Okay, those are the formulas. You will get all of those on the test, plus a couple more that I will give you today.

Those will be the basic formulas of the Laplace transform. If I think you need anything else, I'll give you other stuff, too.

So, I'm going to leave those on the board all period.

The basic test for today is to see how Laplace transforms are used to solve linear differential equations with constant coefficients. Now, to do that, we're going to have to take the Laplace transform of a derivative. And, in order to make sense of that procedure, we're going to have to ask, I apologize in advance, but a slightly theoretical question, namely, we have to have some guarantee in advance that the Laplace transform is going to exist.

Now, how could the Laplace transform fail to exist?

Can't I always calculate this? And the answer is, no, you can't always calculate it because this is an improper integral. I'm integrating all the way up to infinity, and you know that improper integrals don't always converge. You know, if the integrand for example just didn't have the exponential factor there, were simply t dt, that it might look like it made sense, but that integral doesn't converge.

And, anyway, it has no value.

So, I need conditions in advance, which guarantee that the Laplace transforms will exist.

Only under those circumstances will the formulas make any sense. Now, there is a standard condition that's in your book. But, I didn't get a chance to talk about it last time. So, I thought I'd better spent the first few minutes today talking about the condition because it's what we're going to need in order to be able to solve differential equations. The condition that makes the Laplace transform definitely exist for a function is that f of t shouldn't grow too rapidly.

It can grow rapidly. It can grow because the e to the minus s t is pulling it down, trying hard to pull it down to zero to make the integral converge. All we have to do is to guarantee that it doesn't grow so rapidly that the e to the minus s t is powerless to pull it down.

Now, the condition is it's what's called a growth condition. These are very important in applications, and unfortunately, it's always taught in 18.01, but it's not always taught in high school calculus. And, it's a question of how fast the function is allowed to grow.

And, the condition is universally said this way, should be of exponential type. So, what I'm defining is the phrase "exponential type." I'll put it in quotation marks for that reason. What does this mean?

It's a condition, a growth condition on a function, says how fast it can get big.

It says that f of t in size, since f of t might get negatively very large, and that would hurt, make the integral hard to converge, not likely to converge, use the absolute value.

In other words, I don't care if f of t is going up or going down very low. Whichever way it goes, its size should not be bigger than a rapidly growing exponential. And, here's a rapidly growing exponential. c is some positive constant, for some positive constant c and some positive constant k.

And, this should be true for all values of t.

All t greater than or equal to zero.

I don't have to worry about negative values of t because the integral doesn't care about them.

I'm only doing the integration as t runs from zero to infinity.

In other words, f of t could have been an extremely wild function, sewn a lot of oats or whatever functions do for negative values of t, and we don't care.

It's only what's happening from now from time zero onto infinity. As long as it behaves now, from now on, it's okay.

All right, so, the way it should behave is by being an exponential type. Now, to try to give you some feeling for what this means, these functions, for example, if k is 100, do you have any idea what the plot of e to the 100t looks like? It goes straight up.

On every computer you try to plot it on, e to the 100t goes like that unless, of course, you make the scale t equals zero to, over here, is one millionth. Well, even that won't do.

Okay, so these functions really can grow quite rapidly.

Let's take an example and see what's of exponential type, and then perhaps even more interestingly, what isn't. The function sine t, is that of exponential type? Well, sure.

Its absolute value is always less than or equal to one.

So, it's also this paradigm. If I take c equal to one, and what should I take k to be? Zero.

Take k to be zero, c equals one, and in fact sine t plays that condition.

Here's one that's more interesting, t to the n.

Think of t to the 100th power.

Is that smaller than some exponential with maybe a constant out front? Well, t to the 100th power goes straight up, also.

Well, we feel that if we make the exponential big enough, maybe it will win out. In fact, you don't have to make the exponential big. k equals one is good enough. In other words, I don't have to put absolute value signs around the t to the n because I'm only thinking about t as being a positive number, anyway.

I say that that's less than or equal to some constant M, positive constant M times e to the t will be good enough for some M and all t. Now, why is that?

Why is that? The way to think of that, so, what this proves is that, therefore, t to the n is of exponential type, which we could have guessed because after all we were able to calculate its Laplace transform. Now, just because you can calculate the Laplace transform doesn't mean it's of exponential type, but in practical matters, it almost always does.

