Problems: Directional Derivatives

The function $T = x^2 + 2y^2 + 2z^2$ gives the temperature at each point in space.

1. At the point $P = (1, 1, 1)$, in which direction should you go to get the most rapid decrease in T? What is the directional derivative in this direction?

Answer: We know that the fastest *increase* is in the direction of $\boldsymbol{\nabla} T = \langle 2x, 4y, 4z\rangle$. At P, the fastest *decrease* is in the direction of $-\boldsymbol{\nabla} T|_{(1,1,1)} = -\langle 1, 2, 2\rangle$. The unit vector in this direction is $\widehat{\mathbf{u}} = -\langle 1/3, 2/3, 2/3\rangle$.

The rate of change in this direction is $-|\boldsymbol{\nabla} T| = -3$. Equivalently, you could compute:

$$\left.\frac{dT}{ds}\right|_{P,\widehat{\mathbf{u}}} = \boldsymbol{\nabla} T|_P \cdot \widehat{\mathbf{u}} = -3.$$

2. At P, about how far should you go in the direction found in part (1) to get a decrease of 0.3?

Answer: The directional derivative is a true derivative describing the limit of a ratio. In this case it equals $\displaystyle\lim_{\Delta s \to 0} \frac{\Delta T}{\Delta s}$, where Δs is the distance moved in the $\widehat{\mathbf{u}}$ direction. Thus, we can write $\dfrac{\Delta T}{\Delta s} \approx \dfrac{dT}{ds}$.

In this problem we have $\dfrac{dT}{ds} = -3$ and $\Delta T = -0.3$.

$$\frac{-0.3}{\Delta s} \approx -3 \ \Rightarrow\ \Delta s \approx 0.1.$$