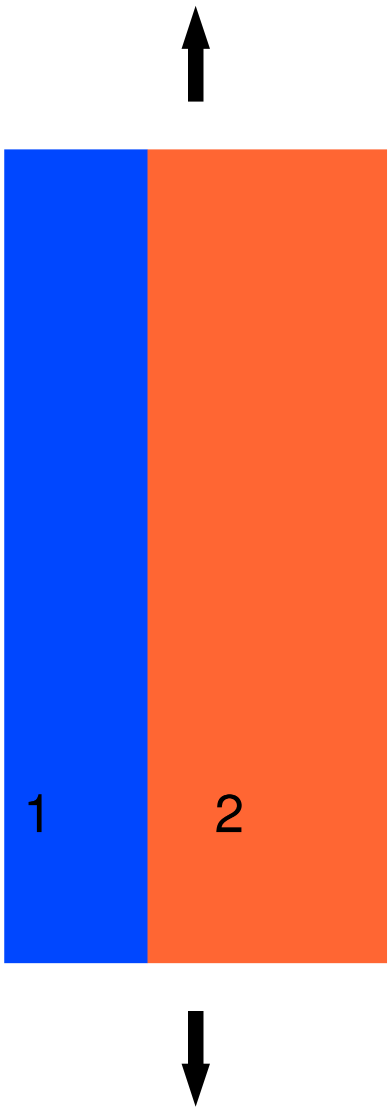
But what about larger inclusions?





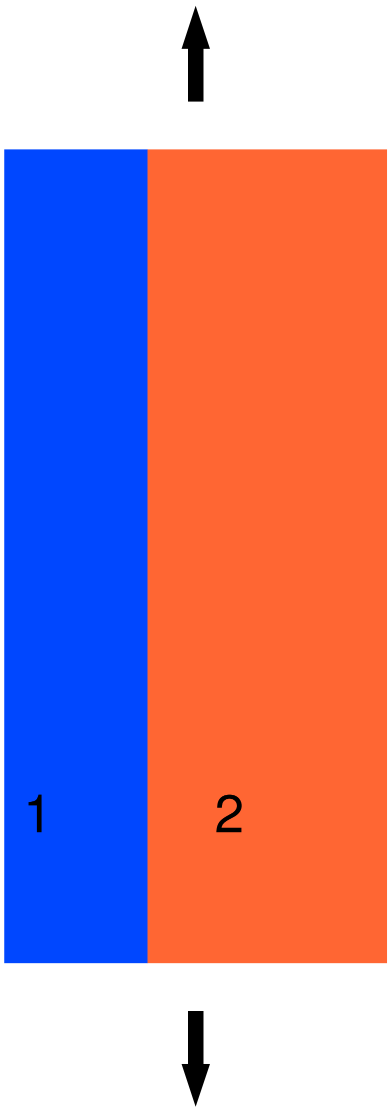
For particle size greater than about 1 μm, it is necessary to take into account the stress distribution **inside** the particle.

Bounding Case - Isostrain



- Examine simplified cases to determine a range of possible outcomes
- Assume the following:
  - Two materials with different E and ν (or G and K)
  - Perfect bonding between zones 1 and 2
  - Constant cross-section
- Given these assumptions:
  - What is E for the composite?
  - How are loads shared between the two zones?

Bounding Case - Isostrain



$$\epsilon_1 = \epsilon_2 = \epsilon_{tot}$$

$$\sigma_1 = E_1 \epsilon_1 = E_1 \epsilon_{tot} \quad ; \quad \sigma_2 = E_2 \epsilon_2 = E_2 \epsilon_{tot}$$

$$P_1 = A_1 \sigma_1 = A_1 E_1 \epsilon_{tot} \quad ; \quad P_2 = A_2 \sigma_2 = A_2 E_2 \epsilon_{tot}$$

$$P_{tot} = P_1 + P_2 = \epsilon_{tot} (A_1 E_1 + A_2 E_2)$$

$$\sigma_{tot} = \frac{P_{tot}}{A_1 + A_2} = \epsilon_{tot} \left(\frac{A_1}{A_1 + A_2} E_1 + \frac{A_2}{A_1 + A_2} E_2 \right)$$

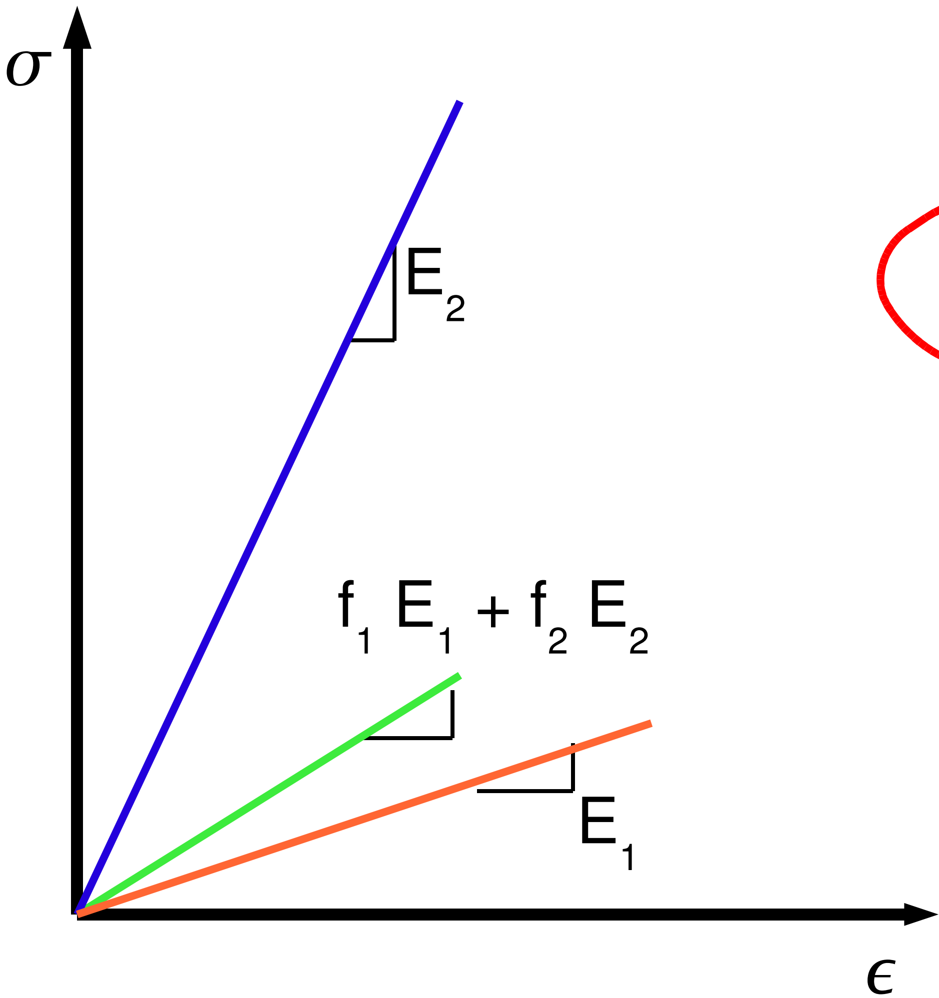
$$\sigma_{tot} = (f_1 E_1 + f_2 E_2) \epsilon_{tot}$$

