CHRISTINE BREINER: Welcome back to recitation.

In this video I'd like us to do two things. The first thing we're going to do is we're going to graph the curve r equals 1 plus cosine theta over 2, for theta between 0 and 4 pi, and we're going to graph it in the xy-plane. And then after we've done that, we're going to take a look at some components of that curve and we're going to calculate the area of some components that close up.

So what I'd like you to do first is get a good picture of this curve, in the xy-plane. I'll give you a little while to do that. So why don't you pause the video, get a good picture of that curve, then come back when you're ready and I'll show you how I graph it, and then we'll get into these area problems.

OK. Welcome back. So the goal, again, was to graph a certain curve described by r and theta, but in the xy-plane. For theta between 0 and pi over 4. And when I do these problems, we want to make sure that we understand how r depends on theta. That's sort of the main goal to graph this curve.

So what I do, actually, is I look at this not in the xy-plane, which was what I said to do in the problem, but I first look at it in what we see as the r, theta plane. And what we mean by that is we're going to graph this just like we would if this variable was x and this variable was y. So we move out of what we know about how r relates to x and y, and how theta relates to x and y, and we just look at how r relates to theta. So let me draw that first and we'll see if we can sort of understand what I mean by that, and then we'll put that picture, use that curve to put that into the xy-plane.

OK. So, the first thing we do is I'm going to let this be the theta-axis and this be the r-axis. And let's look at--

I want to write down over here what the equation is so I don't have to keep turning around.

So again, this is not, I don't want to think about this as the xy-plane. Because in the xy-plane, theta has certain values at each point that are fixed based on the angle. But now I'm letting theta vary in this direction, and r is varying in this direction. And my theta, I said, was between 0 and pi over 4-- I'm sorry, not pi over 4. 4*pi. 0 and 4*pi. pi over 4 would be a very small component of this that I'm interested in.

And let's think about what happens, what kind of transformations have been done to the normal cosine function. So if I took the normal cosine function and I take it-- instead of cosine theta, I look it cosine theta over 2, what that's doing is that's stretching it horizontally out. So think about the period of the cosine function usually is 2*pi. But notice what happens when I put in 2*pi for theta, I'm getting cosine of pi. If I want to get cosine of 2*pi, I have to let theta go to 4*pi, which is why I'm letting theta be between 0 and 4*pi.

So dividing the input value by 2 doubles your period. So the period is now 4*pi. So to get all the way through, I'm going to have to go up to 4*pi. So let me just make a 2*pi here, and this is about a 4*pi here. So here's around 3 pi, and here's around pi.

So that now, instead of the usual cosine function, it's going to take twice as long to get all the way through. That's one thing we know. What else have we done to the usual cosine function? We've moved it up by 1. And so instead of starting out when your input is 0, starting out at height 1, when you're input is 0, you start out at height 1 plus 1. You start out at height 2.

So in fact, this function, let me point out, it's going to start at 2, which means it also is going to end over here. Because it's periodic, it's going to end at 2.

And let's think about what else we know. We know that the usual cosine function goes down to negative 1. But I've added 1 to it, so now it only goes down to 0. OK?

Hopefully that makes sense. Maybe I should even-- hmm. I don't want to draw the actual cosine function again right on here. But let me draw the regular cosine function here. So we have it, the regular cosine function-- because I keep talking about it-- does something like this. It goes at 1 here, and it's at 1 again here. And so it's at minus 1 at pi. And so, very roughly, it looks something like this. Right? So I keep referencing the cosine function, so this is the part I'm referencing.

So we have to stretch it by 2, and then shift it up by 1. And so we see what was at pi, negative 1, I'm now going to be at 2*pi, 0.

And then where do these points go? This was pi over 2. The x-value is going to be doubled. I'm going to be at pi, and the y-value is going to go up by 1. So I'm at (pi, 1) and (3*pi, 1). And so the curve will look something like this. I'm not an expert here, but hopefully that looks something like the cosine function. But with a stretch and a shift.

So that is the curve r equals 1 plus cosine theta over 2 in what we consider the r, theta plane. So theta is varying in this direction and r is varying in this direction. But how do I transfer that to the xy-plane? That's the real point that I want to make about this problem.

So let's look at what's happening in the xy-plane. So this will be x and this will be y. Let's pick some points and try to figure out what's happening. So where is this point? This is the point zero comma two. When theta is 0, r equals 2. Where is theta equal to 0 in this picture? It's in the x-direction. Right? That's theta equals 0, and also 2*pi and 4*pi. Any multiple of 2*pi, theta is pointing in this direction. And r equals 2 there. So in a strange twist, this is the point (2, 0) on the x, y plane. But that's no going to, that's just a coincidence, OK? Don't think, oh, I'm just going to flip the values everywhere. That's not going to happen. OK?

