As a matter of fact, it plots them very accurately.

But it is something you also need to learn to do yourself, as you will see when we study nonlinear equations.

It is a skill. And since a couple of important mathematical ideas are involved in it, I think it is a very good thing to spend just a little time on, one lecture in fact, plus a little more on the problem set that I will give out. The last problem set that I will give out on Friday. I thought it might be a little more fun to, again, have a simple-minded model.

No romance this time. We are going to have a little model of war, but I have made it sort of sublimated war. Let's take as the system, I am going to let two of those be parameters, you know, be variable, in other words.

And the other two I will keep fixed, so that you can concentrate on them better. I will take a and d to be negative 1 and negative 3. And the other ones we will leave open, so let's call this one b times y, and this other one will be c times x.

I am going to model this as a fight between two states, both of which are trying to attract tourists.

Let's say this is Massachusetts and this will be New Hampshire, its enemy to the North. Both are busy advertising these days on television. People are making their summer plans. Come to New Hampshire, you know, New Hampshire has mountains and Massachusetts has quaint little fishing villages and stuff like that.

So what are these numbers? Well, first of all, what do x and y represent? x and y basically are the advertising budgets for tourism, you know, the amount each state plans to spend during the year. However, I do not want zero value to mean they are not spending anything.

It represents departure from the normal equilibrium.

x and y represent departures -- -- from the normal amount of money they spend advertising for tourists. The normal tourist advertising budget.

If they are both zero, it means that both states are spending what they normally spend in that year.

If x is positive, it means that Massachusetts has decided to spend more in the hope of attracting more tourists and if negative spending less. What is the significance of these two coefficients? Those are the normal things which return you to equilibrium. In other words, if x gets bigger than normal, if Massachusetts spends more there is a certain poll to spend less because we are wasting all this money on the tourists that are not going to come when we could be spending it on education or something like that. If x gets to be negative, the governor tries to spend less.

Then all the local city Chamber of Commerce rise up and start screaming that our economy is going to go bankrupt because we won't get enough tourists and that is because you are not spending enough money. There is a push to always return it to the normal, and that is what this negative sign means. The same thing for New Hampshire. What does it mean that this is negative three and that is negative one?

It just means that the Chamber of Commerce yells three times as loudly in New Hampshire. It is more sensitive, in other words, to changes in the budget.

Now, how about the other? Well, these represent the war-like features of the situation.

Normally these will be positive numbers.

Because when Massachusetts sees that New Hampshire has budgeted this year more than its normal amount, the natural instinct is we are fighting. This is war.

This is a positive number. We have to budget more, too. And the same thing for New Hampshire. The size of these coefficients gives you the magnitude of the reaction.

If they are small Massachusetts say, well, they are spending more but we don't have to follow them.

We will bucket a little bit. If it is a big number then oh, my God, heads will roll. We have to triple them and put them out of business. This is a model, in fact, for all sorts of competition.

It was used for many years to model in simper times armaments races between countries. It is certainly a simple-minded model for any two companies in competition with each other if certain conditions are met. Well, what I would like to do now is try different values of those numbers.

And, in each case, show you how to sketch the solutions at different cases. And then, for each different case, we will try to interpret if it makes sense or not.

My first set of numbers is, the first case is -- -- x prime equals negative x plus 2y.

And y prime equals, this is going to be zero, so it is simply minus 3 times y.

Now, what does this mean? Well, this means that Massachusetts is behaving normally, but New Hampshire is a very placid state, and the governor is busy doing other things. And people say Massachusetts is spending more this year, and the Governor says, so what. The zero is the so what factor.

In other words, we are not going to respond to them. We will do our own thing.

What is the result of this? Is Massachusetts going to win out? What is going to be the ultimate effect on the budget? Well, what we have to do is, so the program is first let's quickly solve the equations using a standard technique. I am just going to make marks on the board and trust to the fact that you have done enough of this yourself by now that you know what the marks mean.

I am not going to label what everything is.

I am just going to trust to luck.

The matrix A is negative 1, 2, 0, negative 3.

The characteristic equation, the second coefficient is the trace, which is minus 4, but you have to change its sign, so that makes it plus 4.

And the constant term is the determinant, which is 3 minus 0, so that is plus 3 equals zero. This factors into lambda plus 3 times lambda plus one. And it means the roots therefore are, one root is lambda equals negative 3 and the other root is lambda equals negative 1.

