Parametric Equations of Lines

1. Give parametric equations for x, y, z on the line through $(1, 1, 2)$ in a direction parallel to $\langle 2, -3, -1 \rangle$.

<u>Answer:</u> We're given the basic data for a line of a point and a direction:

$$\langle x, y, z \rangle = \langle 1 + 2t, 1 - 3t, 2 - t \rangle \Leftrightarrow x = 1 + 2t, \quad y = 1 - 3t, \quad z = 2 - t$$

2. Give parametric equations for the intersection of the planes $x + y + z = 1$ and $x + 2y + 3z = 2$.

<u>Answer:</u> We need to find the basic data.

A point on the intersection: we take $z = 0$ and solve for x and y $\Rightarrow$ $P_0 = (0, 1, 0)$.

The line of intersection is perpendicular to both normals (to the planes), thus

$$\mathbf{v} = \langle 1, 1, 1 \rangle \times \langle 1, 2, 3 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = \langle 1, -2, 1 \rangle.$$

We get parametric equations

$$x = t, \quad y = 1 - 2t, \quad z = t.$$

Remark: The parametrization is not unique. You might have described the same line using a different point P_0 or a scaled version of $\mathbf{v}$.

18.02SC Multivariable Calculus
Fall 2010