PROFESSOR: Hi. Welcome back to recitation.

In last lecture we talked about finding the derivatives of trigonometric functions. In particular, the sine function and the cosine function. So today let's do an example of putting that into practice.

So here's a function: h of x equal to sine of x plus square root of 3 times cosine of x. And I'm asking you to find which values of x have the property that the derivative of h of x is equal to 0. So why don't you take a minute to think about that, work it out on your own, pause the video, and we'll come back and we'll work it out together.

All right. So you've hopefully had a chance to look over this problem, try it out for yourself. Now let's see how to go about it.

So we have the function h of x-- it's equal to sine x plus square root of 3 cosine x-- and we want to know when its derivative is equal to 0. So in order to answer that question we should figure out what its derivative actually is and try and write down a formula for its derivative.

So in this case that's not that bad. If we take a derivative of h, well h is a sum of two functions-- sine x and square root of 3 cosine x. And we know that the derivative of a sum is just the sum of the derivatives. So we have the h prime of x is equal to d over dx of sine x plus d over dx of square root of 3 times cosine x.

Now we learned last time in lecture that the derivative of sine x is cosine of x, and we learned the derivative of cosine of x is minus sine x. So here we have a constant multiple, but by the constant multiple rule that just gets pulled out. So this is equal to cosine x minus square root of 3 times sine x.

So this is h prime of x and now we want to solve the equation h prime of x equals 0. So we want to find those values of x such that cosine x minus square root of 3 sine x is equal to 0.

Now there are a couple different ways to go about this. I think my preferred way is I would add the square root of 3 sine x to one side, and then I want to get my x's together, so I would divide by cosine x. So that gives me-- so on the left side I'll be left with cosine x divided by cosine x, so that's just 1. And on the right side I'll have square root of 3 times sine x over cosine x. So that's just square root of 3 times tan x. Or, and I can rewrite this as tan of x is equal to 1 divided by square root of 3.

Now to find x here, either you can remember your special trig angles and know which values of x make this work. Or you could apply the arc tangent function here. So in either case, the simplest solution here is x equals pi over 6. So if you like, you can draw a little right triangle. You know, if this is x, if tan x is 1 over square root of 3, we should have this side being 1 and this side being square root of 3. And in-- OK, in that case, in this right triangle the hypotenuse would be 2. And so then you, you know, would recognize this is a 30 degree angle, or pi over 6 radian angle. But one thing to remember is that tangent of x is a periodic function with period pi, so not only is pi over 6 a solution, but pi over 6 plus pi is a solution. So that's 7*pi over 6, or pi over 6 plus 2*pi, which is 13*pi over 6, or pi over 6 minus pi, which is minus 5*pi over 6, et cetera. So there are actually infinitely many solutions. They're given by pi over 6 plus an arbitrary multiple of pi.

So then we're done. So I do want to mention, though, that there's another approach to this question, which is, we can start by multiplying this expression for h of x. So if you look at h of x-- that's sine of x plus square root of 3 times cosine x-- it resembles closely one of your trigonometric identities that you know about. So in particular, to make it resemble it even more I can multiply and divide by 2. So I can rewrite h of x equals 2 times 1/2 sine x plus square root of 3 over 2 times cosine x.

And now 1/2 is equal to cosine of pi over 3. And square root of 3 over 2 is equal to sine of pi over 3. So I can rewrite this as 2 times-- what did I say-- I said cosine pi over 3 sine x plus sine pi over 3 cosine x. And this is exactly what you get when you do the angle addition formula for sine. This is the expanded out form, and so we can apply it in reverse and get that this is equal to 2 times sine of x plus pi over 3.

So, so far we haven't done any calculus. We've just done-- so in this solution, our first solution we did some calculus first and then some algebra and trigonometry. So, so far we've just done some algebra and trigonometry. Now the points where h prime of x is equal to 0 are the points where the graph of this function has a horizontal tangent line. So either you can compute its derivative using your rules or by the definition. Or you can just say, oh, we know what this graph looks like. So I've sort of drawn a schematic up here.

So this is a graph, this graph is-- OK, so this is the graph, y equals 2 sine of x plus pi over 3. It's what you get if you take the graph y equals sine x, and you shift it left by pi over 3 and you scale it up by a factor of 2. So this here is at x equals minus pi over 3. This root is x equals 2*pi over 3. And the points we're interested in are the points where there's a horizontal tangent line, where the derivative is 0. And so there's one of these right at this value, which is pi over 6.

And then the second one is this, is that minimum there. So that happens at x equals 7*pi over 6. pi over 6 because for the usual sine function it happens at pi over 2, but we shifted everything left by pi over 3. And so pi over 2 minus pi over 3 is pi over 6. And here for this, for just y equals sine x, this minimum would happen at 3*pi over 2, but we've shifted it left by pi over 3. And so on. you know, every, there's another trough over here, and another peak over there, and so on.

So that's the second way you can do this question using this cute trig identity here.

And that's that.