These are the eigenvalues. With each eigenvalue goes an eigenvector. The eigenvector is found by solving an equation for the coefficients of the eigenvector, the components of the eigenvector.

Here I used negative 1 minus negative 3, which makes 2.

The first equation is 2a1 plus 2a2 is equal to zero.

The second one will be, in fact, in this case simply 0a1 plus 0a2 so it won't give me any information at all.

That is not what usually happens, but it is what happens in this case. What is the solution?

The solution is the vector alpha equals, well, 1, negative 1 would be a good thing to use. That is the eigenvector, so this is the e-vector. How about lambda equals negative 1? Let's give it a little more room. If lambda is negative 1 then here I put negative 1 minus negative 1.

That makes zero. I will write in the zero because this is confusing. It is zero times a1.

And the next coefficient is 2 a2, is zero.

People sometimes go bananas over this, in spite of the fact that this is the easiest possible case you can get.

I guess if they go bananas over it, it proves it is not all that easy, but it is easy. What now is the eigenvector that goes with this? Well, this term isn't there.

It is zero. The equation says that a2 has to be zero. And it doesn't say anything about a1, so let's make it 1.

Now, out of this data, the final step is to make the general solution. What is it?

(x, y) equals, well, a constant times the first normal mode. The solution constructed from the eigenvalue and the eigenvector.

That is going to be 1, negative 1 e to the minus 3t.

And then the other normal mode times an arbitrary constant will be (1, 0) times e to the negative t.

The lambda is this factor which produces that, of course. Now, one way of looking at it is, first of all, get clearly in your head this is a pair of parametric equations just like what you studied in 18.02. Let's write them out explicitly just this once. x equals c1 times e to the negative 3t plus c2 times e to the negative t.

And what is y?

y is equal to minus c1 e to the minus 3t plus zero.

I can stop there.

In some sense, all I am asking you to do is plot that curve. In the x,y-plane, plot the curve given by this pair of parametric equations.

And you can choose your own values of c1, c2. For different values of c1 and c2 there will be different curves.

Give me a feeling for what they all look like.

Well, I think most of you will recognize you didn't have stuff like this. These weren't the kind of curves you plotted. When you did parametric equations in 18.02, you did stuff like x equals cosine t, y equals sine t.

Everybody knows how to do that. A few other curves which made lines or nice things, but nothing that ever looked like that. And so the computer will plot it by actually calculating values but, of course, we will not. That is the significance of the word sketch. I am not asking you to plot carefully, but to give me some general geometric picture of what all these curves look like without doing any work.

Without doing any work. Well, that sounds promising.

Okay, let's try to do it without doing any work.

Where shall I begin? Hidden in this formula are four solutions that are extremely easy to plot.

So begin with the four easy solutions, and then fill in the rest. Now, which are the easy solutions? The easy solutions are c1 equals plus or minus 1, c2 equals zero, or c1 equals zero, or c1 = 0, c2 equals plus or minus 1. By choosing those four values of c1 and c2, I get simple solutions corresponding to the normal mode.

If c1 is one and c2 is zero, I am talking about (1, negative 1) e to the minus 3t, and that is very easy plot. Let's start plotting them.

What I am going to do is color-code them so you will be able to recognize what it is I am plotting.

Let's see. What colors should we use?

We will use pink and orange. This will be our pink solution and our orange solution will be this one.

Let's plot the pink solution first.

The pink solution corresponds to c1 equals 1 and c2 equals zero. Now, that solution looks like-- Let's write it in pink.

No, let's not write it in pink. What is the solution?

It looks like x equals e to the negative 3t, y equals minus e to the minus 3t.

Well, that's not a good way to look at it, actually.

The best way to look at it is to say at t equals zero, where is it? It is at the point 1, negative 1.

And what is it doing as t increases?

Well, it keeps the direction, but travels.

The amplitude, the distance from the origin keeps shrinking. As t increases, this factor, so it is the tip of this vector, except the vector is shrinking.

It is still in the direction of 1, negative 1, but it is shrinking in length because its amplitude is shrinking according to the law e to the negative 3t.

In other words, this curve looks like this.

At t equals zero it is over here, and it goes along this diagonal line until as t equals infinity, it gets to infinity, it reaches the origin. Of course, it never gets there.

It goes slower and slower and slower in order that it may never reach the origin. What was it doing for values of t less than zero? The same thing, except it was further away. It comes in from infinity along that straight line. In other words, the eigenvector determines the line on which it travels and the eigenvalue determines which way it goes.