So, t to the n is of exponential type.

How do you prove that? Well, the weighted secret is to look at t to the n divided by e to the t.

In other words, look at the quotient.

What I'd like to argue is that this is bounded by some number, capital M. That's the question I'm asking.

Now, why is this so? Well, I think I can convince you of it without having to work very hard.

What does the graph of this function look like?

It starts here, so I'm graphing this function, this ratio. When t is equal to zero, its value is zero, right, because of the numerator. What happens as t goes to infinity? What happens to this?

What does it approach? Zero.

And, why? By L'Hop.

By L'Hopital's rule. Just keep differentiating, reapply the rule over and over, keep differentiating it n times, and finally you'll have won the numerator down to t to the zero, which isn't doing anything much. And, the denominator, no matter how many times you differentiate it, it's still t, to the t all the time.

So, by using Lopital's rule n times, you change the top to one or n factorial, actually; the bottom stays e to the t, and the ratio clearly approaches zero, and therefore, it approached zero to start with.

So, I don't know what this function's doing in between.

It's a positive function. It's continuous because the top and bottom are continuous, and the bottom is never zero.

So, it's a continuous function which starts out at zero and is positive, and as t goes to infinity, it gets closer and closer to the t-axis, again.

Well, what does t to the n do?

It might wave around. It doesn't actually.

But, the point is, because it's continuous, starts at zero, ends at zero, it's bounded. It has a maximum somewhere.

And, that maximum is M. So, it has a maximum.

All you have to know is where it starts, and where it ends up, and the fact that it's continuous.

That guarantees that it has a maximum.

So, it is less than some maximum, and that shows that it's of exponential type. Now, of course, before you get the idea that everything's of exponential type, let's see what isn't. I'll give you two functions that are not of exponential type, for different reasons.

One over t is not of exponential type.

Well, of course, it's not defined that t equals zero. But, you know, it's okay for an integral not to be defined at one point because you're measuring an area, and when you measure an area, what happened to one point doesn't really matter much.

That's not the thing. What's wrong with one over t is that the integral doesn't converge at zero times one over t dt. That integral, when t is near zero, this is approximately equal to one, right? If t is zero, this is one. So, it's like the function, integral from zero to infinity of one over t, near zero it's close to, it's like the integral from zero to someplace of no importance, dt over t.

But, this does not converge. This is like log t, and log zero is minus infinity. So, it doesn't converge.

So, one over t is not of exponential type.

So, what's the Laplace transform of one over t?

It doesn't have a Laplace transform. Well, what if I put t equals negative n? What about t to the minus one?

Well, that only works for positive integers, not negative integers.

Okay, so it's not of exponential type.

However, that's because it never really gets started properly. It's more fun to look at a function which is not of exponential type because it grows too fast. Now, what's a function that grows faster that it grows so rapidly that you can't find any function e to the k t which bounds it?

A function which grows too rapidly, a simple one is e to the t squared, grows too rapidly to be of exponential type. And, the argument is simple.

No matter what you propose, it's always, for the K, no matter how big a number, use Avogadro's number, use anything you want. Ultimately, this is going to be bigger than k t no matter how big k is, no matter how big k is. When is this going to happen?

This will happen if t squared is bigger than k t.

In other words, as soon as t becomes bigger than k, you might have to wait quite a while for that to happen, but, as soon as t gets bigger than 10 to the 10 to the 23, this e to the t squared will be bigger than e to the 10 to the 10 to the 23 times t.

So, e to the t squared, it's a simple function, a simple elementary function. It grows so rapidly it doesn't have a Laplace transform. Okay, so how are we going to solve differential equations if e to the t squared?

I won't give you any. And, the reason I won't give you any: because I never saw one occur in real life.

Nature, like sines, cosines, exponentials, are fine, I've never seen a physical, you know, this is just my ignorance. But, I've never seen a physical problem that involved a function growing as rapidly as e to the t squared. That may be just my ignorance.

But, I do know the Laplace transform won't be used to solve differential equations involving such a function.

How about e to the minus t squared?

That's different. It looks almost the same, but e to the minus t squared does this.

