The Most Natural Logarithmic Function

At times in your life you might find yourself tempted to use logarithmic functions with bases other than e; for example, $\log_2 n$ or $\log_{10} r$. We claim that $\ln x$, the natural logarithm or log base e, is the most natural choice of logarithmic function.

Today's example is from the field of economics. Imagine that the price of a stock you own goes down by a dollar; how does that affect you?

That depends on a lot of things. In particular, it depends on whether the original stock price was a dollar or a hundred dollars.

As another example, at the time this lecture was given the London Exchange FTSE index had closed down 27.9 points. This statistic is almost meaningless unless you know the actual total of the index, which was 6,432 at the time.

A more meaningful statistic is the change in the price divided by the price:

$$\frac{\Delta p}{p} = \frac{27.9}{6432} \approx 0.43\%.$$

This tells us that the FTSE index dropped by 0.43% of its total value today.

A day is a relatively short time in the life of an investment so if price p is a function of time t with $\Delta t = 1$ day,

$$\frac{\Delta p}{1 \text{ day}} \approx \frac{dp}{dt} = p'.$$

So instead of looking at $\frac{\Delta p}{p}$ we could discuss $\frac{p'}{p}$, which is just the derivative of the natural log of p.

$$\frac{p'}{p} = (\ln p)'$$

This is the formula for logarithmic differentiation, and it is used all the time by economists and people who model prices of things.

There's no point in using log base ten or log base two, because when you take the derivative of those functions you get an extra constant factor in the denominator:

$$(\log_{10} p)' = \frac{p'}{p \ln 10}.$$

This is just one example — any variable that has to do with ratios is going to involve logarithms. We'll see more of this when we study applications of derivatives. You may have guessed this already; the derivative of $\ln x$ is quite elegant and simple. The most elegant and simple ideas that are often the most powerful, as well.

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