Partial Fractions

Today we'll learn how to integrate functions of the form:

$$\frac{P(x)}{Q(x)}$$

where $P(x)$ and $Q(x)$ are polynomials. Functions of this type are called *rational functions*. The technique for integrating functions of this type is called the method of *partial fractions*.

The method of partial fractions works by algebraically splitting $P(x)/Q(x)$ into pieces that are easier to integrate.

Example:

$$\int \left(\frac{1}{x-1} + \frac{3}{x+2} \right) dx = \ln|x-1| + 3\ln|x+2| + c$$

That was an easy integral. Next we'll see how the same integral might become difficult; if we add the two fractions together we get the same problem in a more challenging form:

$$\begin{aligned}
\frac{1}{x-1} + \frac{3}{x+2} &= \frac{1}{x-1} \cdot \frac{x+2}{x+2} + \frac{3}{x+2} \cdot \frac{x-1}{x-1} \\
&= \frac{(x+2)+3(x-1)}{(x-1)(x+2)} \\
&= \frac{4x-1}{x^2+x-2} \\
\int \frac{4x-1}{x^2+x-2}\, dx &= \ ?
\end{aligned}$$

This integral is algebraically the same as the one we just computed; that was easy, this one looks harder. The method of partial fractions involves algebraically manipulating integrals like the harder one to make them easier.