$$E_{tot} = f_1 E_1 + f_2 E_2$$

P_1, P_2 are the loads on 1 and 2.

f_1, f_2 are the volume fractions of 1 and 2.

Bounding Case - Isostrain



Rule of Mixtures

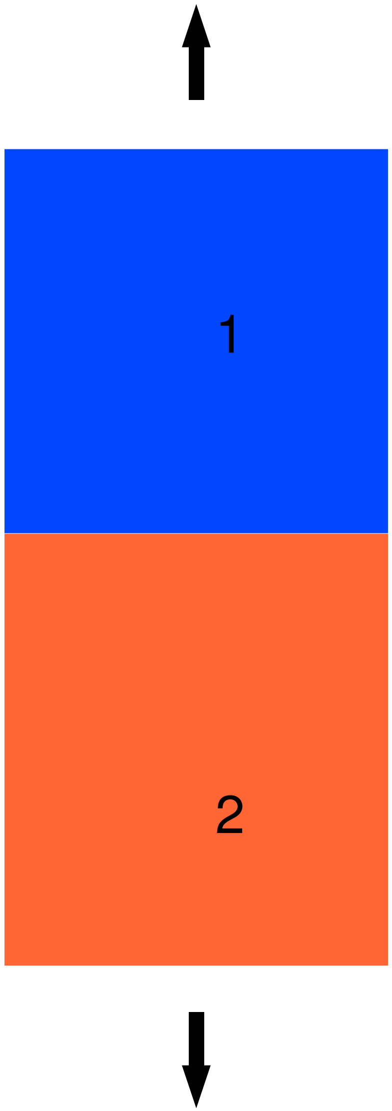
$$E_{tot} = f_1 E_1 + f_2 E_2$$

$$\frac{P_2}{P_1} = \frac{A_2 E_2}{A_1 E_1} = \frac{f_2 E_2}{f_1 E_1}$$

Load Sharing

The phase with the higher Young's Modulus has the higher stress.

Bounding Case - Isostress



$$\sigma_1 = \sigma_2 = \sigma_{tot}$$

$$\sigma_1 = E_1 \epsilon_1 \quad ; \quad \sigma_2 = E_2 \epsilon_2$$

$$\epsilon_{tot} = f_1 \epsilon_1 + f_2 \epsilon_2 = f_1 \frac{\sigma_{tot}}{E_1} + f_2 \frac{\sigma_{tot}}{E_2}$$

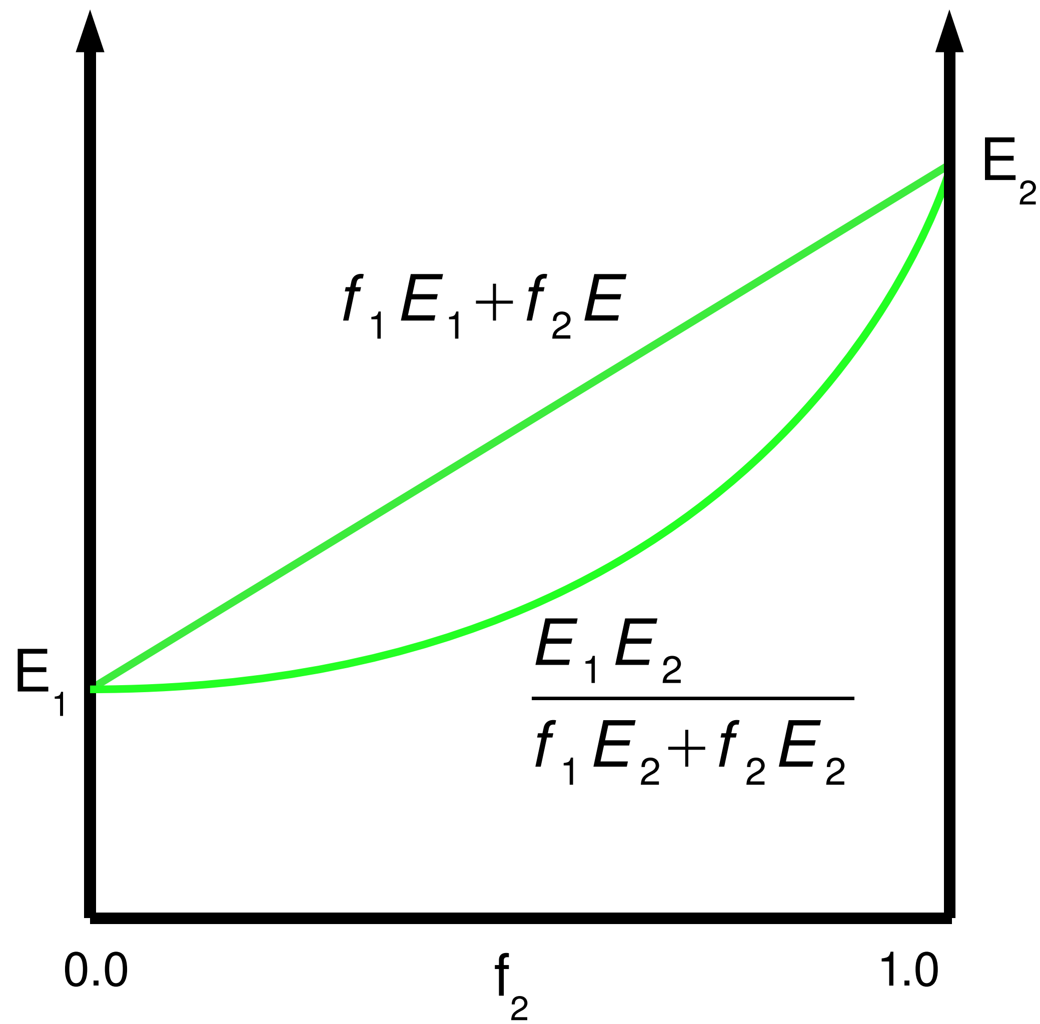
$$E = \frac{\sigma_{tot}}{\epsilon_{tot}} = \frac{1}{\frac{f_1}{\epsilon_1} + \frac{f_2}{\epsilon_2}} = \frac{E_1 E_2}{f_1 E_2 + f_2 E_1}$$

