18.01 Calculus
Due by 2:00pm sharp
Friday, Dec. 2, 2005

Jason Starr
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Problem Set 7

Late homework policy. Late work will be accepted only with a medical note or for another Institute-approved reason.

Cooperation policy. You are encouraged to work with others, but the final write-up must be entirely your own and based on your own understanding. You may not copy another student's solutions. And you should not refer to notes from a study group while writing up your solutions (if you need to refer to notes from a study group, it isn't really "your own understanding").

Part I. These problems are mostly from the textbook and reinforce the basic techniques. Occasionally the solution to a problem will be in the back of the textbook. In that case, you should work the problem first and only use the solution to check your answer.

Part II. These problems are not taken from the textbook. They are more difficult and are worth more points. When you are asked to "show" some fact, you are not expected to write a "rigorous solution" in the mathematician's sense, nor a "textbook solution". However, you should write a clear argument, using English words and complete sentences, that would convince a typical Calculus student. (Run your argument by a classmate; this is a good way to see if your argument is reasonable.) Also, for the grader's sake, try to keep your answers as short as possible (but don't leave out *important* steps).

Part I(20 points)

(a)	(2 points)	p. 318,	Section 9.5,	Problem 26
(b)	(2 points)	p. 348,	Section 10.4,	Problem 10
(c)	(2 points)	p. 348,	Section 10.4,	Problem 13
(d)	(2 points)	p. 348,	Section 10.4,	Problem 14
(e)	(2 points)	p. 350,	Section 10.5,	Problem 10
(f)	(2 points)	p. 356,	Section 10.6,	Problem 11
(g)	(2 points)	p. 356,	Section 10.6,	Problem 12
(h)	(2 points)	p. 362,	Section 10.7,	Problem 14
(i)	(2 points)	p. 362,	Section 10.7,	Problem 22(a)
(j)	(2 points)	p. 362,	Section 10.7,	Problem 22(b)

Part II(30 points)

Problem 1(15 points) This problem sketches a systematic method for finding antiderivatives of expressions $F(\sin(\theta), \cos(\theta))$, where F is a fraction of polynomials.

(a)(8 points) With the substitution $\tan(\theta/2) = z$, verify that,

$$\sin(\theta) = \frac{2z}{1+z^2}, \quad \cos(\theta) = \frac{1-z^2}{1+z^2}, \quad d\theta = \frac{2dz}{1+z^2}.$$

Please try this on your own first. However, if you are very stuck, take a look at Problem 5E-12 in the course reader.

(b)(7 points) By Example 3 on p. 571 of the textbook, the polar equation of the conic section with *eccentricity* e and *focal parameter* $p \cdot e$ is,

$$r = f(\theta) = \frac{ep}{1 - e\cos(\theta)}$$

Set up the integral for the area of the region bounded by $0 \leq r \leq f(\theta)$ for $a \leq \theta \leq A$. Use **(a)** to turn this into an integral involving only $z = \tan(\theta/2)$. Use the nonnegative constant,

$$b = \sqrt{\frac{|1-e|}{1+e}},$$

to simplify your answer. You need not evaluate the integral.

Problem 2(15 points)

(a)(5 points) Read through Section 10.3 (it is only 3 and a half pages). Everybody will get credit for this part.

(b)(5 points) Use **Problem 1 (a)** to rewrite the integral,

$$\int \sin^m(\theta)\cos^n(\theta)d\theta,$$

as an integral involving only $z = \tan(\theta/2)$.

(c)(5 points) Assume $n \geq 2$. Use the following integration by parts,

$$\int \sin^m(\theta)\cos^n(\theta)d\theta, \ u = \sin^m(\theta)\cos^{n-1}(\theta), \ dv = \cos(\theta)d\theta,$$

to find a reduction formula of the form,

$$\int \sin^m(\theta)\cos^n(\theta)d\theta = F(\sin(\theta), \cos(\theta) + A\int \sin^{m+2}(\theta)\cos^{n-2}(\theta)d\theta.$$

Not to be turned in: Of these three methods for evaluating the integral, which would be easiest to use and fastest on an exam?