It's very well-behaved. That's the curve, of course, that you're all afraid of.

Don't panic. Okay.

So, I'd like to explain to you now how differential equations, maybe I should save-- I'll tell you what.

We need more formulas. So, I'll put them, why don't I save this board, and instead, I'll describe to you the basic way Laplace transforms are used to solve differential equations, what are they called, a paradigm. I'll show you the paradigm, and then we'll fill in the holes so you have some overall view of how the procedure goes, and then you'll understand where the various pieces fit into it.

I think you'll understand it better that way.

So, what do we do? Start with the differential equation. But, right away, there's a fundamental difference between what the Laplace transform does, and what we've been doing up until now, namely, what you have to start with is not merely the differential equation.

Let's say we have linear with constant coefficients.

It's almost never used to solve any other type of problem.

And, let's take second order so I don't have to do, because that's the kind we've been working with all term.

But, it's allowed to be inhomogeneous, so, f of t. Let's call the something else, another letter, h of t.

I'll want f of t for the function I'm taking the Laplace transform of. All right, now, the difference is that up to now, you know techniques for solving this just as it stands. The Laplace transform does not know how to solve this just doesn't stands.

The Laplace transform must have an initial value problem.

In other words, you must supply from the beginning the initial conditions that the y is to satisfy.

Now, I don't want to say any specific numbers, so I'll use generic numbers. Well, but look, what do we do if we get a problem and there are no initial conditions; does that mean we can't use the Laplace transform?

No, of course you can use it. But, you will just have to assume the initial conditions are on the numbers.

You'll say it but the initial conditions be y sub zero and y zero prime, or whatever, a and b, whatever you want to call it.

And now, the answer, then, will involve the a and the b or the y zero and the y zero prime.

But, you must, at least, give lip service to the initial conditions, whereas before we didn't have to do that. Now, depending on your point of view, that's a grave defect, or it is, so what?

Let's adopt the so what point of view.

So, there's our problem. It's an initial value problem.

How is it solved by the Laplace transform?

Well, the idea is you take the Laplace transform of this differential equation and the initial conditions.

So, I'm going to explain to you how to do that.

Not right now, because we're going to need, first, the Laplace transform of a derivative, the formula for that. You don't know that yet.

But when you do know it, you will be able to take the Laplace transform of the initial value problem.

So, I'll put the little l here, and what comes out is, well, y of t is the solution to the original problem. If y of t is the function which satisfies that equation and these initial conditions, its Laplace transform, let's call it capital Y, that's our standard notation, but it's going to be of a new variable, s. So, when I take the Laplace transform of the differential equation with the initial conditions, what comes out is an algebraic-- the emphasis is on algebraic: no derivatives, no transcendental functions, nothing like that, an algebraic equation, m Y of s.

And, now what? Well, now, in the domain of s, it's easy to solve this algebraic equation.

Not all algebraic equations are easy to solve for the capital Y.

But, the ones you will get will always be, not because I am making life easy for you, but that's the way the Laplace transform works. So, you will solve it for Y.

And, the answer will always come out to be Y equals, Y of s equals some rational function, some quotient of polynomials in s, a polynomial in s divided by some other polynomial in s.

And, now what? Well, this is the Laplace transform of the answer. This is the Laplace transform of the solution we are looking for.

So, the final step is to go backwards by taking the inverse Laplace transform of this guy. And, what will you get?

Well, you will get y equals the y of t that we are looking for. It's really a wildly improbable procedure. In other words, instead of going from here to here, you have to imagine there's a mountain here. And, the only way to get around it is to go, first, here, and then cross the stream here, and then go back up, and go back up.

It looks like a senseless procedure, what do they call it, going around Robin Hood's barn, it was called when I was a, I don't know why it's called that.

But that's what we used to call it; not Laplace transform.

That was just a generic thing when you had to do something like this. But, the answer is that it's hard to go from here to here, but trivial to go from here to here. This solution step is the easiest step of all. This is not very hard.

It's easy, in fact. This is easy and straightforward. This is trivial, essentially, yeah, trivial.

But, this step is the hard step.

This is where you have to use partial fractions, look up things in the table to get back there so that most of the work of the procedure isn't going from here to here.

Going from here to there is a breeze.