Actually, and also, before I make a mistake, I'm going to make this a little bit bigger. I want it to be bigger. So I don't want this to be 2. I want this to be 1. I'm going to make this (2, 0). I want to have a little bigger picture.

OK. So it's going to be (2, 0) at theta equals 0 and r equal 2. Is it ever hit this point again? Well, it is. And it's going to hit that point again because it's periodic and I've gone out to 4*pi. If I rotate all the way around, I'm at 2*pi for theta. If I rotate all the way around again I'm at 4*pi for theta, and my radius there is 2, also. So this (2, 0) happens again when I have this point. So it's going to close up.

OK. And then what else happens? Well, as theta is rotating, let's take theta between 0 and pi. theta rotates from 0 to pi going like this. Notice what's happening to the r-value. The r-value is going from 2 down to 1.

Now, I'm not going to be totally exact, but-- here's minus 1, OK, in the xy-plane. I'm not going to be totally exactly, but this curve is going to look something like-- there's 2-- it's going to look something like-- oops, I overshot, we'll make that negative 1-- something like this. And what's the point? The point is that I start at radius 2, and by the time I get to theta equals pi I've gone down. And so my radius is 1. This has radius 1 and angle pi. So that represents this part of the curve. That's this part of the curve.

Now, what's nice about this drawing is that we know this part of the curve and this part of the curve should look exactly the same. So once I've drawn half of this, I'm going to know everything. Once I've gone from 0 to 2*pi, I can just reflect it, basically. We'll see what I mean by reflect in this case, but the same radii are happening again and some sort of symmetric fashion.

OK. So we've got 0 to pi. Now what happens between pi and 2*pi? Notice pi to 2*pi in the theta direction on the xy-plane is I start in this direction-- I don't know if you can see that, let me come over to this side-- I start in this direction for pi, and I'm going to rotate down. This is 3*pi over 2, and this is 2*pi. Those are my angles.

So what are my radii? Well, I start at radius 1 and I'm going to radius 0. And so what happens is I'm coming through this negative 1 and I'm coming around, and then by the time I get-- it's going to be something like this-- by the time I get to 2*pi, my radius is 0.

Now just to make sure this curve makes sense to you, you could pick a place, an angle, maybe like right here. I don't know if that's easy to see, but maybe that angle right there. That angle is between 3*pi over 2 and 2*pi. Is there positive radius there? Yeah, there is positive radius there. So in fact, this curve does come into this fourth quadrant here and then curve back in. OK? It does curve back in.

All right. So then what happens between 2*pi and 3*pi?

OK. Hopefully this picture is clear so far. I'm going to come back to the other side.

OK. What happens between 2*pi and 3*pi? 2*pi, we're here. And 3*pi, we're back over here again. And notice that the radius is going to be doing the same kind of growth that it did between 2*pi and 3*pi as it did decay from pi to 2*pi. OK? Because the radii now, there's a symmetry with how the radii behave. So from 2*pi to 3*pi, I start off with radius 0 and I have a small radius. And then once I get to 3*pi over here, I'm going to have radius 1 again. I'm going to be at radius 1, which is going to correspond to this point.

So I'm going to have exactly the same picture, which is dangerous because I probably would do it wrong. But I'll try and draw it this way and then talk about it. Hopefully that looks about the same.

So this curve coming through here was from pi to 2*pi. This curve coming through here was from to 2*pi to 3*pi. And then to finish it off, 3*pi to 4*pi is going to look like this curve here. So I come through-- ooh, this is where it starts to get really dangerous, but let's say that's pretty close-- so there's my 3*pi to 4*pi. It's again, the same growth the way it was decaying between

0 and pi.

So this is your picture in the xy-plane of the curve r equals 1 plus cosine theta over 2.

Now, we haven't calculated any area problems, yet. So what I'd like us to do is I'm going to shade two regions, and I want us to just write down the integral form to find the area for each of these regions.

So the first region of interest is the pink region. I'm going to ask us to find the area of the pink region, and then I'm going to ask us to find the area of everything else, the blue region. So let's think about how to do that. And I think I'm going to have to come over to the other side to write down the integrals, but I'll be coming back and forth. So just to remind you, what you saw in lecture was that dA is equal to r squared over 2 d theta. That's what dA is.

And so this is going to be an integral in theta, and we know what r is as a function of theta. And so if I want to find-- actually, I should even use my colors appropriately. I should say, the pink area is going to be equal to an integral. And I'm going to write the r squared. I know what r squared is. It's 1 plus cosine theta over 2 squared, and then an over 2 d theta.

