Damped Harmonic Oscillators: Introduction

In this session we will look carefully at the equation

$$m\ddot{x} + b\dot{x} + kx = 0$$

as a model of the *damped harmonic oscillator*. When the damping constant b equals zero we know the solution to this equation is

$$x(t) = c_1 \cos(\omega t) + c_2 \sin(\omega t) = A\cos(\omega t - \phi),$$

where $\omega = \sqrt{k/m}$. Since this has a pure sinusoidal solution we call the system a **simple harmonic oscillator**. When $b \neq 0$ we call the system a **damped harmonic oscillator**.

Our goal is to understand the effect of b on the system. We'll see that when b is small the system is *underdamped* and the output is a *damped sinusoid* or *damped oscillation*. When b is large the system is *overdamped* and it no longer oscillates. Right between under and overdamping is a value of b called *critical damping*. We will learn how to find the critical damping value.

Our main tool will be the method of *characteristic roots* discussed in the last session. We will use the mathlet *Damped Vibrations* to visualize what we have learned.