Okay, now, in order to implement this, what is it we have to do? Well, the basic thing is I have to explain to you, you already know at least a little bit, a reasonable amount of technique for taking that step if you went to recitation yesterday and practiced a little bit. This part, I assure you, is nothing. So, all I have to do now is explain to you how to take the Laplace transform of the differential equation. And, that really means, how do you take the Laplace transform of a derivative?

So, that's our problem. What I want to form, in other words, is a formula for the Laplace transform f prime of t.

Now, in terms of what? Well, since f is an arbitrary function, the only thing I could hope for is somehow to express the Laplace transform of the derivative in terms of the Laplace transform of the original function.

So, that's what I'm aiming for. Okay, where are we going to start? Well, starting is easy because we know nothing. If you don't know anything, then there's no place to start but the definition.

Since I know nothing whatever about the function f of t, and I want to calculate the Laplace transform, I'd better start with the definition.

Whatever this is, it's the integral from zero to infinity of e to the minus s t times f prime of t dt.

Now, what am I looking for? I'm looking for somehow to transform this so that what appears here is not f prime of t, which I'm clueless about, but f of t because if this were f of t, this expression would be the Laplace transform of f of t.

And, I'm assuming I know that. So, the question is how do I take this and somehow do something clever to it that turns this into f of t instead of f prime of t?

Now, to first the question that way, I hope I would get 100% response on what to do.

But, I'll go for 1%. So, what should I do?

I want to change that, so that instead of f prime of t, f of t appears there instead.

What should I do? Integrate by parts, the most fundamental procedure in advanced analysis.

Everything important and interesting depends on integration by parts. And, when you consider that integration by parts is nothing more than just the formula for the derivative of a product read backwards, it's amazing.

It never fails to amaze me, but it's okay.

That's what mathematics are so great.

Okay, so let's use integration by parts.

Integration by parts: okay, so, we have to decide, of course, there's no doubt that this is the factor we want to integrate, which means we have to be willing to differentiate this factor.

But that will be okay because it looks practically, like any exponential, it looks practically the same after you've differentiated it. So, let's do the work.

First step is you don't do the differentiation.

You only do the integration. So, the first step is e to the negative s t. And, the integral of f prime of t is just f of t.

And, that's to be evaluated between the limits zero and infinity. And then, minus, again, before you forget it, put down that minus sign.

The integral between the limits of what you get by doing both operations, both the differentiation and the integration. So, the differentiation will be by using the chain rule. Remember, I'm differentiating with respect to t.

The variable is t here, not s. s is just a parameter.

It's just a constant, a variable constant, if you get my meaning. That's not an oxymoron.

A variable constant: a parameter is a variable constant, variable because you can manipulate the little slider and make a change its value, right?

That's why it's variable. It's not a variable.

It's variable, if you get the distinction.

Okay, well, I mean, it becomes a variable [LAUGHTER]. But right now, it's not a variable. It's just sitting there in the integral. All right, so, minus s, e to the negative s t, f of t dt.

Now, this part's easy.

The interesting thing is this expression.

So, and the most interesting thing is I have to evaluate it at infinity. Now, of course, that means take the limit as you go towards, as you let t goes to infinity. Now, so what I'm interested in knowing is what's the limit of that expression?

I'll write it as f of t divided by e to the s t.

Remember, s is a positive number. s t goes to infinity, and I want to know what the limit of that is.

Well, the limit is what it is. But really, if that limit isn't zero, I'm in deep trouble since the whole process is out of control. What will make that limit zero?

Well, that f of t should not grow faster than e to the s t if s is a big enough number.

And now, that's just what will happen if f of t is of exponential type. It's for this step right here that is the most crucial place at which we need to know that f of t is of exponential type. So, that limit is zero since f of t is of exponential type, in other words, that the value, the absolute value of f of t, becomes less than, let's say, put in the c if you want, but it's not very important, c e to the k t efor all values of t. And, therefore, this will go to zero as soon as s becomes bigger than that k.

In other words, if f of t isn't growing any faster than e to the k t , then as soon as s is a number, that parameter has the value bigger than k, this ratio is going to go to zero because the denominator will always be bigger than the numerator, and getting bigger faster. So, this goes to zero if s is bigger than k. At the upper limit, therefore, this is zero. Again, assuming that s is bigger than that k, the k of the exponential type, how about at the lower limit? We're used to seeing zero there, but we're not going to get zero.

