

## Substitution When $u'$ Changes Sign

We've been told that changing variables of integration only works if  $u(x)$  is either always increasing or always decreasing on the interval of integration. Let's see what goes wrong by trying to calculate  $\int_{-1}^1 x^2 dx$ .

We'll try plugging in  $u(x) = x^2$ ; then we get:

$$\begin{aligned} du &= 2x dx \\ dx &= \frac{1}{2x} du = \frac{1}{2\sqrt{u}} du \\ u_1 &= (-1)^2 \quad \text{and} \\ u_2 &= (-1)^2 \end{aligned}$$

Thus:

$$\int_{-1}^1 x^2 dx = \int_1^1 u \frac{1}{2\sqrt{u}} du = 0.$$

But we know that  $\int_{-1}^1 x^2 dx$  is not zero; it's the area under a parabola. Our conclusion is **not true**.

The reason for this is that  $u'(x) = 2x$  is negative when  $x < 0$  and positive when  $x > 0$ ; the sign change causes us trouble. If we break the integral into two halves so that  $u'$  has a consistent sign on each half, we'll be able to compute the integral without difficulty.

We could actually have caught this early; there is a mistake in our calculation of the expression for  $dx$ . In fact, when we wrote:

$$\frac{1}{2x} du = \frac{1}{2\sqrt{u}} du$$

we should have noticed that in fact:

$$\frac{1}{2x} du = \frac{1}{\pm 2\sqrt{u}} du.$$

It's possible to use this formula to get the correct answer, but not recommended. Instead, just split your integral into intervals over which  $u'$  is always either positive or negative.

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