PRACTICE EXAM 1

(1) Compute $\int_{99}^{103}(2x-198)^2\left[\sqrt{x-99}\,\right]dx$ where here $[x]$ is defined to be the largest integer $\leq x$.
(2) Let S be a square pyramid with base area r^2 and height h. Using Cavalieri's Theorem, determine the volume of the pyramid.
(3) Let f be an integrable function on $[0,1]$. Prove that $|f|$ is integrable on $[0,1]$.
(4) The well ordering principle states that every non-empty subset of the natural numbers has a least element. Prove the well ordering principle implies the principle of mathematical induction. (Hint: Let $S \subset \mathbb{P}$ be a set such that $1 \in S$ and if $k \in S$ then $k+1 \in S$. Consider $T = \mathbb{P} - S$. Show that $T = \emptyset$.)
(5) Suppose $\lim_{x\to p^+} f(x) = \lim_{x\to p^-} f(x) = A$. Prove $\lim_{x\to p} f(x) = A$.

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