18.03SC Practice Problems 12

Gain and phase lag

Exponential response formula (ERF): A solution to $p(D)x = Ae^{rt}$ is given by $x_p = A\frac{e^{rt}}{p(r)}$, as long as $p(r) \neq 0$.

ERF with resonance: Assume that $p(r) = 0$. A solution to $p(D)x = Ae^{rt}$ is given by $x_p = A\frac{te^{rt}}{p'(r)}$, as long as $p'(r) \neq 0$.

1. Explain the notation in the ERF.

2. Consider the system,

$$m\ddot{x} + b\dot{x} + kx = ky.$$

In this system regard $y(t)$ as the input signal and $x(t)$ as the system response. Take $m = 1$, $b = 3$, $k = 4$, $y(t) = A\cos t$. Replace the input signal by a complex exponential y_{cx} of which it is the real part, and use the ERF to compute the exponential ("steady state") system response z_p. Compute H such that $z_p = Hy_{cx}$; H is the *complex gain*. Find $|H|$ and ϕ such that $H = |H|e^{-i\phi}$. Use this information to compute the gain and the phase lag of the original system. What is the steady state (sinusoidal) solution? Is the amplitude of vibration of the mass greater than or less than the amplitude A of the motion of the far end of the spring?

3. Find a solution of $\ddot{x} + 4x = \cos(2t)$. You may have solved similar problems before by guessing a sinusoidal solution and solving for the coefficient. This time solve by making a complex replacement with the appropriate complex exponential on the right hand side and attempt to use an exponential response formula.