

Problem Set 4

Late homework policy. Late work will be accepted only with a medical note or for another Institute-approved reason.

Cooperation policy. You are encouraged to work with others, but the final write-up must be entirely your own and based on your own understanding. You may not copy another student's solutions. And you should not refer to notes from a study group while writing up your solutions (if you need to refer to notes from a study group, it isn't really "your own understanding").

Part I. These problems are mostly from the textbook and reinforce the basic techniques. Occasionally the solution to a problem will be in the back of the textbook. In that case, you should work the problem first and only use the solution to check your answer.

Part II. These problems are not taken from the textbook. They are more difficult and are worth more points. When you are asked to "show" some fact, you are not expected to write a "rigorous solution" in the mathematician's sense, nor a "textbook solution". However, you should write a clear argument, using English words and complete sentences, that would convince a typical Calculus student. (Run your argument by a classmate; this is a good way to see if your argument is reasonable.) Also, for the grader's sake, try to keep your answers as short as possible (but don't leave out *important* steps).

Part I(20 points)

- (a) (4 points) p. 170, Section 5.2, Problem 12
- (b) (4 points) p. 177, Section 5.3, Problem 45
- (c) (4 points) p. 206, Section 6.5, Problem 1
- (d) (4 points) p. 212, Section 6.6, Problem 4
- (e) (4 points) p. 213, Section 6.6, Problem 30

Part II(30 points)

Problem 1(5 points) A function $f(x)$ is defined for all $x > 0$ to be,

$$f(x) = \int_0^{\sqrt{x}} e^{-t^2} dt.$$

Using the Fundamental Theorem of Calculus, compute df/dx . Do **not** attempt to evaluate $f(x)$ directly.

Problem 2(10 points) Using the definition of the Riemann integral, upper sums, and the formulas for special sums in the textbook, compute that

$$\int_{-a}^a (a-x)(a+x)dx = \frac{4a^3}{3},$$

for all $a > 0$.

Problem 3(15 points) This problem is a special case of a more general method for computing antiderivatives that will be developed systematically later in the semester.

(a)(10 points) Compute the antiderivative $\int \frac{1}{1-u^2} du$ as follows. Consider the sum $\frac{a}{u+1} + \frac{b}{u-1}$, where a and b are unspecified constants, and clear denominators. Find a choice of a and b so the sum simplifies to $\frac{1}{1-u^2}$. Now replace $\frac{1}{1-u^2}$ in the integrand by your expression $\frac{a}{u+1} + \frac{b}{u-1}$. Use $\ln(u+1)$, $\ln(u-1)$ to compute the antiderivative.

(b)(5 points) Reduce the computation of the antiderivative $\int \frac{1}{\cos(x)} dx$ to (a) by multiplying numerator and denominator of the integrand by $\cos(x)$, using the trig identity $(\sin(x))^2 + (\cos(x))^2 = 1$ and substituting $u = \sin(x)$. Don't forget to back-substitute. Also, say for what range of x your antiderivative is valid ($\ln(y)$ isn't defined if y is negative!).