Modulus Limits

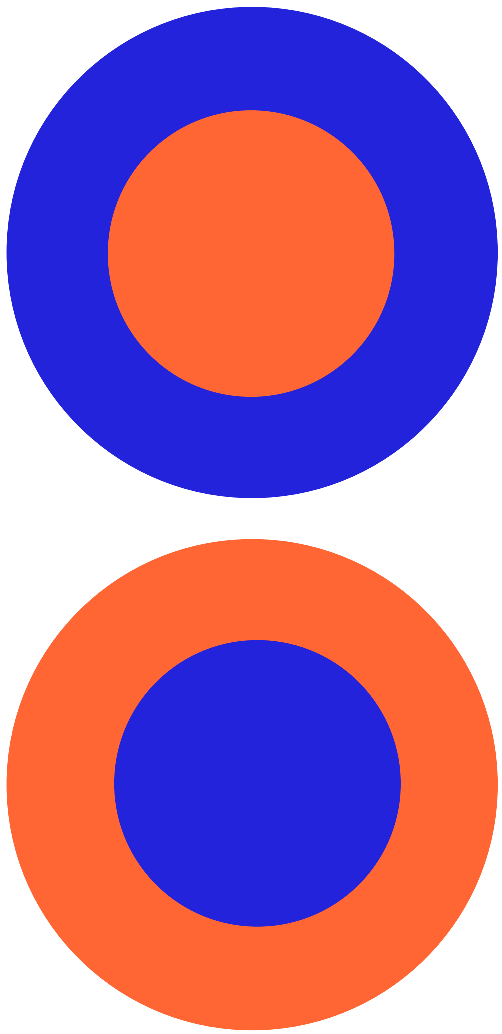
Isostrain case: $E_{tot} = f_1 E_1 + f_2 E_2$

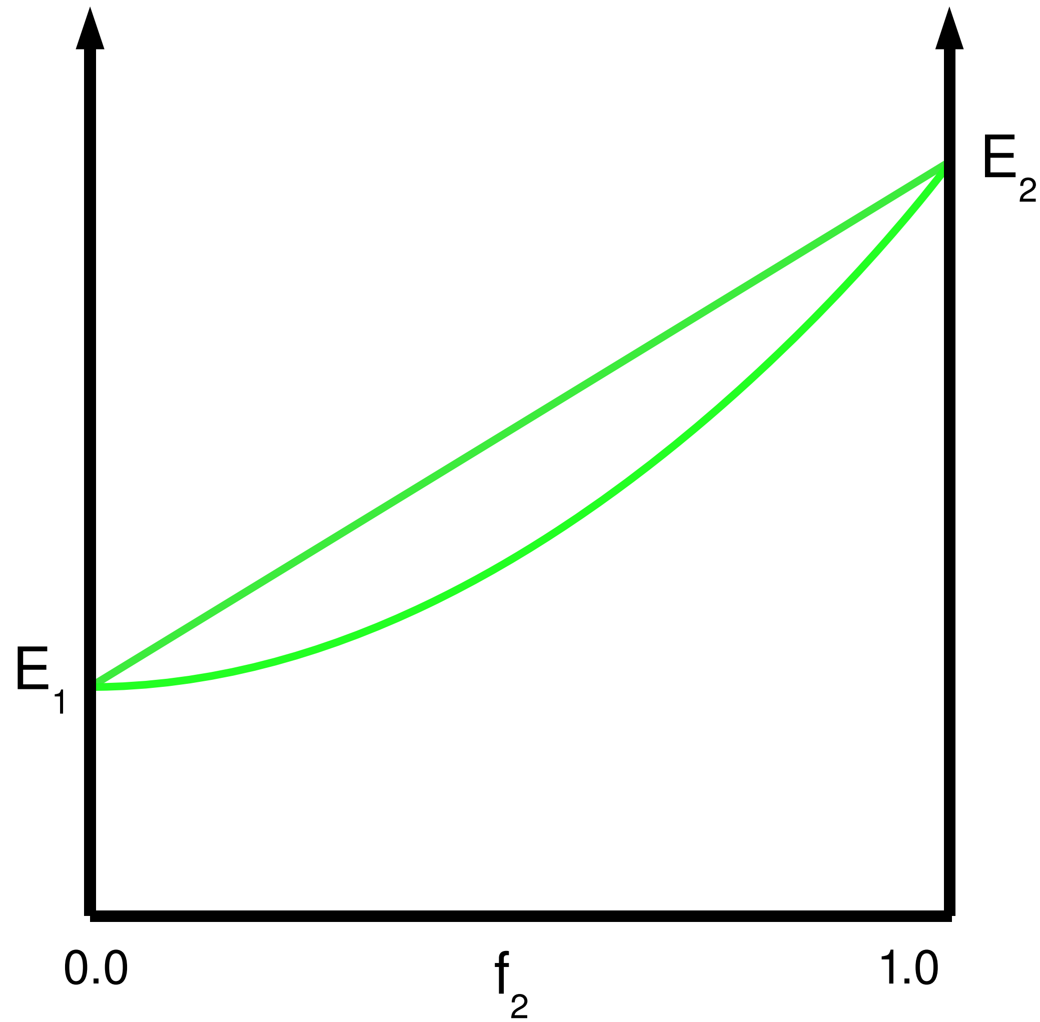
Isostress case: $E_{tot} = \frac{E_1 E_2}{f_1 E_2 + f_2 E_2}$