If the eigenvalue is negative, it is approaching the origin as t increases. How about the other one?

Well, if c1 is negative 1, then everything is the same except it is the mirror image of this one.

If c1 is negative 1, then at t equals zero it is at this point. And, once again, the same reasoning shows that it is coming into the origin as t increases. I have now two solutions, this one corresponding to c1 equals 1, and the other one c2 equals zero.

This one corresponds to c1 equals negative 1.

How about the other guy, the orange guy?

Well, now c1 is zero, c2 is one, let's say.

It is the vector (1, 0), but otherwise everything is the same. I start now at the point (1, 0) at time zero. And, as t increases, I come into the origin always along that direction.

And before that I came in from infinity.

And, again, if c2 is 1 and if c2 is negative 1, I do the same thing but on the other side.

That wasn't very hard. I plotted four solutions.

And now I roll up my sleeves and waive my hands to try to get others. The general philosophy is the following. The general philosophy is the differential equation looks like this.

It is a system of differential equations.

These are continuous functions. That means when I draw the velocity field corresponding to that system of differential equations, because their functions are continuous, as I move from one (x, y) point to another the direction of the velocity vectors change continuously.

It never suddenly reverses without something like that.

Now, if that changes continuously then the trajectories must change continuously, too. In other words, nearby trajectories should be doing approximately the same thing. Well, that means all the other trajectories are ones which come like that must be going also toward the origin. If I start here, probably I have to follow this one.

They are all coming to the origin, but that is a little too vague. How do they come to the origin?

In other words, are they coming in straight like that? Probably not.

Then what are they doing? Now we are coming to the only point in the lecture which you might find a little difficult.

Try to follow what I am doing now.

If you don't follow, it is not well done in the textbook, but it is very well done in the notes because I wrote them myself. Please, it is done very carefully in the notes, patiently follow through the explanation. It takes about that much space.

It is one of the important ideas that your engineering professors will expect you to understand.

Anyway, I know this only from the negative one because they say to me at lunch, ruin my lunch by saying I said it to my students and got nothing but blank looks.

What do you guys teach them over there?

Blah, blah, blah. Maybe we ought to start teaching it ourselves. Sure.

Why don't they start cutting their own hair, too?

Here is the idea. Let me recopy that solution.

The solution looks like (1, negative 1) e to the minus 3t plus c2, (1, 0) e to the negative t.

What I ask is as t goes to infinity, I feel sure that the trajectories must be coming into the origin because these guys are doing that. And, in fact, that is confirmed. As t goes to infinity, this goes to zero and that goes to zero regardless of what the c1 and c2 are. That makes it clear that this goes to zero no matter what the c1 and c2 are as t goes to infinity, but I would like to analyze it a little more carefully. As t goes to infinity, I have the sum of two terms. And what I ask is, which term is dominant? Of these two terms, are they of equal importance, or is one more important than the other? When t is 10, for example, that is not very far on the way to infinity, but it is certainly far enough to illustrate.

Well, e to the minus 10 is an extremely small number. The only thing smaller is e to the minus 30. The term that dominates, they are both small, but relatively-speaking this one is much larger because this one only has the factor e to the minus 10, whereas, this has the factor e to the minus 30, which is vanishingly small.

In other words, as t goes to infinity -- Well, let's write it the other way.

This is the dominant term, as t goes to infinity.

Now, just the opposite is true as t goes to minus infinity.

t going to minus infinity means I am backing up along these curves. As t goes to minus infinity, let's say t gets to be negative 100, this is e to the 100, but this is e to the 300, which is much, much bigger.

So this is the dominant term as t goes to negative infinity.

Now what I have is the sum of two vectors.

Let's first look at what happens as t goes to infinity.

As t goes to infinity, I have the sum of two vectors.

This one is completely negligible compared with the one on the right-hand side. In other words, for a all intents and purposes, as t goes to infinity, it is this thing that takes over.

Therefore, what does the solution look like as t goes to infinity? The answer is it follows the yellow line. Now, what does it look like as it backs up? As it came in from negative infinity, what does it look like?

Now, this one is a little harder to see.

This is big, but this is infinity bigger.

I mean very, very much bigger, when t is a large negative number.

Therefore, what I have is the sum of a very big vector.

You're standing on the moon looking at the blackboard, so this is really big. This is a very big vector.

This is one million meters long, and this is only 20 meters long. That is this guy, and that is this guy. I want the sum of those two.