If I plug in t equals zero, this factor becomes one.

And, what happens to that one? f of zero.

You mean, I'm going to have to know what f of zero is before I can take the Laplace transform of this derivative?

The answer is yes, and that's why you have to have an initial value problem. You have to know in advance what the value of the function that you are looking for is at zero because it enters into the formula.

I didn't make up these rules; I'm just following them.

So, what's the rest? The two negatives cancel, and you get plus s. It's just a parameter, so I can pull it out of the integral.

I'm integrating with respect to t, and what's left is, well, what is left? If I take out minus s, combine it there, I get what's left is just the Laplace transform of the function I started with.

So, it's F of s. And, that's the magic formula for the Laplace transform of the derivative.

So, it's worth putting up on our little list.

So, f prime of t, assuming it's of exponential type, has as its Laplace transform, well, what is it? Let's put down the major part of it is s times whatever the Laplace transform of the original function, F of t, was. So, I take the original Laplace transform. When I multiply it by s, that corresponds to taking the derivative.

But there's also that little extra piece.

I must know the value of the starting value of the function.

This is the formula you'll used to take a Laplace transform of the differential equation. Now, but you see I'm not done yet because that will take care of the term a y prime.

But, I don't know what the Laplace transform of the second derivative is. Okay, so, we need a formula for the Laplace transform of a second derivative as well as the first. Now, the hack method is to say, secondary, all right. I've got to do this.

I'll second derivative here, second derivative here, what do I do with that? Ah-ha, I integrate by parts twice. Yes, you can do that.

But that's a hack method. And, of course, I know you're too smart to do that.

What you would do instead is-- How are we going to fill that in? Well, a second derivative is also a first derivative. A second derivative is the first derivative of the first derivative.

Okay, now, we'll just call this glop, something.

So, it's glop prime. What is the formula for the Laplace transform of glop prime? It is, well, I have my formula. It is the glop prime.

The formula for it is s times the Laplace transform of glop, okay, glop. Well, glop is f prime of t.

I'm not done yet, minus glop evaluated at zero. What's glop evaluated at zero?

Well, f prime of zero.

Now, I don't want the formula in that form, but I have to have it in that form because I know what the Laplace transform of f prime of t is.

I just calculated that. So, this is equal to s times the Laplace transform of f prime of t, which is s times F of s, capital F of s, minus f of zero.

All that bracket stuff corresponds to this guy. And, don't forget the stuff that's tagging along, minus f prime of zero.

And now, put that all together.

What is it going to be? Well, there's the principal term which consists of s squared multiplied by F of s.

That's the main part of it.

And, the rest is the sort of fellow travelers.

So, we have minus s times f of zero, little term tagging along. This is a constant times s.

And then, we've got another one, still another constant.

So, what we have is to calculate the Laplace transform of the second derivative, I need to know both f of zero and f prime of zero, exactly the initial conditions that the problem was given for the initial value problem. But, notice, there's a principal part of it. That's the s squared F of s.

That's the guts of it, so to speak. The rest of it, you know, you might hope that these two numbers are zero.

It could happen, and often it is made to happen and problems to simplify them. And I case, you don't have to worry; they're not there. But, if they are there, you must put them in or you get the wrong answer.

So, that's the list of formulas.

So, those formulas on the top board and these two extra ones, those are the things you will be working with on Friday.

But I stress, the Laplace transform won't be a big part of the exam. The exam, of course, doesn't exist, let's say a maximum of 20%, maybe 15. I don't know, give or take a few points. Yeah, what's a point or two?

Okay, let's solve, yeah, we have time.

We have time to solve a problem.

Let's solve a problem. See, I can't touch that.

It's untouchable. Okay, this, we've got to keep.

Problem? Okay.

Okay, now you know how to solve this problem by operators.

Let me just briefly remind you of the basic steps.

You have to do two separate tasks.

You have to first solve the homogeneous equation, putting a zero there. That's the first thing you learned to do. That's easy.

You could almost do that in your head now.