These two cases provide the bounds for more complicated microstructures.



Modulus Limits (Hashin-Shtrikman)

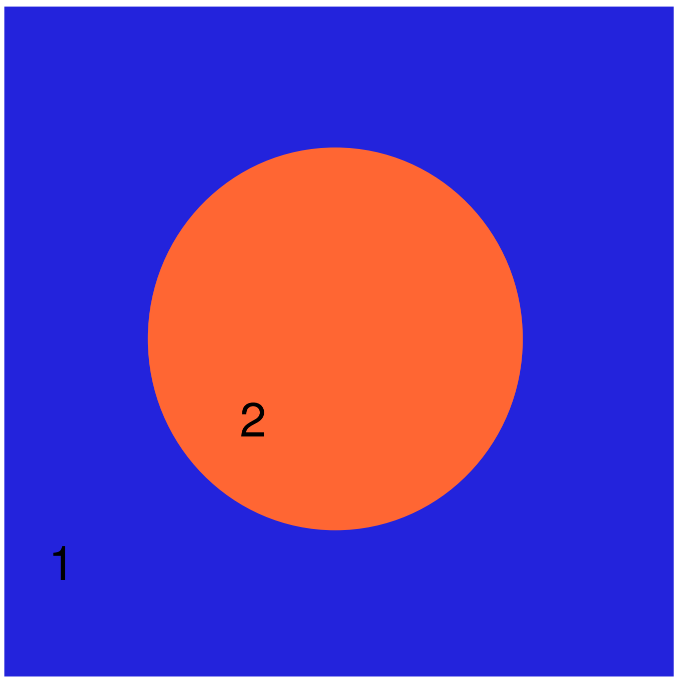




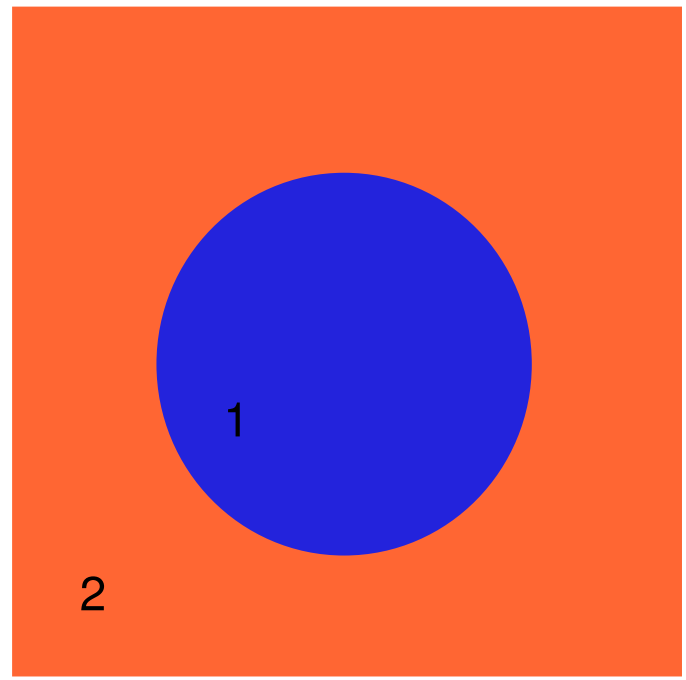
Assumption of isotropy leads to tighter bounding curves.

Contiguity

Take two systems, each with $f_2 = 0.5$: Which bounding line is a particular system closest to? Look at the connectivity:

