PROFESSOR: Welcome back to recitation. In this video I want us to work on the following problem. So we're going to assume little f is a continuous function. And if we know that the integral from 0 to x of f of t dt is equal to x squared sine pi x, I want us to find little f of 2. So I'm going to give you a little while to work on the problem. When you're ready, come back and we'll take a look at how I do the problem.

Welcome back. So again, we're working on a problem where we know that f is a continuous function. We know that the integral from 0 to x of f of t dt is equal to a certain function. And we want to find f of 2. So what we're going to use, because we know little f is continuous, we can actually use the fundamental theorem of calculus. And we know that d/dx of this whole expression here is actually little f of x.

So let's just remind us-- I'll write it down formally. FTC implies that d/dx, the interval from 0 to x f of t dt is equal to f of x. Right? We know that. So, we know that d/dx of the right-hand side and d/dx of the left-hand side are the same. And so we know that if we take d/dx of the right-hand side, we'll get little f of x. Because d/dx of the left-hand side of this equation is little f of x. So d/dx of the right-hand side is also little f of x. So now what we have to do is apply what we know about taking derivatives to find little f of x. Then once we find little f of x we can evaluate it at f equal 2, at 2-- sorry-- at x equal 2. Sorry about that.

So let's come over to the right part of the board and we'll find d/dx of this function here. So what do we have to use? We have to use the product rule and then we'll have to use a little chain rule, a really simple chain rule in the second term. I'm going to write it down below, so we have some room here. So the first term, we get 2x sine pi*x. The second term we get x squared. Derivative of sine is cosine. pi*x. And then we have to pull out a pi. Which I'll just put, well I'll put it here for the moment. The pi comes from taking the derivative of pi*x.

And now all we need, this is then f of x, right? So I just will remind us this is f of x. Let me put it right here, a little f of x. So if I want to find f of 2, all I have to do is evaluate this at x equal 2. So f of 2 is going to equal-- 2 times 2 is 4-- sine 2*pi plus 4-- I'll bring the pi in front-- 4*pi cosine 2*pi. So what do I get? Sine 2*pi is 0. Cosine 2*pi is 1. So ultimately, I just get 4*pi.

So, hopefully you were able to solve this problem and get the same answer. Now let me remind you, one more time what we did. Let's come over here. We started with a function that we knew was continuous. We knew the integral from 0 to x of that function was equal to a

certain function of x. And we wanted to evaluate this function at a certain, at a certain point. So what we exploited was the fundamental theorem of calculus. And the fundamental theorem of calculus tells us that d/dx of the left-hand side is f of x. And so that we can take d/dx of the right-hand side and figure out what f of x is. And then we just evaluate. So I think I'll stop there.