Boiling Cauldron: Introduction

Now let's fill the cauldron from our example with water and light a fire under it to get the water to boil (at 100°C). Let's say it's a cold day: the temperature of the air outside the cauldron is 0°C. How much energy does it take to boil this water, i.e. to raise the water's temperature from 0°C to 100°C?

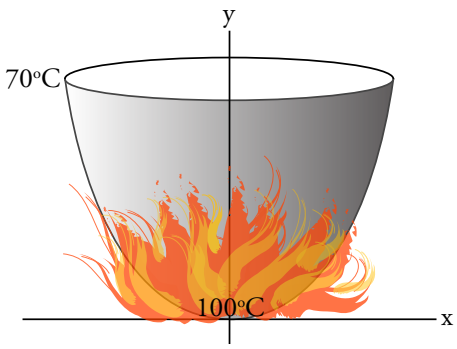

Figure 1: The boiling cauldron.

The temperature of the water is not the same at each level in the kettle. At the bottom of the kettle, where you're heating it up, it's at its highest temperature — 100 degrees Celsius. At the top, it's going to be, say, 70 degrees Celsius. The temperature is varying in height; if it varies linearly the temperature at height y will be $100 - \frac{30}{a}y$ degrees Celsius.

The total amount of heat you need to add is going to be temperature times volume, and some places will get more heat than others. The base of the cauldron will be hottest, but it also has the least volume. The cauldron is widest at the top, so we have more water to heat at that level.

At each horizontal level, the temperature is constant, so we'll use horizontal rectangles in this calculation. We'll revolve these short horizontal rectangles about the y-axis to get disks, calculate the amount of heat needed for each disk, then integrate that value with respect to dy.

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