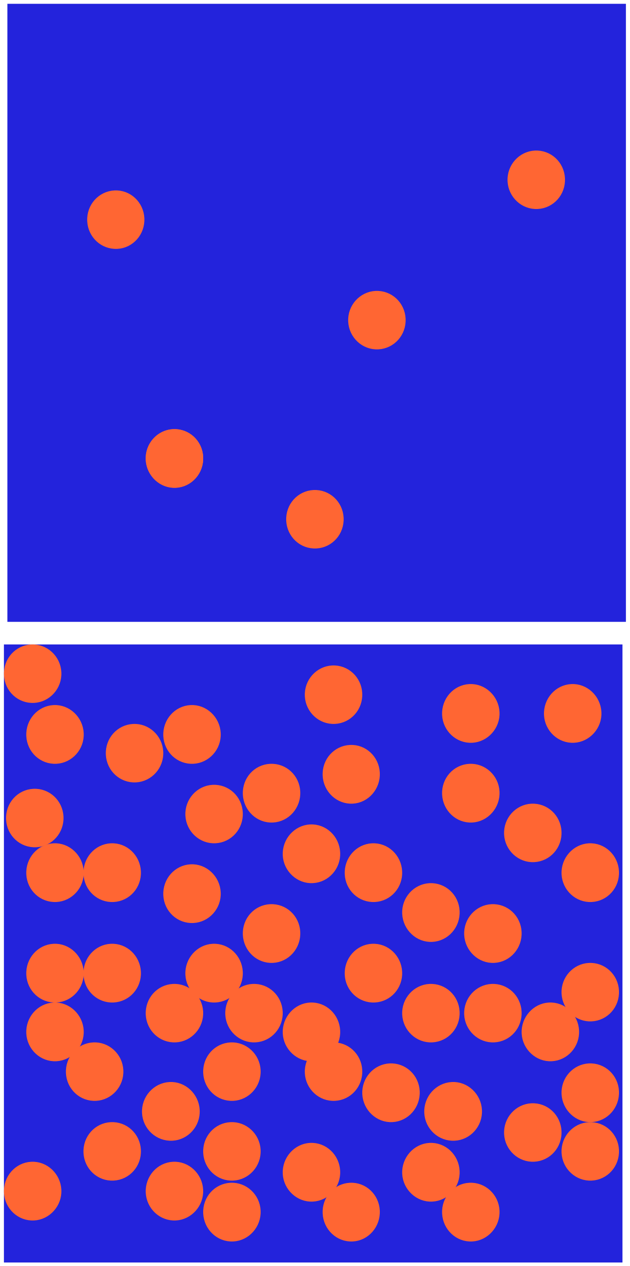


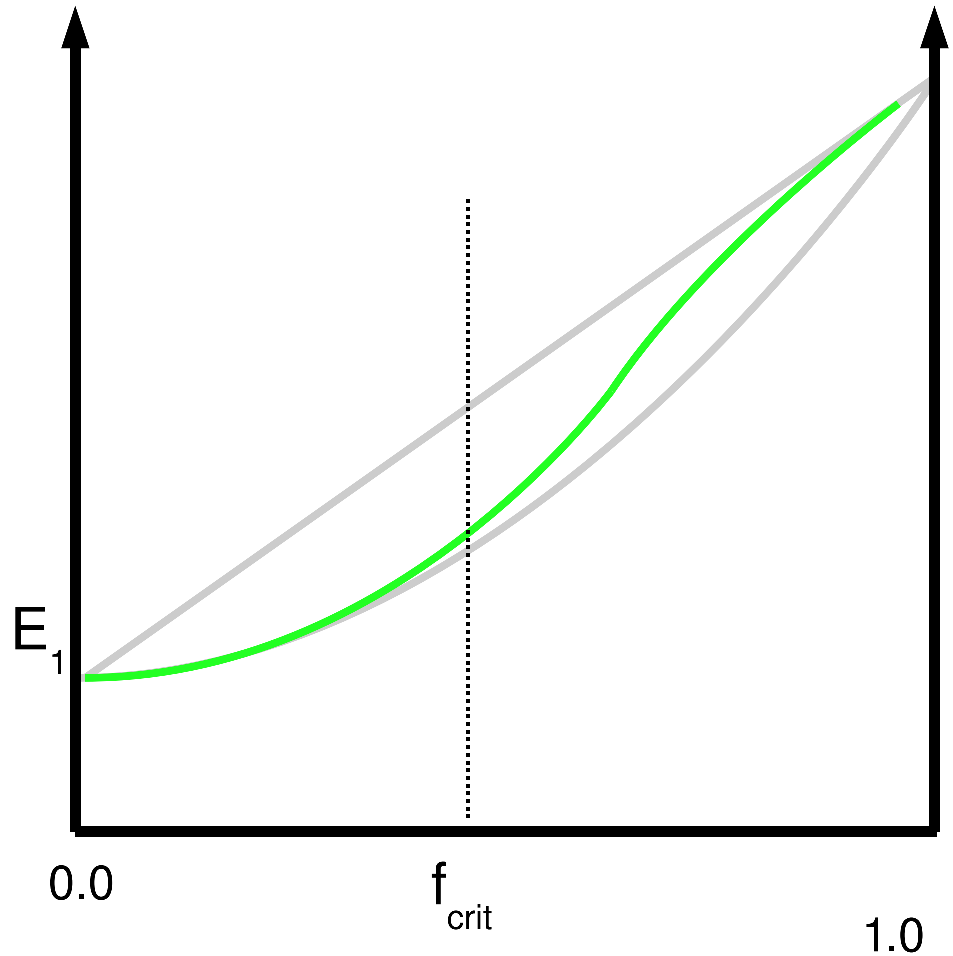
Closer to E_1



Closer to E_2

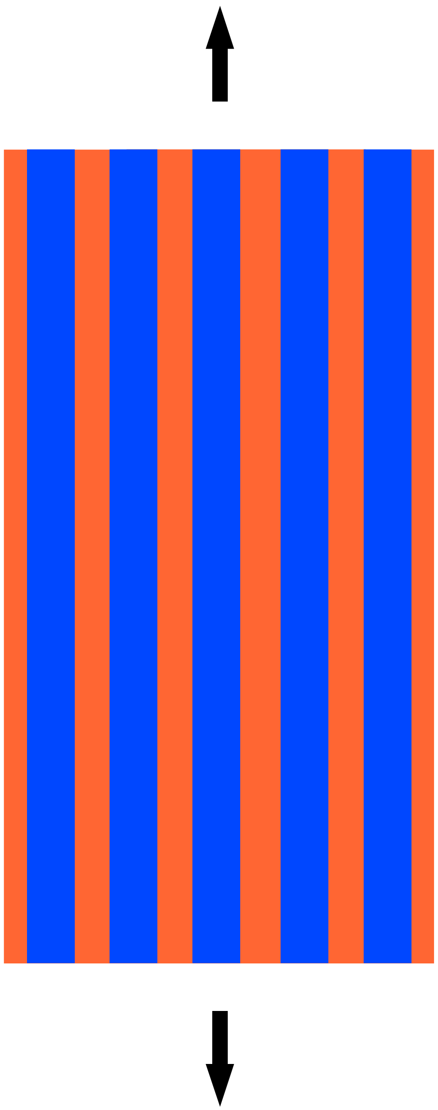
Contiguity and Percolation





