Chapter 8. SOLUTIONS

8.1. The first costs more energy because the wall has a larger area than if it were perpendicular to the top surface. The second wall violates the condition of continuity of the normal component of magnetization across the wall. Basically, it is a magnetically "charged" wall. Charged walls can exist under some conditions when the medium is magnetically hard. A longitudinal recording medium is a series of charged "head-to-head" walls.

8.2. The anisotropy constants in J/m^3 are listed below with the calculated wall width, Eq. 8.15, and wall energy density, Eq. 8.16.

	Co	Ni$_{80}$Fe$_{20}$	Nd$_2$Fe$_{14}$B
K_u (J/m^3)	4.1×10^5	3×10^2	5×10^6
δ_{dw}(nm)	22	810	6.3
σ_{dw}(mJ/m^2)	11	0.3	40

8.3. The magnetoelastic energy, $B_1(e_{xx}\alpha_x^2 + ...)$, simply adds to the anisotropy energy, $K_u sin^2\theta$. For uniaxial strain, the ME energy becomes $-2B_1 e_o sin^2\theta$ where θ is the angle between the direction of magnetization and that of the tensile strain direction. The tensile direction becomes a hard axis and the plane normal to it becomes an easy plane if $B_1 e_o > 0$.

Tensile stress

Case i)   ii)   iii)
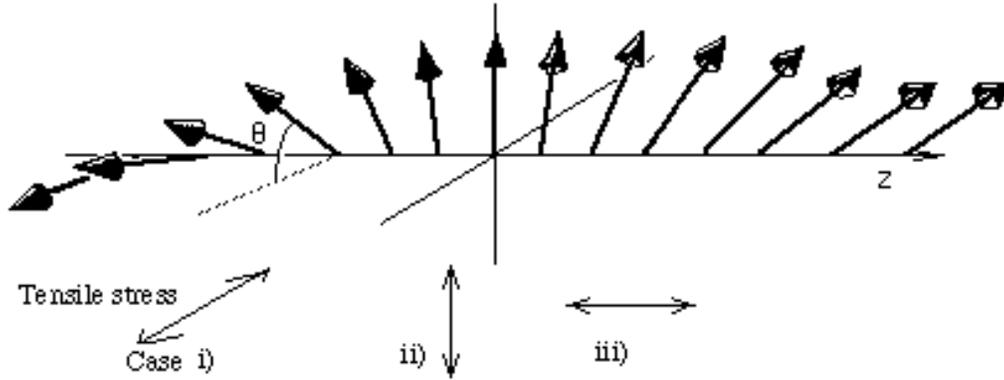
Thus, for the three cases identified above, the net anisotropy can be expressed as follows:

i) $(K_u - 2B_1e_o)sin^2\theta,$ ii) $(K_u + 2B_1e_o)sin^2\theta$ iii) $K_u sin^2\theta + 2B_1e_o sin^2(\pi/2)$

These coefficients replace K in the expressions for wall energy, $4(AK)^{1/2}$, and width, $\pi(A/K)^{1/2}$.

Case i) Wall energy decreases and wall thickness increases for $B_1e_o > 0$;

Case ii) Wall energy increases and wall thickness decreases for $B_1e_o > 0$;

Case iii) The tensile stress leaves the magnetization in all parts of the wall equally affected so there is no change in wall energy or shape for this direction of stress.

8.4. $f_a = Ksin^2\theta$ yields: $(\theta')^2 - (K/A)sin^2\theta = C.$

For boundary conditions: $\theta(-\infty) = 0$: 0 - 0 $= C$

$\theta(-\infty) = -\pi/2$: 0 - (K/A) $= C$

For the first set of boundary conditions the solution is given in the text. For the second set of boundary conditions, the second integral is

$$z - z_o = \sqrt{\frac{A}{K}} \int_{\theta_o}^{\theta} \frac{1}{\sqrt{sin^2\theta + C}} d\theta$$

which has for its solutions the elliptic integrals.

2

8.5 The free energy density is a function of position x across the Néel wall width. It can be written

$$f(x) = A\left(\frac{\partial\theta(x)}{\partial x}\right)^2 + K_u \sin^2\theta(x) + \frac{\mu_o M_s^2}{2}\frac{2t}{\delta_{dw}}$$

We can approximate the average x component of the magnetization across the wall width as $\langle\sin^2\theta\rangle = 1/2$, and the demagnetizing factor of the wall as t/δ_{dw}. Assuming a wall width of order 10^{-7} m and an anisotropy of 10^5 J/m^3, and a magnetization of 1T, the three terms above have magnitudes or 10^4, 10^5 and 4 x 10^5 J/m^3, respectively. None is insignificant. Without integrating this function, the energy per unit area can be approximated as

$$\sigma_{dw} = A\left(\frac{\pi}{\delta_{dw}}\right)\delta_{dw} + \frac{K_u}{2}\delta_{dw} + \mu_o M_s^2 t$$

with the wall width treated as a variable. Clearly this function is minimized with respect to the wall width for $\delta_{dw} = \pi\sqrt{\frac{2A}{K_u}}$, independent of the magnetostatic contribution

because in this approximation, the magnetostatic contribution is independent of the wall width.

8.6 For an electrical circuit the power dissipation is $i^2 R$. For a magnetic circuit (Appendix 2.1) the energy density is $\phi^2 R_m = \phi^2 l/(\mu A) = B^2 lA/\mu = BHV$. In the free space about the sample BHV is equal to $\mu_o H^2 V$. Inside the sample, using $H \approx -NM_r$, BHV is equal to $\mu H^2 V \approx \mu N^2 M_r^2 V$. This quantity is positive definite and minimized by virtue of the product NM_r which is allows larger M_r if N is small but demands smaller M_r when N is large.

8.7 a) Equating total magnetostatic energy for a sphere to wall energy, σ_{dw} , gives:

$$(^1/_6)\mu_0 M_s^2 \, (^4/_3)\pi r_c^3 = 4(AK)^{1/2}(\pi r_c^2)$$

$$r_c = 18(AK)^{1/2}/(\mu_0 M_s^2)$$

b) For iron, $A = 2 \times 10^{-11}$ J/m, $K_1 = 5 \times 10^4$ J/m^3 (but this is not a uniaxial anisotropy), $\mu_0 M_s = 2.2$ T, so $r_c \approx 4.7$ nm

c) $\delta_{dw} = \pi(A/K)^{1/2} = 63$ nm

d) $\sigma_{dw} = 4(AK)^{1/2}$ so $\delta_{dw} = \pi\sigma_{dw} /(4 K)$

$$r_c = (3/4)\sigma_{dw} /\{(1/6) \, \mu_0 M_s^2\}$$

δ_{dw} is proportional to the ratio of σ_{dw} to the anisotropy energy density.

r_c is proportional to the ratio of σ_{dw} to the magnetostatic energy density for a sphere. In the domain wall, it is the strength of the anisotropy energy that increases the energy inside the domain wall and therefore reduces the wall width. In the particle, it is the magnetostatic energy that drives the creation of the domain wall; the greater the magnetostatic energy, the less stable is a single-domain particle and the smaller is its critical radius.