You get the characteristic polynomial, get its roots, get the two functions, e to the t and e to the negative t, which are the solutions.

You make up c1 times one, and c2 times the other.

That's the complementary function that solves the homogeneous problem. And then you have to find a particular solution. Can you see what would happen if you try to find the particular solution?

The number here is negative one, right?

Negative one is a root of the characteristic polynomial, so you've got to use that extra formula.

It's okay. That's why I gave it to you.

You've used the exponential input theorem with the extra formula. Then, you will get the particular solution. And now, you have to make the general solution. The particular solution plus the complementary function, and now you are ready to put in the initial conditions. At the very end, when you've got the whole general solution, now you put in, not before, you put in the initial conditions. You have to calculate the derivative of that thing and substitute this.

You take it as it stands to substitute this.

You get a pair of simultaneous equations for c1 and c2.

You solve them: answer.

It's a rather elaborate procedure, which has at least three or four separate steps, all of which, of course, must be done correctly.

Now, the Laplace transform, instead, feeds the entire problem into the Laplace transform machine.

You follow that little blue pattern, and you come out with the answer. So, let's do the Laplace transform way. Okay, so, the first step is to say, if here's my unknown function, y of t, it obeys this law, and here are its starting values, a bit of its derivative. What I'm going to take is the Laplace transform of this equation.

In other words, I'll take the Laplace transform of this side, and this side also.

And, they must be equal because if they were equal to start with, the Laplace transforms also have to be equal.

Okay, so let's take the Laplace transform of this equation.

Okay, first ID the Laplace transform of the second derivative. Okay, that's going to be, don't forget the principal terms.

There is some people who get so hypnotized by this.

I just know I'm going to forget this.

So, they read it. Then they forget this.

But that's everything. That's the important part.

Okay, so it's s times, I'm calling the Laplace transform not capital F but capital Y because my original function is called little y. So, it's s squared Y.

It's Y of s, but I'm not going to put that, the of s in because it just makes the thing look more complicated. And now, before you forget, you have to put in the rest. So, minus s times the value at zero, which is one, minus the value of the derivative. But, that's zero.

So, this is not too hard a problem.

So, minus s minus zero, so I don't have to put that in. So, all this is the Laplace transform of y double prime.

And now, minus the Laplace transform of y, well, that's just capital Y. What's that equal to?

The Laplace transform of the right-hand side.

Okay, look up the formula. It is e to the negative t, a is minus one, so, it's one over s minus minus one; so, it is s plus one.

This is that. Okay, the next thing we have to do is solve for Y. That doesn't look too hard.

Solve it for y. Okay, the best thing to do is put s squared, group all the Y terms together unless you're really quite a good calculator.

Maybe make one extra line out of it.

Yeah, definitely do this. And then, the extra garbage I refer to as the garbage, this stuff, and this stuff, the stuff, the linear polynomials which are tagging along move to the right-hand side because they don't involve capital Y. So, this we will move to the other side. And so, that's equal to (one over (s plus one)) plus s.

Now, you have a basic choice. About half the time, it's a good idea to combine these terms.

The other half of the time, it's not a good idea to combine those terms. So, how do we know whether to do it or not to do it? Experience, which you will get by solving many, many problems.

Okay, I'm going to combine them because I think it's a good thing to do here. So, what is that?

That's s squared plus s plus one.

So, it's s squared plus s plus one divided by s plus one, okay? I'm still not done because now we have to know, what's Y?

All right, now we have to think.

What we're going to do is get Y in this form.

But, I want it in the form in which it's most suited for using partial fractions. In other words, I want the denominator as factored as I possibly can be.

Okay, well, the numerator is going to be just what it was.

How should I write the denominator?

Well, the denominator is going to have the factor s plus one in it from here. But after I divide through, the other factor will be s squared minus one, right? But, s squared minus one is s minus one times s plus one.

So, I have to divide this by s squared minus one.

Factored, it's this. So, the end result is there are two of these and one of the other.

And now, it's ready to be used. It's better to be a partial fraction. So, the final step is to use a partial fraction's decomposition on this so that you can calculate its inverse Laplace transform.

So, let's do that. Okay, (s squared plus s plus one) divided by that thing, (s plus one) squared times (s minus one) equals s plus one squared plus s plus one plus s minus one.

