The First Fundamental Theorem of Calculus

Our first example is the one we worked so hard on when we first introduced definite integrals:

Example: $F(x) = \frac{x^3}{3}$.

When we differentiate $F(x)$ we get $f(x) = F'(x) = x^2$. The fundamental theorem of calculus tells us that:

$$\int_a^b x^2\,dx = \int_a^b f(x)\,dx = F(b) - F(a) = \frac{b^3}{3} - \frac{a^3}{3}$$

This is more compact in the new notation. We'll use it to find the definite integral of x^2 on the interval from 0 to b, to get exactly the result we got before:

$$\int_0^b x^2\,dx = \int_0^b f(x)\,dx = F(x)\big|_0^b = \left.\frac{x^3}{3}\right|_0^b = \frac{b^3}{3}.$$

By using the fundamental theorem of calculus we avoid the elaborate computations, difficult sums, and evaluation of limits required by Riemann sums.

Example: Area under one "hump" of $\sin(x)$.

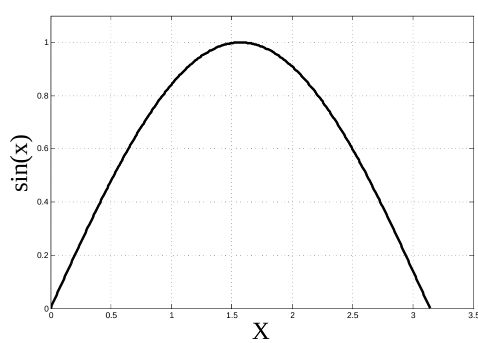

Figure 1: $\sin(x)$ for $0 < x < \pi$

The area under the curve $y = \sin x$ between 0 and π is given by the definite integral $\int_0^\pi \sin(x)\,dx$. The antiderivative of $\sin(x)$ is $-\cos(x)$, so we apply the fundamental theorem of calculus with $F(x) = -\cos(x)$ and $f(x) = \sin(x)$:

$$\int_0^\pi \sin(x)\,dx = -\cos(x)|_0^\pi .$$

Be careful with the arithmetic on the next step; it's easy to make a mistake:

$$-\cos(x)|_0^\pi = -\cos(\pi) - (-\cos(0)) = -(-1) - (-1) = 2.$$

So the area under one hump of the graph of $\sin(x)$ is simply 2 square units.

Example: $\int_0^1 x^{100}\,dx$

$$\int_0^1 x^{100}\,dx = \left.\frac{x^{101}}{101}\right|_0^1 = \frac{1}{101} - 0 = \frac{1}{101}$$