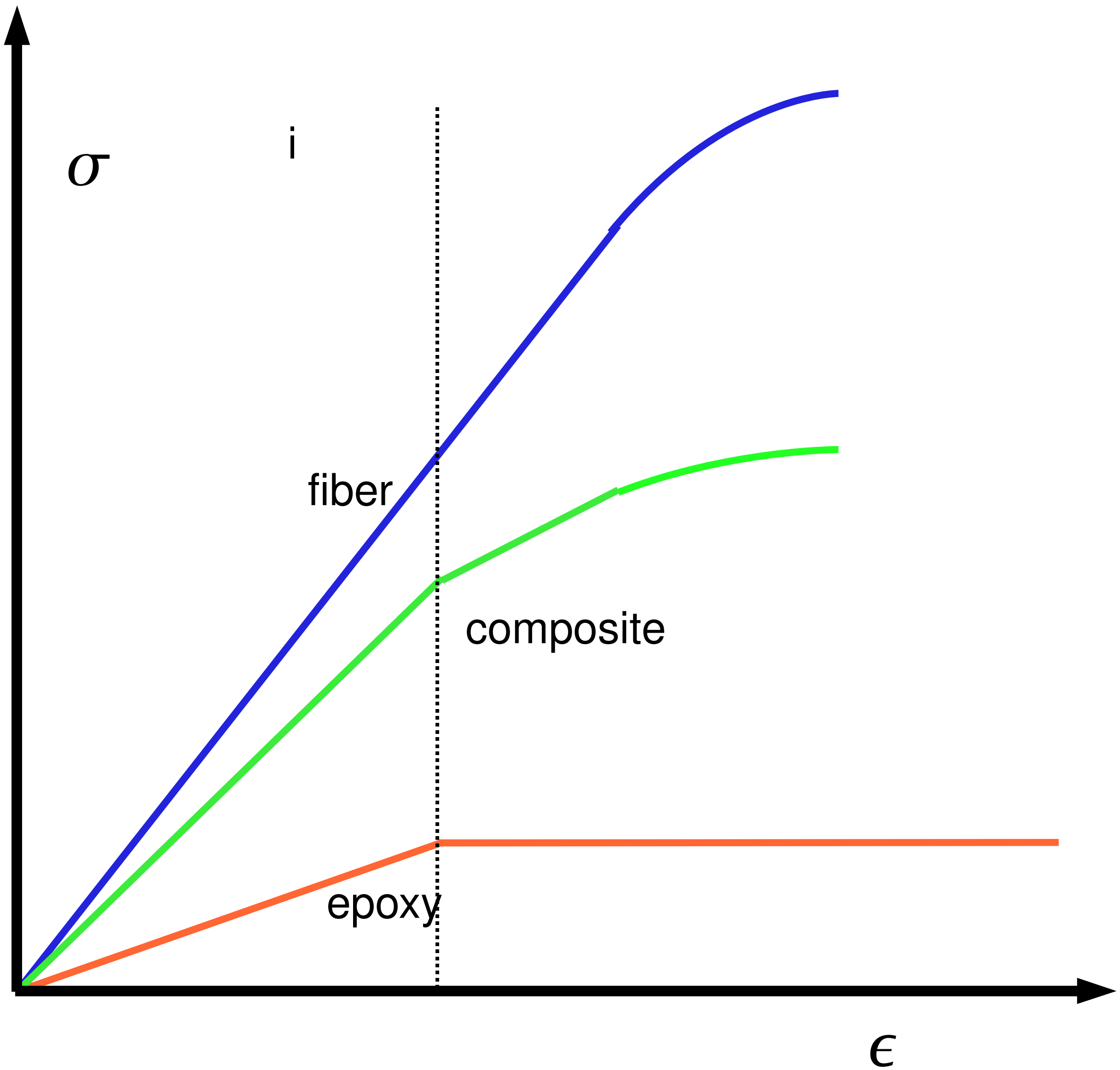
f_{crit} (the “percolation threshold”) is a critical volume fraction of f_2 above which there is a connected network of (randomly scattered) material 2 throughout the matrix.

