How High Can A Tree Grow?

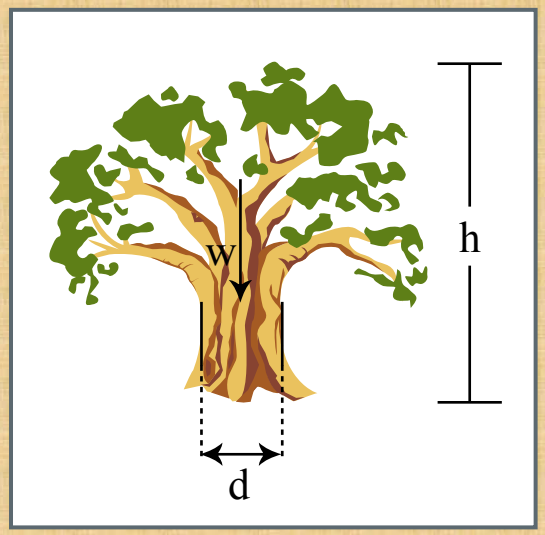


Figure by MIT OCW.

Important Dimensions:

Tree Height : **h**

Trunk Diameter : **d**

Wood weight per unit volume : **G**

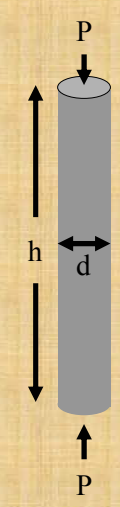
Volume of a cylinder : $C_1 d^2 h$

Volume of the tree trunk : $C_2 d^2 h$

Volume of tree trunk and crown: $C_3 d^2 h$

(C_1,C_2, and C_3 are constants)

Tree trunk is loaded by its own weight, $W = GC_3 d^2 h$ (equation #1)



Column Buckling:

$$P_{cr} = \frac{C_4 E d^4}{h^2}$$ Equation #2

E: is the stiffness of the material (how much it deforms for a given load) and is called "Young's Modulus"

(C_4 is a constant)

The equation says that if material and diameter of the column are fixed, and the height is doubled, the buckling load decreases by a factor of 2^2 or 4.

If the material and height of the column are fixed, and the diameter is doubled, the buckling load increases by a factor of 2^4 or a factor of 16.

Tree Height is Controlled by Column Buckling

As the tree gets taller, its weight increases

At some point the trunk will buckle under its own weight. This happens when the weight (equation #1) equals the buckling load (equation #2)

$$W = P_{cr}$$

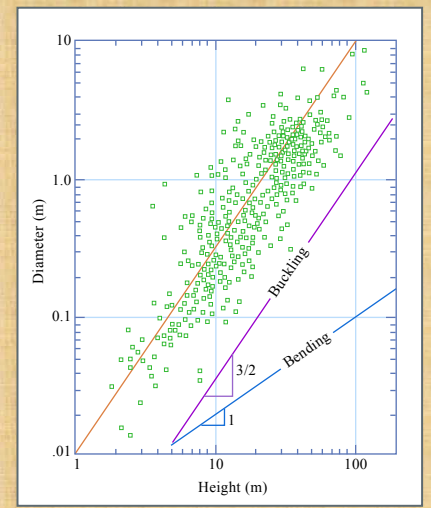
$$GC_3 d^2 h = \frac{C_4 E d^4}{h^2}$$

Rearranging: $\frac{G}{E}\frac{C_3}{C_4} h^3 = d^2$

For different types of trees G/E is a constant

(See poster on wood structure and properties)

combining $\sqrt{\frac{G}{E}\frac{C_3}{C_4}} = C_5$ We have $d = C_5 h^{3/2}$



On a log-log plot
is a straight line with slope 3/2

$d = C_5 h^{3/2}$

From Bonner, J.T. and McMahon, T.A. (1983) *On Size and Life*. Scientific American Books

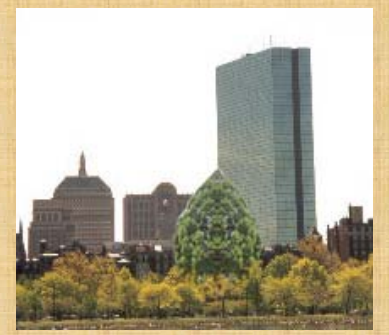
Figure by MIT OCW. After Bonner and McMahon (1983).

The National Register of Big Trees:

The American Forestry Association records the diameter and the height of the tallest individual trees of different species in the National Register of Big Trees. Each point on the plot (see left) corresponds to one individual record tree of a particular type.

For record trees: $d = C_5 h^{3/2}$

Their height is limited by column buckling.



What is the tallest tree in the world?

The Mendocino Tree is a 367.5 feet high redwood and is 600-800 years old. It is in the remote Montgomery Woods State Reserve in California. This tree would be almost half the height of the Hancock Tower in Boston (see artist's rendition).

To learn more about the structural principles behind many natural materials visit the website of the MIT class that created this poster.