Antiderivative of $\frac{1}{x}$

We return to our theme of using $f = F'$ to understand F.

We recently found the antiderivative of t^{-2}. For the most part, it's easy to antidifferentiate x^n, except for the tricky case of $\frac{1}{x}$.

Example: Solve the differential equation $L'(x) = \frac{1}{x}$; $L(1) = 0$.

The second fundamental theorem of calculus tells us that the solution is:

$$L(x) = \int_1^x \frac{dt}{t} = \ln x.$$

We also know that this antiderivative must be the logarithm function, but it turns out that this way of looking at the function makes many calculations much easier.

One very interesting thing about this solution is that we started with a ratio of polynomials ($\frac{1}{x}$) and ended with a transcendental function. The natural log function can't be written in terms of our usual algebraic operations.