Introduction to Polar Coordinates

Polar coordinates involve the geometry of circles. Just as Professor Jerison loves the number zero, the rest of MIT loves circles.

Polar coordinates are another way of describing points in the plane. Instead of giving x and y coordinates, we'll describe the location of a point by:

- r = distance to origin
- θ = angle between the ray from the origin to the point and the horizontal axis.

(This is the geometric idea, but is not a perfect match for how polar coordinates are actually used.)

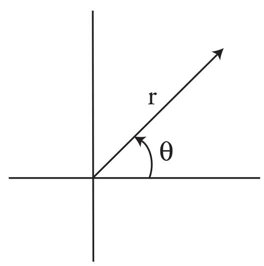

Figure 1: Polar coordinates describe a radius r and angle θ.

If we wish to relate polar coordinates back to rectangular coordinates (i.e. find the x and y coordinates of a point (r, θ)), we use the following formulas:

$$x = r\cos\theta, \qquad y = r\sin\theta.$$

This is the official, unambiguous definition of polar coordinates, from which we get the geometric description above and also the following:

To convert rectangular coordinates to polar coordinates, use:

$$r = \sqrt{x^2 + y^2}, \qquad \theta = \tan^{-1}\left(\frac{y}{x}\right).$$

This is close to being a good formula, and it's useful.

The ambiguity in these formulas comes from the fact that r could be negative $\sqrt{x^2 + y^2}$, and θ could also be $\tan^{-1}\left(\frac{-y}{-x}\right)$. You must refer to your diagram when using these formulas to convert from rectangular to polar coordinates.

Two Coordinate Systems

The coordinate system that we're used to is the rectangular coordinate system. The notation (x, y) describes a location in that plane that is x units to the right of the origin and y units above the origin. As shown in Figure 2, the (green) lines $y = k$ are lines of constant height; the (red) lines $x = c$ are made up of all the points that are exactly c units to the right of the origin.

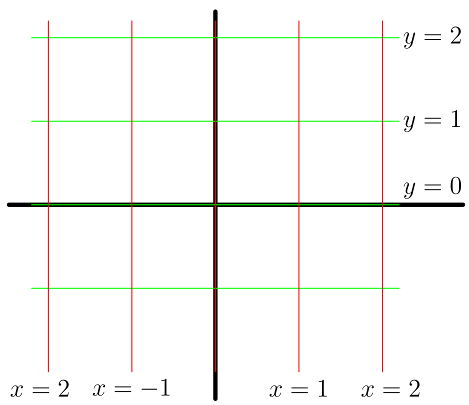

Figure 2: Lines $y = k$ and $x = c$ in a rectangular coordinate system.

In the polar coordinate system, the notation (r, θ) describes a point r units away from the origin at an angle of θ degrees. In Figure 3, each ray $\theta = c$ radiating from the origin is made up of points (r, θ) which all have the same angle θ. The circles $r = k$ about the origin are made up of points which are all the same distance k from the origin.

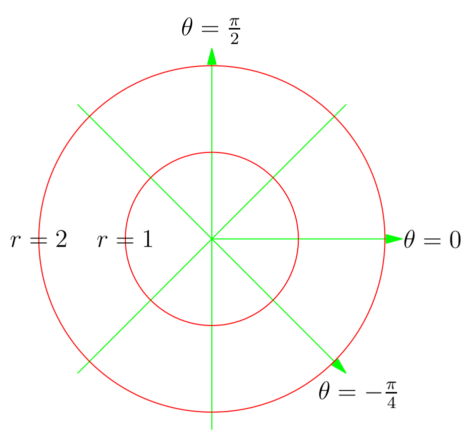

Figure 3: Lines $r = k$ and $\theta = c$ in a polar coordinate system.

The polar coordinate system is just a different way of describing the locations of points in the plane.