Fiber-Epoxy Example



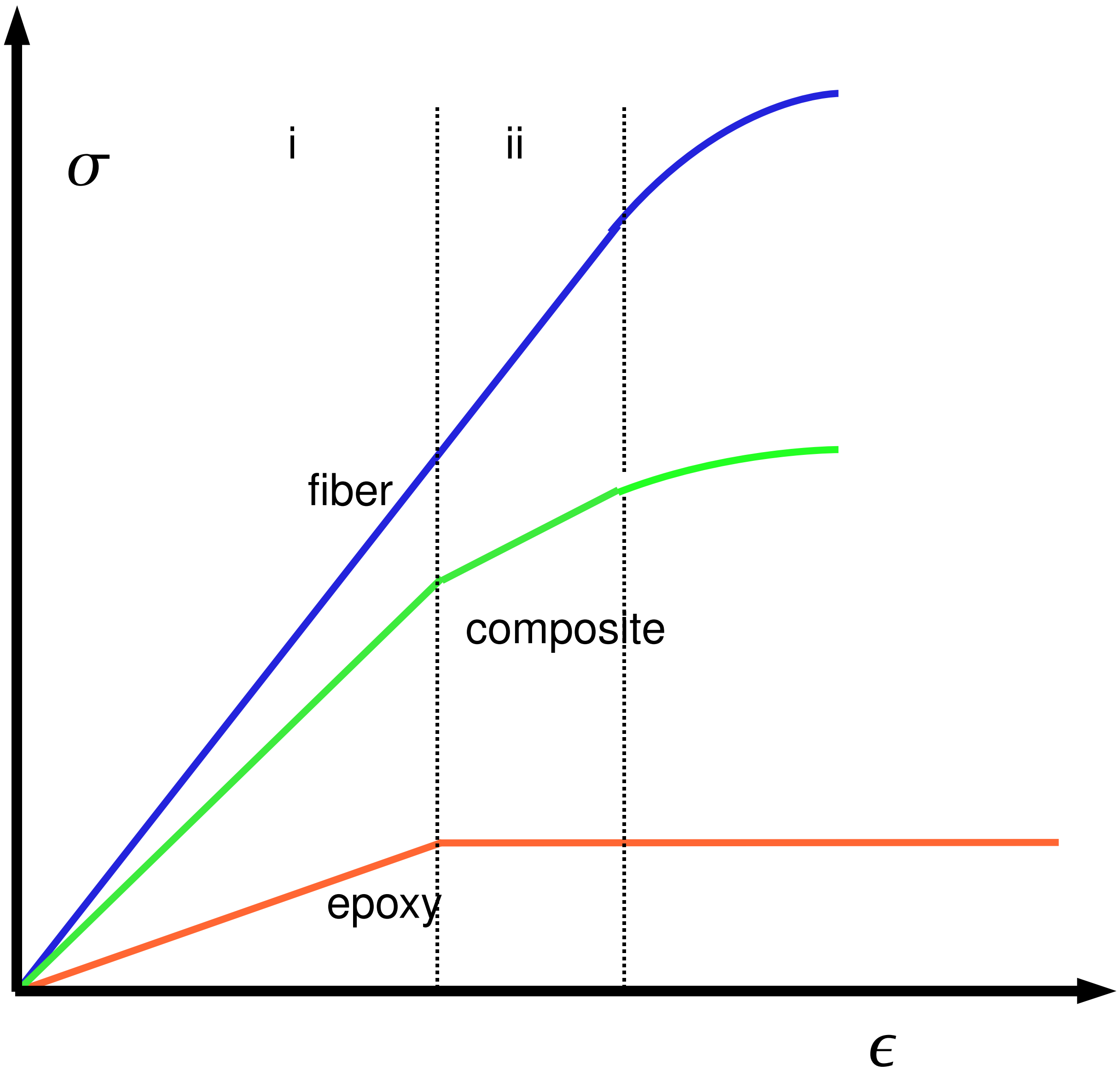
- Compromise between strength and ductility:
  - For better load transfer, you want long fibers, which lead to better long-range connectivity
  - For better ductility, you want short fibers (a "dispersed structure"). A broken short fiber has less effect on the macroscopic properties than a broken long fiber

Fiber-Epoxy Example



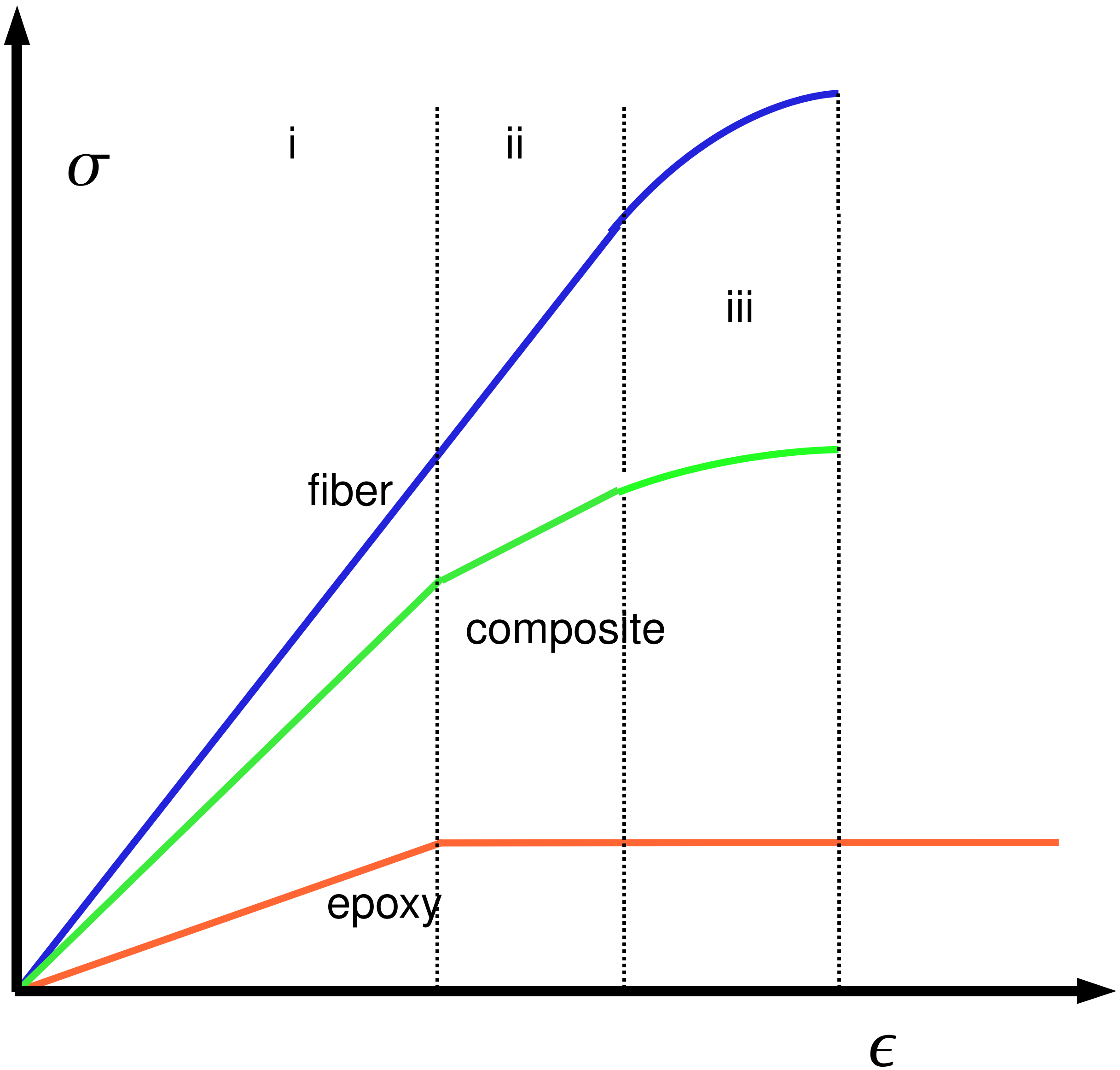
	σ Epoxy	σ Fiber
$i:$	$f_1 E_1 \epsilon$	$f_2 E_2 \epsilon$