What does the sum look like? The answer is a sum is approximately parallel to the long guy because this is negligible. This does not mean they are next to each other. They are slightly tilted over, but not very much. In other words, as t goes to negative infinity it doesn't coincide with this vector. The solution doesn't, but it is parallel to it. It has the same direction.

I am done. It means far away from the origin, it should be parallel to the pink line.

Near the origin it should turn and become more or less coincident with the orange line. And those were the solutions.

That's how they look.

How about down here? The same thing, like that, but then after a while they turn and join.

Here, they have to turn around to join up, but they join.

And that is, in a simple way, the sketches of those functions.

That is how they must look. What does this say about our state? Well, it says that the fact that the governor of New Hampshire is indifferent to what Massachusetts is doing produces ultimately harmony.

Both states revert ultimately their normal advertising budgets in spite of the fact that Massachusetts is keeping an eye peeled out for the slightest misbehavior on the part of New Hampshire. Peace reins, in other words. Now you should know some names.

Let's see. I will write names in purple.

There are two words that are used to describe this situation.

First is the word that describes the general pattern of the way these lines look. The word for that is a node.

And the fact that all the trajectories end up at the origin for that one uses the word sink.

This could be modified to nodal sink.

That would be better. Nodal sink, let's say.

Nodal sink or, if you like to write them in the opposite order, sink node.

In the same way there would be something called a source node if I reversed all the arrows. I am not going to calculate an example. Why don't I simply do it by giving you -- For example, if the matrix A produced a solution instead of that one.

Suppose it looked like 1, negative 1 e to the 3t.

The eigenvalues were reversed, were now positive. And I will make the other one positive, too. c2 1, 0 e to the t.

What would that change in the picture?

The answer is essentially nothing, except the direction of the arrows. In other words, the first thing would still be 1, negative 1.

The only difference is that now as t increases we go the other way.

And here the same thing, we have still the same basic vector, the same basic orange vector, orange line, but it has now traversed the solution.

We traverse it in the opposite direction.

Now, let's do the same thing about dominance, as we did before. Which term dominates as t goes to infinity? This is the dominant term.

Because, as t goes to infinity, 3t is much bigger than t.

This one, on the other hand, dominates as t goes to negative infinity.

How now will the solutions look like?

Well, as t goes to infinity, they follow the pink curve.

Whereas, as t starts out from negative infinity, they follow the orange curve.

As t goes to infinity, they become parallel to the pink curve, and as t goes to negative infinity, they are very close to the origin and are following the yellow curve. This is pink and this is yellow. They look like this.

Notice the picture basically is the same.

It is the picture of a node. All that has happened is the arrows are reversed. And, therefore, this would be called a nodal source.

The word source and sink correspond to what you learned in 18.02 and 8.02, I hope, also, or you could call it a source node.

Both phrases are used, depending on how you want to use it in a sentence. And another word for this, this would be called unstable because all of the solutions starting out from near the origin ultimately end up infinitely far away from the origin.

This would be called stable. In fact, it would be called asymptotically stable. I don't like the word asymptotically, but it has become standard in the literature. And, more important, it is standard in your textbook.

And I don't like to fight with a textbook.

It just ends up confusing everybody, including me.

That is enough for nodes. I would like to talk now about some of the other cases that can occur because they lead to completely different pictures that you should understand.

Let's look at the case where our governors behave a little more badly, a little more combatively.

It is x prime equals negative x as before, but this time a firm response by Massachusetts to any sign of increased activity by stockpiling of advertising budgets. Here let's say New Hampshire now is even worse. Five times, quintuple or whatever increase Massachusetts makes, of course they don't have an income tax, but they will manage.

Minus 3y as before. Let's again calculate quickly what the characteristic equation is.

Our matrix is now negative 1, 3, 5 and negative 3.

The characteristic equation now is lambda squared. What is that?

Again, plus 4 lambda. But now the determinant is 3 minus 15 is negative 12.

And this, because I prepared very carefully, all eigenvalues are integers. And so this factors into lambda plus 6 times lambda minus 2, does it not? Yes.

6 lambda minus 2 is four lambda.

Good. What do we have?

Well, first of all we have our eigenvalue lambda, negative 6. And the eigenvector that goes with that is minus 1. This is negative 1 minus negative 6 which makes, shut your eyes, 5. We have 5a1 plus 3a2 is zero.

And the other equation, I hope it comes out to be something similar.

I didn't check. I am hoping this is right.

The eigenvector is, okay, you have been taught to always make one of the 1, forget about that.

Just pick numbers that make it come out right.

