Problems: Vector Fields

1. Sketch the following vector fields. Pay attention to their names because we will be encountering these fields frequently.

(a) Force, constant gravitational field $\mathbf{F}(x, y) = -g\mathbf{j}$.

(b) Velocity $\mathbf{V}(x, y) = \dfrac{x}{x^2 + y^2}\mathbf{i} + \dfrac{y}{x^2 + y^2}\mathbf{j} = \langle x, y\rangle / r^2$. (This is a shrinking radial field –like water pouring from a source at (0,0).)

(c) Unit tangential field $\mathbf{F} = \langle -y, x\rangle / r$.

(d) Gradient $\mathbf{F} = \boldsymbol{\nabla} f$, where $f(x, y) = \dfrac{xy}{3}$ and $\boldsymbol{\nabla} f = \left\langle \dfrac{y}{3}, \dfrac{x}{3} \right\rangle$.

2. Compute the gradient field of $f(x, y) = xy^2$.

18.02SC Multivariable Calculus
Fall 2010