Fiber-Epoxy Example



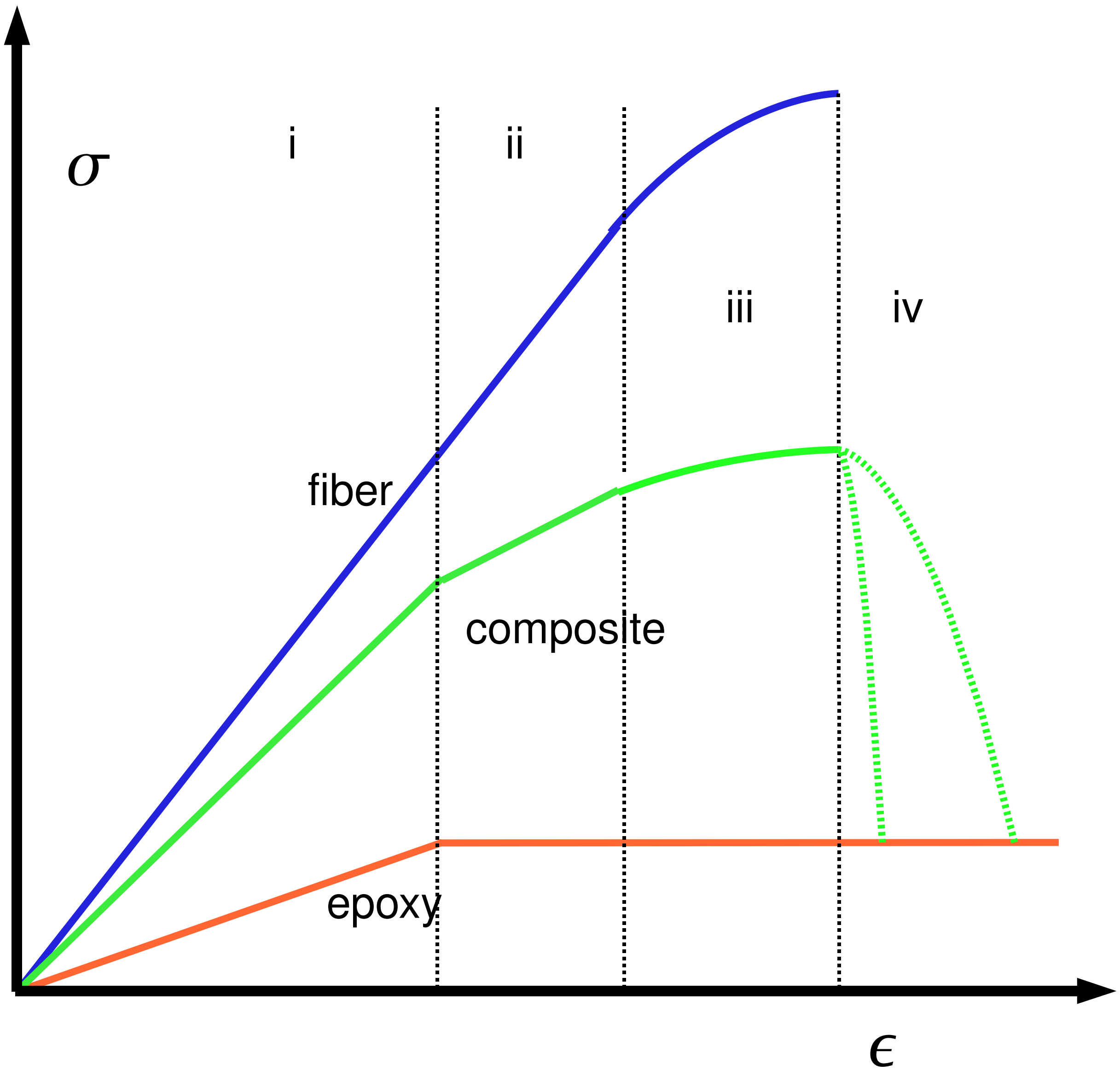
	σ Epoxy	σ Fiber
$i:$	$f_1 E_1 \epsilon$	$f_2 E_2 \epsilon$
$ii:$	$\sigma_1(\epsilon)$	$f_2 E_2 \epsilon$

Fiber-Epoxy Example



	σ Epoxy	σ Fiber
i :	$f_1 E_1 \epsilon$	$f_2 E_2 \epsilon$
ii :	$\sigma_1(\epsilon)$	$f_2 E_2 \epsilon$
iii :	$\sigma_1(\epsilon)$	$\sigma_2(\epsilon)$

Fiber-Epoxy Example



	σ Epoxy	σ Fiber
i :	$f_1 E_1 \epsilon$	$f_2 E_2 \epsilon$
ii :	$\sigma_1(\epsilon)$	$f_2 E_2 \epsilon$
iii :	$\sigma_1(\epsilon)$	$\sigma_2(\epsilon)$
iv :	$\sigma_1(\epsilon)$	$---$

questions?

3.40J / 22.71J / 3.14 Physical Metallurgy
Fall 2009