I am going to make this one 3, and then I will make this one negative 5. As I say, I have a policy of integers only. I am a number theorist at heart. That is how I started out life anyway. There we have data from which we can make one solution. How about the other one?

The other one will correspond to the eigenvalue lambda equals 2. This time the equation is negative 1 minus 2 is negative 3.

It is minus 3a1 plus 3a2 is zero.

And now the eigenvector is (1, 1).

Now we are ready to draw pictures.

We are going to make this similar analysis, but it will go faster now because you have already had the experience of that. First of all, what is our general solution? It is going to be c1 times 3, negative 5 e to the minus 6t.

And then the other normal mode times an arbitrary constant will be 1, 1 times e to the 2t.

I am going to use the same strategy.

We have our two normal modes here, eigenvalue, eigenvector solutions from which, by adjusting these constants, we can get our four basic solutions.

Those are going to look like, let's draw a picture here.

Again, I will color-code them. Let's use pink again.

The pink solution now starts at 3, negative 5.

That is where it is when t is zero. And, because of the coefficient minus 6 up there, it is coming into the origin and looks like that. And its mirror image, of course, does the same thing. That is when c1 is negative one. How about the orange guy?

Well, when t is equal to zero, it is at 1, 1.

But what is it doing after that? As t increases, it is getting further away from the origin because the sign here is positive. e to the 2t is increasing, it is not decreasing anymore, so this guy is going out. And its mirror image on the other side is doing the same thing.

Now all we have to do is fill in the picture.

Well, you fill it in by continuity.

Your nearby trajectories must be doing what similar thing?

If I start out very near the pink guy, I should stay near the pink guy. But as I get near the origin, I am also approaching the orange guy.

Well, there is no other possibility other than that.

If you are further away you start turning a little sooner.

I am just using an argument from continuity to say the picture must be roughly filled out this way.

Maybe not exactly. In fact, there are fine points.

And I am going to ask you to do one of them on Friday for the new problem set, even before the exam, God forbid. But I want you to get a little more experience working with that linear phase portrait visual because it is, I think, one of the best ones this semester. You can learn a lot from it.

Anyway, you are not done with it, but I hope you have at least looked at it by now. That is what the picture looks like. First of all, what are we going to name this? In other words, forget about the arrows. If you just look at the general way those lines go, where have you seen this before? You saw this in 18.02.

What was the topic? You were plotting contour curves of functions, were you not?

What did you call contours curves that formed that pattern?

A saddle point. You called this a saddle point because it was like the center of a saddle.

It is like a mountain pass. Here you are going up the mountain, say, and here you are going down, the way the contour line is going down.

And this is sort of a min and max point.

A maximum if you go in that direction and a minimum if you go in that direction, say.

Without the arrows on it, it is like a saddle point.

And so the same word is used here.

It is called the saddle. You don't say point in the same way you don't say a nodal point. It is the whole picture, as it were, that is the saddle. It is a saddle.

There is the saddle. This is where you sit.

Now, should I call it a source or a sink?

I cannot call it either because it is a sink along these lines, it is a source along those lines and along the others, it starts out looking like a sink and then turns around and starts acting like a source. The word source and sink are not used for saddle. The only word that is used is unstable because definitely it is unstable.

If you start off exactly on the pink lines you do end up at the origin, but if you start anywhere else ever so close to a pink line you think you are going to the origin, but then at the last minute you are zooming off out to infinity again. This is a typical example of instability. Only if you do the mathematically possible, but physically impossible thing of starting out exactly on the pink line, only then will you get to the origin. If you start out anywhere else, make the slightest error in measure and get off the pink line, you end off at infinity. What is the effect with our war-like governors fighting for the tourist trade willing to spend any amounts of money to match and overmatch what their competitor in the nearby state is spending?

The answer is, they all lose.

Since it is mostly this section of the diagram that makes sense, what happens is they end up all spending an infinity of dollars and nobody gets any more tourists than anybody else.

So this is a model of what not to do.

I have one more model to show you.

Maybe we better start over at this board here.

Massachusetts on top. New Hampshire on the bottom.

x prime is going to be, that is Massachusetts, I guess as before. Let me get the numbers right.

Leave that out for a moment. y prime is 2x minus 3y.

New Hampshire behaves normally.

It is ready to respond to anything Massachusetts can put out. But by itself, it really wants to bring its budget to normal.

Now, Massachusetts, we have a Mormon governor now, I guess. Imagine instead we have a Buddhist governor. A Buddhist governor reacts as follows, minus y. What does that mean?

