18.034 Honors Differential Equations

Spring 2009

18.034 Recitation: February 5th, 2009

1. Consider the equation $y'' + y' = 2y$. For what a is e^{ay} a solution? Find a solution y_0 to $y'''' + y''' = 2y''$ with exponential growth as $x \to -\infty$ and $\lim_{x\to\infty} y_0(x)/x = 1$.

2. Suppose that if y_0 is a solution to $y'' + y' - 2y = F(x)$ on all of $(-\infty, \infty)$, then there is another solution y_1 to the same equation with $|y_1(x) - y_0(x)| \to 0$ as $x \to \infty$.

3. Show that $y = e^x$ and $y = \cos x$ cannot be solutions of the same first-order equation $y' = f(x, y)$ on any interval containing the origin.

4. Solve $ydx + 3xdy = 14y^4dy$.

5. Suppose that a trajectory of $(3x^2 - y)dx + (3y^2 - x)dy = 0$ contains the point $(1, 1)$. Show that it also contains the points $(1, -1)$, $(-1, 1)$, $(0, 1)$, $(1, 0)$.

6. (Birkhoff-Rota: p. 6, # 7) Show that solutions of $y' = g(y)$ are convex up or convex down for given y according as $|g|$ is an increasing or decreasing function there.