In the top will be constants, just constants. Let's do it this way first, and I'll say at the very end, something else.

Maybe now. Many of you are upset.

Some of you are upset. I know this for a fact because in high school, or wherever you learned to do this before, there weren't two terms here.

There was just one term, s plus one squared.

If you do it that way, then it's all right. Then, it's all right, but I don't recommend it. In that case, the numerators will not be constants.

But, if you just have that, then because this is a quadratic polynomial all by itself.

You've got to have a linear polynomial, a s plus b in the top, see?

So, you must have a s plus b here, as I'm sure you remember if that's the way you learned to do it. But, to do cover-up, the best way as much as possible to separate out the terms. If this were a cubic term, God forbid, s plus one cubed, then you'd have to have s plus one cubed, s plus one squared.

Okay, I won't give you anything bigger than quadratic.

[LAUGHTER] You can trust me.

Okay, now, what can we find by the cover up method?

Well, surely this. Cover up the s minus one, put s equals one, and I get three divided by two squared, four. So, this is three quarters.

Now, in this, you can always find the highest power by cover-up because, cover it up, put s equals negative one, and you get one minus one plus one.

So, one up there, negative one here makes negative two here. So, one over negative two.

So, it's minus one half.

Now, this you cannot determine by cover-up because you'd want to cover-up just one of these s plus ones. But then you can't put s equals negative one because you get infinity.

You get zero there, makes infinity.

So, this must be determined some other way, either by undetermined coefficients, or if you've just got one thing, for heaven's sake, just make a substitution. See, this is supposed to be true. This is an algebraic identity, true for all values of the variable, and therefore, it ought to be true when s equals zero, for instance. Why zero?

Well, because I haven't used it yet.

I used negative one and positive one, but I didn't use zero. Okay, let's use zero.

Put s equals zero. What do we get?

Well, on the left-hand side, I get one divided by one squared, negative. So, I get minus one on the left hand side equals, what do I get on the right?

Put s equals zero, you get negative one half.

Well, this is the number I'm trying to find. So, let's write that simply as plus c, putting s equals zero. s equals zero here gives me negative three quarters.

Okay, what's c? This is minus a half, minus three quarters, is minus five quarters.

Put it on the other side, minus one plus five quarters is plus one quarter.

So, c equals one quarter.

And now, we are ready to do the final step. Take the inverse Laplace transform. You see what I said when I said that all the work is in this last step?

Just look how much of the work, how much of the board is devoted to the first two steps, and how much is going to be devoted to the last step? Okay, so we get e to the inverse Laplace transform. Well, the first term is the hardest. Let's let that go for the moment. So, I leave a space for it, and then we will have one quarter.

Well, one over s plus one is, that's just the exponential formula.

One over s plus one would be e to the negative t,e to the minus one times t. So, it's one quarter e to the minus one times t.

And, how about the next thing would be three quarters times, well, here it's negative one, so that's e to the plus t.

Notice how those signs work.

And, that just leaves us the Laplace transform of this thing.

Now, you look at it and you say, this Laplace transform happened in two steps. I took something and I got, essentially, one over s squared.

And then, I changed s to s plus one.

All right, what gives one over s squared?

The Laplace transform of what is one over s squared?

t, you say to yourself, one over s to some power is essentially some power of t. And then, you look at the formula. Notice at the top is one factorial, which is one, of course.

Okay, now, then how do I convert this to one over s plus one squared? That's the exponential shift formula. If you know what the Laplace transform, so the first formula in the middle of the board on the top, there, if you know what, change s to s plus one, corresponds to multiplying by e to the t.

So, it is t times e to the negative t.

Sorry, that corresponds to this.

So, this is the exponential shift formula.

If t goes to one over s squared, then t e to the minus t goes to one over s plus one squared.

Okay, but there's a constant out front. So, it's minus one half t e to the negative t.

Now, tell me, what parts of this solution, oh boy, we're over time. But, notice, this is what would have been the particular solution, (y)p before, and this is the stuff that occurs in the complementary function, but already the appropriate constants have been supplied for the coefficients.

You don't have to calculate them separately.

They were built into the method.

Okay, good luck on Friday, and see you there.