And then what is important about this? It's our bounds. Right? Our bounds are what's important. And so let's go back and let's look at our picture. What are the bounds on theta for the pink region? So where does that pink region start and stop? And maybe we even need to look at this top graph, also.

So if we think about it, we went from 0 to pi to get this outer curve. So how do we get the inner curve? The inner curve started at theta equals pi, went to here, went to here-- that was theta equals 3*pi. So it went all the way from pi to 3*pi.

Now, if you're paying good attention, you can say, well, Christine, we know that this region is totally symmetric. So why don't I just take the area from pi to 2*pi and multiply it by 2? And you can. You can do it either way. So you can either take the integral from all the way from pi to 3*pi, which corresponds to starting at this angle, going all the way around, and coming back to there, which takes you all the way around this curve. Or you can go from pi to 2*pi and multiply that by 2.

So let me come back and let me write that down. It's either pi to 3*pi, this, or you just write it as integral from pi to 2*pi. And if I multiply this by 2, the 2 drops out. The 2 in the denominator drops out.

I'm not going to solve this problem for you, but I do want to point out the kinds of terms you would have. You would have a constant term when you square this. You would have a term that was 2 cosine theta over 2, which is easy to integrate by a u-substitution. And then you would have a cosine squared theta over 2, which you'd want to use the double angle formula or the half angle formula that you've seen used to manipulate these integrals that involve just a cosine squared or a sine squared. So that would be your strategy.

OK, now let's look at the blue area. OK. So to find the blue area, again, I know all that matters is really the bounds. We're going to see we have to do a little extra work. But this is our first setup. And now let's go look at the bounds. OK. So we go back to the curve.

All right. What is the blue area? Well, the blue area, that's a little harder. So let's see what happens. If I were to take theta from 0 to pi, what would happen? I would not only pick up the blue area, but I'd pick up this pink stuff inside. But I don't want the pink stuff. I just want the blue stuff. So what am I going to have to do to find the blue area just, say, from 0 up to pi? I'm going to have to find the area from 0 to pi, and then I'm going to have to subtract off the area of this component. OK? But we know, actually, how to find the area of this component.

OK. So hopefully this makes sense. Because let's think about, when I'm finding area, I'm going from the origin and I'm coming out and I have the radius out there. So when I integrate this dA from 0 to pi for theta, I'm picking up pieces that come out, little sectors that come out like this between pi over 2 and pi. So I'm getting more area than I want if I just let theta go between 0 and pi. So I have to calculate all of it, and then I have to take away the extra stuff.

OK. So the blue area is actually the bigger area subtracting the smaller area. And so how am I going to write this? If we come back over, I'm just going to take 2 times this whole thing. So I'm going to take 2 times this. And I know I have to integrate it from 0 to pi. And I'm taking 2 times because it's symmetric, remember. And then I'm going to subtract track off this one that's 2 times the thing from pi to 2*pi.

Now, you might say, well, Christine, the pink stuff I'm interested in between 0 and pi is actually theta between not pi and 2*pi, but 2*pi and 3*pi. But again, there's all this symmetry in the problem. So it doesn't really matter. But if you're a stickler, I guess I'll even write it this way just to make sure. So I'll write it as 2*pi to 3*pi so that everyone's very happy. OK?

So again, what do we do? These 2's divide out. So from 0 to pi of r squared d theta, that's going to give me the area of the blue plus the pink. And then 2*pi to 3*pi of 1 plus cosine over theta over 2 squared d theta is going to give me the area of the pink.

So the blue plus the pink is over here, and then the pink is over here. So when I subtract that off I just get the blue.

OK. Let me, again, just go back one more time and point out what we did at the very beginning to remind us what was happening. And then I will finish.

So let's come back over here. So the idea was to graph this curve that was in r is a function of theta. And I was supposed to understand what that looked like in the xy-plane. And so my trick was to take the relationship between r and theta and graph that explicitly in an r, theta plane-- so I let theta vary in the horizontal direction and r vary in the vertical direction-- and I can do that very easily. And then translate that into the xy-plane.

So my curve, again, went the big part, the little part here, little part here, big part here. That was the order. So if you need arrows on it, this was the order. And then once I had that, the problem was about areas. And there was a lot of symmetry in this problem, but the main point I wanted to show was just knowing where your theta starts and stops is not enough to determine an area of a region, if that region is excluding some part that would be potentially counted twice. So that was the reason I wanted you to calculate not just the pink area, but also see that the blue is not from theta from 0 to pi, but you have to subtract off this extra stuff that you counted. And I guess that is where I will stop.