EXAM 2 - OCTOBER 29, 2010

(1) (5 points each) Find the derivative of each of the following functions
- $g(x) = \log(\cos(x^2))$

- $h(x) = e^{\sqrt{x}\sin x}$

(2) (10 points) Consider the function
$$g(x) = \frac{\log x}{x^2}.$$

Determine the behavior of g in a neighborhood of $x = 1$. Specifically, is the function increasing or decreasing? Is it convex or concave? Justify your answers.

(3) (10 points) Consider the functions $f(x) = x \sin x$ and $g(x) = (x + 5) \cos x$. Prove there exists $c \in (0, \pi/2)$ such that $f(c) = g(c)$. (If you are using a theorem, make sure you explain why the function or functions you are considering satisfy the hypotheses of the theorem.)

(4) (15 points) Define $f(x)$ such that $f(x) = x$ for every rational value of x and $f(x) = -x$ for every irrational x.
(a) Prove $f(x)$ is continuous at $x = 0$.

(b) Set $a \neq 0$. Prove that $f(x)$ is not continuous at $x = a$.

(5) (15 points) Let f be continuous. Prove that

$$\int_0^x f(t)(x-t)dt = \int_0^x \left(\int_0^t f(u)du \right) dt.$$

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