18.01 EXAM 1
SEPTEMBER 18, 2003

Name: ______________________________________

Problem 1:	__________	/20
Problem 2:	__________	/25
Problem 3:	__________	/10
Problem 4:	__________	/20
Problem 5:	__________	/15
Problem 6:	__________	/10
Total:	__________	/100

Please write the hour of your recitation.

Hour: ____________

Instructions: *Please write your name at the top of every page of the exam.* The exam is closed book, calculators are not allowed, but you are allowed to use your prepared index card. You will have approximately 50 minutes for this exam. The point value of each problem is written next to the problem – use your time wisely. Please show all work, unless instructed otherwise. Partial credit will be given only for work shown.

You may use either pencil or ink. If you have a question, need extra paper, need to use the restroom, etc., raise your hand.

Date: Spring 2001.

Name: ______________________________ **Problem 1**: ____________ /20

Problem 1(20 points) Use the limit definition of the derivative to compute the derivative of $y = \frac{1}{x^2}$ for all points $x > 0$. Show all work.

Name: ________________________ **Problem 2**: __________ /25

Problem 2(25 points) The point $P = (0, 1)$ lies on two distinct lines tangent to the parabola with equation $y = x^2 + 2$. Find the equations of *both* tangent lines. Show all work and circle the final answer.

Extra credit(5 points) Let $Q = (x_0, y_0)$ and $R = (x_1, y_1)$ be two distinct points on the parabola above. There is a unique point $S = (x_2, y_2)$ which lies on both the tangent line at Q and on the tangent line at R. Show that for *every* pair of distinct points Q and R on the parabola, $2x_2 = x_1 + x_2$, i.e., the line passing through S parallel to the y-axis bisects the line segment QR.

Name: ______________________________ **Problem 3**: ____________ /10

Problem 3(10 points) For each of the following functions, compute the derivative. Show all work, including each step in your derivation, but you do not need to state the rules you are using. Circle the final answer.

(a)(2 points) $y = \frac{1}{\sqrt{1+x^4}}$

(b)(2 points) $y = x\ln(x) + \frac{1}{e^{-x}}$

(c)(3 points) $y = (1 + x^{1000})^{\frac{1}{1000}}$

(d)(3 points) $y = 10^x + \log_{10}(x^2 - x)$

Name: ____ **Problem 4**: _________________ /20

Problem 4(20 points) The point $T = (\frac{-4\sqrt{10}}{5}, \sqrt{10})$ lies on the ellipse with equation

$$9(x+y)^2 + (x-y)^2 = 36.$$

Using implicit differentiation, determine the equation of the tangent line to the ellipse at the point T; *do not simply find the slope of the tangent line*, you must write the equation of the tangent line. Show all work and circle the final answer.

Name: ____________________ **Problem 5**: ____________ /15

Problem 5(15 points) Let $u(x) = xe^{-x}$.

(a)(5 points) Find the first, second and third derivatives of $u(x)$ with respect to x. Show all work and circle the answers.

(b)(10 points) Find a formula for the n^{th} derivative of $u(x)$ with respect to x. Show all work, but you do not need to *prove* your answer by induction. In writing your final answer either use brace notation (depending on whether n is even or odd), or use $(-1)^n$ which is $+1$ for n even and -1 for n odd.

Name: ____________________ **Problem 6**: __________ /10

Problem 6(10 points) A stone is dropped from the edge of a cliff so that the height $s(t)$ in meters after t seconds is given by

$$s(t) = 80 - 5t^2.$$

(a)(5 points) Find the velocity v_f of the stone when it strikes the ground, i.e., when $s(t) = 0$ and $t > 0$. Show all work and circle the final answer.

(b)(5 points) Determine at what height the stone has velocity $\frac{1}{2}v_f$. Show all work and circle the final answer.