It means that when he sees New Hampshire increasing the budget, his reaction is, we will lower ours.

We will show them love. It looks suicidal, but what actually happens? Well, our little program is over. Our matrix a is negative 1, negative 1, 2, negative 3.

The characteristic equations is lambda squared plus 4 lambda.

And now what is the other term? 3 minus negative 2 makes 5.

This is not going to factor because I tried it out and I know it is not going to factor. We are going to get lambda equals, we will just use the quadratic formula, negative 4 plus or minus the square root of 16 minus 4 times 5, that is 16 minus 20 or negative 4 all divided by 2, which makes minus 2, pull out the 4, that makes it a 2, cancels this 2, minus 1 inside. It is minus 2 plus or minus i.

Complex solutions.

What are we doing to do about that?

Well, you should rejoice when you get this case and are asked to sketch it because, even if you calculate the complex eigenvector and from that take its real and imaginary parts of the complex solution, in fact, you will not be able easily to sketch the answer anyway.

But let me show you what sort of thing you can get and then I am going to wave my hands and argue a little bit to try to indicate what it is that the solution actually looks like.

You are going to get something that looks like -- A typical real solution is going to look like this.

This is going to produce e to the minus 2t times e to the i t. e to the minus 2 plus i all times t. This will be our exponential factor which is shrinking in amplitude.

This is going to give me sines and cosines.

When I separate out the eigenvector into its real and imaginary parts, it is going to look something like this. a1, a2 times cosine t, that is from the e to the it part. Then there will be a sine term.

And all that is going to be multiplied by the exponential factor e to the negative 2t.

That is just one normal mode. It is going to be c1 times this plus c2 times something similar. It doesn't matter exactly what it is because they are all going to look the same.

Namely, this is a shrinking amplitude.

I am not going to worry about that.

My real question is, what does this look like?

In other words, as a pair of parametric equations, if x is equal to a1 cosine t plus b1 sine t and y is a2 cosine plus b2 sine, what does it look like?

Well, what are its characteristics?

In the first place, as a curve this part of it is bounded. It stays within some large box because cosine and sine never get bigger than one and never get smaller than minus one. It is periodic.

As t increases to t plus 2pi, it comes back to exactly the same point it was at before.

We have a curve that is repeating itself periodically, it does not go off to infinity. And here is where I am waving my hands. It satisfies an equation.

Those of you who like to fool around with mathematics a little bit, it is not difficult to show this, but it satisfies an equation of the form A x squared plus B y squared plus C xy equals D.

All you have to do is figure out what the coefficients A, B, C and D should be. And the way to do it is, if you calculate the square of x you are going to get cosine squared, sine squared and a cosine sine term.

You are going to get those same three terms here and the same three terms here. You just use undetermined coefficients, set up a system of simultaneous equations and you will be able to find the A, B, C and D that work. I am looking for a curve that is bounded, keeps repeating its values and that satisfies a quadratic equation which looks like this.

Well, an earlier generation would know from high school, these curves are all conic sections.

The only curves that satisfy equations like that are hyperbola, parabolas, the conic sections in other words, and ellipses. Circles are a special kind of ellipses. There is a degenerate case.

A pair of lines which can be considered a degenerate hyperbola, if you want. It is as much a hyperbola as a circle, as an ellipse say. Which of these is it?

Well, it must be those guys. Those are the only guys that stay bounded and repeat themselves periodically.

The other guys don't do that. These are ellipses.

And, therefore, what do they look like?

Well, they must look like an ellipse that is trying to be an ellipse, but each time it goes around the point is pulled a little closer to the origin. It must be doing this, in other words. And such a point is called a spiral sink. Again sink because, no matter where you start, you will get a curve that spirals into the origin. Spiral is self-explanatory.

And the one thing I haven't told you that you must read is how do you know that it goes around counterclockwise and not clockwise? Read clockwise or counterclockwise. I will give you the answer in 30 seconds, not for this particular curve.

That you will have to calculate.

All you have to do is put in somewhere.

Let's say at the point (1, 0), a single vector from the velocity field. In other words, at the point (1, 0), when x is 1 and y is 0 our vector is minus 1, 2, which is the vector minus 1, 2, it goes like this.

Therefore, the motion must be counterclockwise.

And, by the way, what is the effect of having a Buddhist governor? Peace.

Everything spirals into the origin and everybody is left with the same advertising budget they always had.

Thanks.